

# A small work on Collatz Conjecture

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**Abstract - In this paper a proof for Collatz Conjecture is given. Here I try to give very simple Mathematic, proof and explanation.**

## I. INTRODUCTION

This work is study of Collatz Conjecture that concerns a particular sequence. It is also known as the  $3n+1$  problem. Jefferey Loganasin 2010 claimed that based only on know information about this problem, this is an extra ordinary difficult problem, completely out of reach of present day mathematics as of 2020 the Conjecture has been checked by computer for all starting values up to the power of 68. All the initial values tested so far eventually end in the repeating cycle (4,2,1) which has only three terms from this lower bound can also be obtained for the number of terms a repeating cycle other than (4,2,1) must have. But this is a computer evidence and is not a proof. Overall this thesis aims to prove that the Collatz Sequence eventually reaches 1 for all positive integer.

The Collatz Conjecture is a Conjecture in Mathematics that concerns a sequence defined as fallows: Start with any positive integer  $N$  and each term is obtained from the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matters what value of  $N$ . the sequence will always reach 1.

For instance, starting with  $N = 12$ , one gets the sequence 12,6,3,10,5,16,8,4,2,1

$N = 19$  for example takes longer to reach 1: 19,58,29,88,44,22,11,34,17,52,26,13,40,20,10,5,16,8,4,2,1.

3. Proof of the conjecture:

Consider the following operation on an arbitrary positive integer. If the number is even, divide it by two, If the number is odd, triple it and add one, in modular arithmetic notation, define the function as follows.

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } N \equiv 0 \pmod{2} \\ 3n+1 & \text{if } N \equiv 1 \pmod{2} \end{cases}$$

The Collatz Conjecture is : This process with eventually reach the number 1, regardless of which positive integer is chosen initially.

Not all the odd numbers converge into unique odd number only  $\frac{2^n-1}{3} = \text{odd number}$ , where  $n=2,4,6,8,10$ .

We can judge this binary form as two series as  $S_4$  and  $S_5$  where  $S_4$  is a even series and  $S_5$  is the odd series both are converging each other.

$$S_4 = \{4,16,64,256,1024,\dots\} \text{ or } \{2^2, 2^4, 2^6, 2^8, 2^{10}, \dots\}$$

Introducing some sequences

$$S_1 = \{1,2,3,4,5,6,\dots\}$$

$$S_2 = \{2,4,8,16,32,64,\dots\}$$

$$S_1-S_2 = \{1,2,3,4,5,6,7,8,10,\dots\}$$

$$S_1-S_2 = \{1,3,5,6,7,9,10,\dots\}$$

$$S_3 = \{6,10,12,14,18,\dots\}$$

$$\frac{S_3}{2} = \{3,5,6,7,9,10,\dots\}$$

$$\frac{S_3}{2} - S_3 = \{3,5,7,9,11,13,\dots\}$$

$$S_1-(S_2+S_3) = \{1,3,5,7,9,11,\dots\}$$

5. Proving the Collatz Conjecture through sequences  
Consider the sequence  $S_1$  of positive integer.

$$S_1 = \{1,2,3,4,5,6,\dots\}$$

$$S_2 = \{2^n / N \in \mathbb{Z}^+\}$$

$$S_3 = \{6,10,12,14,\dots\}$$

$$S_2+S_3 = \{2,4,6,8,10,\dots\}$$

Then  $S_3 = \{6,10,12,14,\dots\}$  The terms in  $S_3$  when we divide by 2 and subtract from  $S_3$  gives an odd number

which is in  $\frac{S_3}{2} - S_3$  There are 3 cases.

Case 1:  $n$  is an integer of the form  $2^n$  for some in this case it is easy to see that the series decreases to 1

Case 2:  $N$  is an odd number then  $3n+1$  is an even number if it is of the form  $2^n$  it reduces to case 1.

For example : if  $n=5$  then  $\frac{3(5)+1}{2} = 16$

We can write 16 as  $2^n$  form.

$$N=21 \text{ then } \frac{3(21)+1}{2} = 64 = 2^6$$

In case no 2 there are some unique odd numbers which are binary form of 4,16,64.... when odd number converge into this unique odd number then it converge into 4,16,64....

$$S_5 = \{1, 5, 21, 85, 341, \dots\} \text{ or } \left\{ \frac{2^2-1}{3}, \frac{2^4-1}{3}, \frac{2^6-1}{3}, \dots \right\}$$

If  $2^n$  {n=1,3,5,7,9.....} Then  $2^n-1$  does not divide completely by 3. So, odd powers of 2 fail to satisfy or converge some other odd numbers because  $2^n$  where N is odd give an even number if we subtract 1 from this number the remainder never divide by 3 completely.

$$\text{For Example (1)} \quad \frac{2^2-1}{3} = 0.3333\dots \quad (2) \quad \frac{2^3-1}{3} = \frac{8-1}{3} = \frac{7}{3} = 2.33333$$

Case 3: If  $3n+1$  is not in the form  $2^n$  for any N then it must be in the series  $S_3$ . If it is in series  $S_3$  then it must converge into series  $S_1-(S_2+S_3)$  which eventually leads to an  $S_5$  series.

Lets check a multiply table  $N \times 2$

N	N <sub>1</sub>	N <sub>2</sub>	N <sub>3</sub>	N <sub>4</sub>	N <sub>5</sub>	N <sub>6</sub>	N <sub>7</sub>	N <sub>8</sub>	N <sub>9</sub>	N <sub>10</sub>	...
1	2	4	8	16	32	64	128	256	512	1024	...
2	4	8	16	32	64	128	256	512	1024	2048	...
3	6	12	24	48	96	192	384	768	1536	3072	...
4	8	16	32	64	128	256	512	1024	2048	4096	...
5	10	20	40	80	160	320	640	1280	2560	5120	...
6	12	24	48	96	192	384	768	1536	3072	6144	...
7	14	28	56	112	224	448	896	1792	3584	7168	...
8	16	32	64	128	256	512	1024	2048	4096	8192	...
9	18	36	72	144	288	576	1152	2304	4608	9216	...
10	20	40	80	160	320	640	1280	2560	5120	10240	...
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.

In this table N is a natural number and  $N_1, N_2, N_3, \dots$  are mutation number of its own. Taking any number in  $N_1, N_2, N_3, \dots$  is like taking the its odd N number.

Example

$$\left( \left( \left( \frac{1792}{2} \right) \right) \right) \quad 7 \text{ taking } 1792 \text{ is equivalent to taking } 7.$$

This example proves taking the odd number is more convenience than taking

$N_1, N_2, N_3, N_4, \dots$  numbers, if we taking any  $N_1, N_2, N_3, \dots$  number the conjecture reduce

This number into N. We have already achieved this in series  $S_1-(S_2+S_3)$ .

6. Odd form route or typical way of odd numbers

In this way we see some unique odd numbers, which in series  $S_5$ . We know that we can also put those numbers in  $\frac{2^n-1}{3}$  form. Now lets take a tour in typical way of odd number, in this way all numbers beyond number 1, will reach the number 5.

So we know that number 5 is form of  $\frac{2^3-1}{3} = 5$  i.e 16, series  $S_4$  and  $S_5$  which eventually reach 16, 8, 4, 2, 1.

In this typical way we consider only odd number (odd steps) of  $\frac{3n+1}{2}$

$$P_1 \quad 1 \rightarrow \frac{2^2-1}{3} = 1 \text{ (} S_5 \text{ series unique number)}$$

$$P_2 \quad 3 \rightarrow 5 \left( \frac{4^2-1}{3} = \frac{15}{3} = 5, S_5 \text{ series unique number} \right)$$

$$P_3 \quad 5 \rightarrow P_2 \text{ (Achieved in } P_2)$$

- $P_4$   $7 \rightarrow 11 \rightarrow 17 \rightarrow 13 \rightarrow 5$   
 $P_5$   $9 \rightarrow 7 \rightarrow 11 \rightarrow 17 \rightarrow 13 \rightarrow 5$   
 $P_6$   $11 \rightarrow 17 \rightarrow 13 \rightarrow 5$   
 $P_7$   $13 \rightarrow 5$  (Because we observed that 13 goes to 5 in  $P_4$ ,  $P_5$  and  $P_6$ )  
 $P_8$   $15 \rightarrow 23 \rightarrow 35 \rightarrow 53 \rightarrow 5$   
 $P_9$   $17 \rightarrow 5$  ( $P_4$ ,  $P_5$ ,  $P_6$ )  
 $P_{10}$   $19 \rightarrow 29 \rightarrow 11 \rightarrow 5$  ( $P_4$ ,  $P_5$ ,  $P_6$ )  
 $P_{11}$   $21 \rightarrow 5$  ( $\frac{2^6-1}{3} = \frac{64-1}{3} = 21$  a unique number)  
 $P_{12}$   $23 \rightarrow 5$  ( $P_8$ )  
 $P_{13}$   $25 \rightarrow 5$  ( $P_{10}$ )  
 $P_{14}$   $27 \rightarrow 41 \rightarrow 31 \rightarrow 47 \rightarrow 71 \rightarrow 107 \rightarrow 161 \rightarrow 121 \rightarrow 91 \rightarrow 137 \rightarrow 103 \rightarrow 155 \rightarrow 233 \rightarrow 175 \rightarrow 263 \rightarrow 395 \rightarrow 593 \rightarrow 445 \rightarrow 167 \rightarrow 251 \rightarrow 377 \rightarrow 283 \rightarrow 425 \rightarrow 319 \rightarrow 479 \rightarrow 719 \rightarrow 1079 \rightarrow 1619 \rightarrow 2429 \rightarrow 911 \rightarrow 1367 \rightarrow 2051 \rightarrow 3077 \rightarrow 577 \rightarrow 433 \rightarrow 325 \rightarrow 61 \rightarrow 5$  ( $P_8$ ,  $P_{12}$ )  
 $P_{15}$   $29 \rightarrow 5$  ( $P_4$ ,  $P_6$ ,  $P_{10}$ )  
 $P_{16}$   $31 \rightarrow 47 \rightarrow 5$  ( $P_{14}$ )  
 $P_{17}$   $33 \rightarrow 25 \rightarrow 5$  ( $P_{13}$ ,  $P_{10}$ )  
 $P_{18}$   $35 \rightarrow 53 \rightarrow 5$  ( $P_8$ )  
 $P_{19}$   $37 \rightarrow 7 \rightarrow 5$  ( $P_4$ )  
 $P_{20}$   $39 \rightarrow 59 \rightarrow 89 \rightarrow 67 \rightarrow 101 \rightarrow 19 \rightarrow 5$  ( $P_{10}$ )  
 $P_{21}$   $41 \rightarrow 31 \rightarrow 5$  ( $P_{16}$ )  
 $P_{22}$   $43 \rightarrow 65 \rightarrow 49 \rightarrow 37 \rightarrow 5$  ( $P_{19}$ )  
 $P_{23}$   $45 \rightarrow 17 \rightarrow 5$  ( $P_6$ )  
 $P_{24}$   $47 \rightarrow 71 \rightarrow 5$  ( $P_{14}$ )  
 $P_{25}$   $49 \rightarrow 37 \rightarrow 5$  ( $P_{19}$ )  
 $P_{26}$   $51 \rightarrow 77 \rightarrow 29 \rightarrow 5$  ( $P_{15}$ )  
 $P_{27}$   $53 \rightarrow 5$  ( $P_{18}$ )  
 $P_{28}$   $55 \rightarrow 83 \rightarrow 125 \rightarrow 47 \rightarrow 5$  ( $P_{24}$ )  
 $P_{29}$   $57 \rightarrow 43 \rightarrow 65 \rightarrow 5$  ( $P_{22}$ )  
 $P_{30}$   $59 \rightarrow 5$  ( $P_{20}$ )  
 $P_{31}$   $61 \rightarrow 23 \rightarrow 5$  ( $P_{12}$ )  
 $P_{32}$   $63 \rightarrow 95 \rightarrow 143 \rightarrow 215 \rightarrow 323 \rightarrow 485 \rightarrow 91 \rightarrow 137 \rightarrow 5$  ( $P_{14}$ )  
 $P_{33}$   $65 \rightarrow 49 \rightarrow 5$  ( $P_{25}$ )  
 $P_{34}$   $67 \rightarrow 101 \rightarrow 5$  ( $P_{20}$ )  
 $P_{35}$   $69 \rightarrow 13 \rightarrow 5$  ( $P_7$ )  
 $P_{36}$   $71 \rightarrow 107 \rightarrow 161 \rightarrow 121 \rightarrow 5$  ( $P_{14}$ )  
 $P_{37}$   $73 \rightarrow 55 \rightarrow 83 \rightarrow 125 \rightarrow 47 \rightarrow 5$  ( $P_{23}$ )  
 $P_{38}$   $75 \rightarrow 113 \rightarrow 85 \rightarrow 5$  ( $\frac{2^8-1}{3} = \frac{256-1}{3} = \frac{255}{3} = 85$  a unique odd number)  
 $P_{39}$   $77 \rightarrow 29 \rightarrow 5$  ( $P_{15}$ )  
 $P_{40}$   $79 \rightarrow 119 \rightarrow 179 \rightarrow 269 \rightarrow 101 \rightarrow 19 \rightarrow 29 \rightarrow 11 \rightarrow 17 \rightarrow 13 \rightarrow 5$  ( $P_{20}$ )  
 $P_{41}$   $81 \rightarrow 61 \rightarrow 5$  ( $P_{31}$ )  
 $P_{42}$   $83 \rightarrow 125 \rightarrow 47 \rightarrow 5$  ( $P_{24}$ )  
 $P_{43}$   $85 \rightarrow 5$  ( $P_{34}$  a unique number)  
 $P_{44}$   $87 \rightarrow 131 \rightarrow 197 \rightarrow 37 \rightarrow 5$  ( $P_{19}$ )  
 $P_{45}$   $89 \rightarrow 67 \rightarrow 5$  ( $P_{34}$ )  
 $P_{46}$   $91 \rightarrow 137 \rightarrow 5$  ( $P_{14}$ )  
 $P_{47}$   $93 \rightarrow 35 \rightarrow 53 \rightarrow 5$  ( $P_8$ )  
 $P_{48}$   $95 \rightarrow 143 \rightarrow 5$  ( $P_{32}$ )  
 $P_{49}$   $97 \rightarrow 73 \rightarrow 5$  ( $P_{37}$ )  
 $P_{50}$   $99 \rightarrow 149 \rightarrow 7 \rightarrow 5$  ( $P_4$ )  
 $P_{51}$   $101 \rightarrow 5$  ( $P_{20}$ )  
 $P_{52}$   $103 \rightarrow 155 \rightarrow 5$  ( $P_{14}$ )  
 $P_{53}$   $105 \rightarrow 79 \rightarrow 5$  ( $P_{40}$ )  
 $P_{54}$   $107 \rightarrow 5$  ( $P_{14}$ )  
 $P_{55}$   $107 \rightarrow 5$  ( $P_{21}$ )

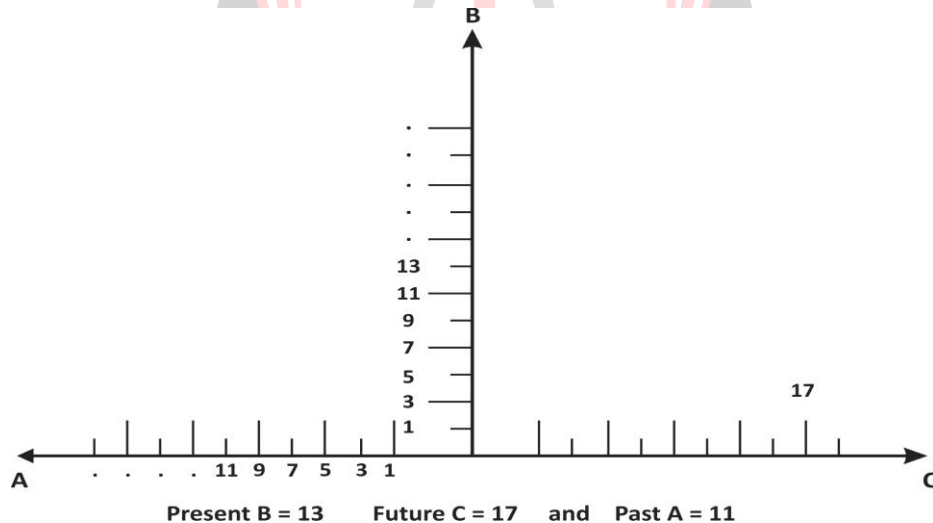
- $P_{56} \quad 111 \rightarrow 167 \rightarrow 251 \rightarrow 377 \rightarrow 283 \rightarrow 425 \rightarrow 319 \rightarrow 479 \rightarrow 719 \rightarrow 1079 \rightarrow 5 \quad (P_{14})$   
 $P_{57} \quad 113 \rightarrow 85 \rightarrow 5 \quad (P_{38}, P_{43})$   
 $P_{58} \quad 115 \rightarrow 173 \rightarrow 65 \rightarrow 5 \quad (P_{22}) (P_{33})$   
 $P_{59} \quad 117 \rightarrow 11 \rightarrow 5 \quad (P_6)$   
 $P_{60} \quad 119 \rightarrow 179 \rightarrow 79 \rightarrow 5 \quad (P_{40})$   
 $P_{61} \quad 121 \rightarrow 91 \rightarrow 5 \quad (P_{46})$   
 $P_{62} \quad 123 \rightarrow 185 \rightarrow 139 \rightarrow 209 \rightarrow 157 \rightarrow 59 \rightarrow 5 \quad (P_{20})$   
 $P_{63} \quad 125 \rightarrow 47 \rightarrow 5 \quad (P_{24})$   
 $P_{64} \quad 127 \rightarrow 191 \rightarrow 287 \rightarrow 431 \rightarrow 647 \rightarrow 971 \rightarrow 1457 \rightarrow 1093 \rightarrow 205 \rightarrow 77 \rightarrow 5 \quad (P_{39})$   
 $P_{65} \quad 129 \rightarrow 97 \rightarrow 5 \quad (P_{49})$   
 $P_{66} \quad 131 \rightarrow 197 \rightarrow 37 \rightarrow 5 \quad (P_{19})$   
 $P_{67} \quad 133 \rightarrow 25 \rightarrow 5 \quad (P_{10}, P_{13})$   
 $P_{68} \quad 135 \rightarrow 203 \rightarrow 305 \rightarrow 229 \rightarrow 43 \rightarrow 5 \quad (P_{22})$   
 $P_{69} \quad 137 \rightarrow 103 \rightarrow 5 \quad (P_{52})$   
 $P_{70} \quad 139 \rightarrow 209 \rightarrow 157 \rightarrow 5 \quad (P_{62})$   
 $P_{71} \quad 141 \rightarrow 53 \rightarrow 5 \quad (P_{18}, P_{27})$   
 $P_{72} \quad 143 \rightarrow 215 \rightarrow 323 \rightarrow 485 \rightarrow 95 \rightarrow 5 \quad (P_{48})$   
 $P_{73} \quad 145 \rightarrow 109 \rightarrow 5 \quad (P_{21}, P_{55})$   
 $P_{74} \quad 147 \rightarrow 221 \rightarrow 83 \rightarrow 5 \quad (P_{42})$   
 $P_{75} \quad 149 \rightarrow 7 \rightarrow 5 \quad (P_4)$

$$P_{76} \quad 151 \rightarrow 227 \rightarrow 341 \quad \left( \frac{2^{10}-1}{3} = \frac{1024-1}{3} = 341 \right) \rightarrow 5 \quad Z_5$$

#### Theory-1

In this typical way we observe odd steps create future odd steps (numbers) which alternatively prove that odd numbers also converge into 4,2,1. This odd steps prove that Collatz Conjecture produce past→present→future numbers.

Here we can take 5 or 4,2,1 as past numbers, what odd n Number you chose is called present number and what number gives more bigger than you chosen odd N number is called future number.



Let we consider that numbers as A = Past number, B = Present number, C- Future number.

Then  $A \in B$  and  $B \in C$  then  $C \in A$ ,  $A < B < C$

If we write past number in left side, present number in middle, future number in right side then example for 127 is.

$$\begin{array}{ccc}
 \text{Past (A)} & \text{Present (B)} & \text{Future (C)} \\
 5, 13, 17, 11, 29, 77 & \leftarrow 127 \rightarrow & 191, 287, 431, 647, 971, 1457, 1093, 205
 \end{array}$$

We saw there some numbers goes to high and suddenly falling down to 4,2,1. What is the reason for this? The reason is rate of growth and rate of falling down.

Yes the rate of growth of odd steps  $3 \times N + 1$  ( $N = \text{odd}$ ) is fixed for every step. But rate of falling down is not fixed. Rate of falling down rates 2 for even steps, but not fixed for every number, some times it is random.

For Example :  $N = 5$

Rate of growth is  $3 \times 5 + 1 = 16$

$$\text{Random} \left( \left( \left( \left( \frac{16}{2} \right) \right) \right) \right) = 1$$

The rate of falling down rate depend on even number and his randomness.

### 7. Odd Formation of Conjecture

In this method we can see all odd numbers goes to base unique number of 5.

And we also know 5 goes to 16 and 16 goes to 8,4,2,1. In this way all numbers beyond number 1, will goes through number 5.

$$P_1 \quad 1 \rightarrow \frac{(1+1+1+1)}{2} \rightarrow 2 \rightarrow 1$$

$$P_2 \quad 3 \rightarrow \frac{(3+3+3+1)}{2} \rightarrow \frac{(5+5+5+1)}{2}$$

$$P_3 \quad 5 \rightarrow P_2 \text{ (We already achieved in } P_2 \text{)}$$

$$P_4 \quad 7 \rightarrow \frac{(7+7+7+1)}{2} \rightarrow \frac{(11+11+11+1)}{2} \rightarrow \frac{(17+17+17+1)}{2} \rightarrow \frac{(13+13+13+1)}{2} \rightarrow \frac{(5+5+5+1)}{2}$$

$$P_5 \quad 9 \rightarrow \frac{(9+9+9+1)}{2} \rightarrow (P_4)$$

$$P_6 \quad 11 \rightarrow (P_4)$$

$$P_7 \quad 13 \rightarrow (P_4)$$

$$P_8 \quad 15 \rightarrow \frac{(15+15+15+1)}{2} \rightarrow \frac{(23+23+23+1)}{2} \rightarrow \frac{(35+35+35+1)}{2} \rightarrow \frac{(53+53+53+1)}{2} \rightarrow \frac{(5+5+5+1)}{2}$$

$$P_9 \quad 17 \rightarrow (P_4, P_8)$$

$$P_{10} \quad 19 \rightarrow \frac{(19+19+19+1)}{2} \rightarrow \frac{(29+29+29+1)}{2} \rightarrow \frac{(11+11+11+1)}{2} \rightarrow (P_4, P_6)$$

$$P_{11} \quad 21 \rightarrow \frac{(21+21+21+1)}{2} \rightarrow \frac{(5+5+5+1)}{2}$$

$$P_{12} \quad 23 \rightarrow \frac{(23+23+23+1)}{2} \rightarrow (P_8)$$

$$P_{13} \quad 25 \rightarrow \frac{(25+25+25+1)}{2} \rightarrow \frac{(19+19+19+1)}{2} \rightarrow (P_{10})$$

If n is a odd number then the process began as

$$\left[ \frac{(n+n+n+1)}{2} \dots \dots \dots \frac{(5+5+5+1)}{2} \rightarrow 8,4,2,1 \right]$$

or

$$\left[ \frac{(3n+1)}{2} \dots \dots \dots \frac{(3(5)+11)}{2} \rightarrow 8,4,2,1 \right]$$

Odd formation of N integer is main proof of Collatz Conjecture. In this method we can divide all N integer into two part, Part - 1 and Part - 2. Part 1 is belong to all N integer which we can write odd formation method, Part-2 is those numbers which we cannot write in odd formation method, they are  $2^n$  (where n is 1,3,5,7.....). But they can easily reach 4,2,1. So here we give the strong proof that every N integer goes to 4,2,1 in Collatz Conjecture.

Now the biggest question is why Collatz Conjecture always reach 4,2,1?

Because it follows a hidden number theory. The theory is each pairs of odd steps

divided by each other, step 2 divided by step 1 (odd steps) or  $p/q$  ( $p$ =step 2 and  $q$  is step 1) Then the value is always between 1.666.... to 1.5. When we deal with big number. Then value close to came 1.5 which is similar to  $3/2$ . So if we replace any denominator or nominator of  $3/2$  It will break the average 1.5 to 1.666.... We also know that every even step, previous even step X divided by next even step y then  $x/y=2$ .

Some Examples:  $N=7$

$$\frac{3n+1}{2} \rightarrow 7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

Here 11 and 7 are odd pairs because between this two odd number there are only one even number 22 occurs. This kind of odd numbers called odd pairs of Collatz Conjecture. If we divide this number as  $p/q$  then it gives a average between 1.666 .... to 1.5.

If  $7+7+1 = 22$ , 4 terms equal to next two term  $11+11 = 22$  is called odd pairs.

$a+a+a+1=b+b$  odd pairs.  $a, b$  are odd.

$$\frac{\text{Step 2}}{\text{Step 1}} = \frac{p}{q} = 1.666.... \text{ to } 1.5$$

$$\frac{11}{7} = 1.571428571439 \quad \frac{17}{11} = 1.54545454.....$$

So every pairs of steps will gives result like this.  $N=7 \rightarrow 1.57142857143, 1.54545454... 2,2,2,2,2,1$

Example :  $N=111$

Example :  $N=111$

111  $\rightarrow$  334  $\rightarrow$  167  $\rightarrow$  502  $\rightarrow$  251  $\rightarrow$  754  $\rightarrow$  377  $\rightarrow$  1132  $\rightarrow$  566  $\rightarrow$  283  $\rightarrow$  850  $\rightarrow$  425  $\rightarrow$  1276  $\rightarrow$  638  $\rightarrow$  319  $\rightarrow$  958  $\rightarrow$  479 1438  $\rightarrow$  719  $\rightarrow$  2158  $\rightarrow$  1079  $\rightarrow$  3238  $\rightarrow$  1619  $\rightarrow$  4858  $\rightarrow$  2429  $\rightarrow$  7288  $\rightarrow$  3644  $\rightarrow$  1822  $\rightarrow$  911  $\rightarrow$  2734  $\rightarrow$  1367 4102  $\rightarrow$  2051  $\rightarrow$  6154  $\rightarrow$  3077  $\rightarrow$  9233  $\rightarrow$  4616  $\rightarrow$  2308  $\rightarrow$  1154  $\rightarrow$  577  $\rightarrow$  1732  $\rightarrow$  866  $\rightarrow$  433  $\rightarrow$  1300  $\rightarrow$  650  $\rightarrow$  325  $\rightarrow$  488  $\rightarrow$  244  $\rightarrow$  122  $\rightarrow$  61  $\rightarrow$  184  $\rightarrow$  92  $\rightarrow$  46  $\rightarrow$  23  $\rightarrow$  70  $\rightarrow$  35  $\rightarrow$  53  $\rightarrow$  160  $\rightarrow$  80  $\rightarrow$  40  $\rightarrow$  20  $\rightarrow$  10  $\rightarrow$  5  $\rightarrow$  16  $\rightarrow$  8  $\rightarrow$  4  $\rightarrow$  2  $\rightarrow$  1

$$\frac{167}{111} = 1.50450450....., \quad \frac{251}{167} = 1.50299401198, \quad \frac{377}{251} = 1.50199203187$$

$$\frac{425}{283} = 1.50176678445, \quad \frac{479}{319} = 1.50156739812, \quad \frac{719}{479} = 1.50104384134,$$

$$\frac{1079}{719} = 1.50069541029, \quad \frac{1619}{1079} = 1.50046..., \quad \frac{2429}{1619} = 1.50030883...,$$

$$\frac{1367}{719} = 1.50054....., \quad \frac{2051}{1367} = 1.50036576...., \quad \frac{3077}{2051} = 1.5002437.....,$$

$$\frac{35}{23} = 1.52173913043, \quad \frac{53}{35} = 1.51428571429...,$$

So every  $\frac{3n+1}{2}$  transaction will gives a model result like (1.666 to 1.5) (2) (1.666 to 1.5) (2) (1.666 to 1.5) (2) (2) (2) 1.

This kind of result generate only when  $N$  is multiplied by 3, add 1 and divided by 2 if any changes in this process the increasing rate or average will break the chain and  $N$  does not reach to 4,2,1 and create short cycle or which does not satisfy Collatz Cycle taking the long process which never decrease.

Below the example indicate pairs of odd steps and pairs of even steps taking long process which never average rate is maximum 1.666... to minimum 1.5 for odd steps, even pairs of steps decrease average is always 2 and its randomness so if any continue mathematics operation go on this average that we will clearly reach 4,2,1 perhaps this average system works only 3 x n.

This method only works for Number 1,2,3 depending on its position in Collatz Conjecture. Actually this is not a 4,2,1 problem this is a 1,2,3 solution.

## 8. Another View of Collatz Conjecture

Take any positive integer, if it is odd multiply by 3 and add 1, Now find out the factors of that number. In this factors choose a big factor of odd, and go on the same process. If your number is even then select large odd factor of that number, now multiply by 3 to that large odd factor number, add one, go on the process. In this process if you reach large odd factor like  $S_5 = \{1,5,21,85,341,...\}$  then process end up with 4,2,1. If it fail to reach large odd factor  $S_5 (N>1)$  then eventually reach 4,2,1 because  $S_4 = \{2,4,8,16,...\}$  are only single large odd factor that is 1.

Collatz Conjecture is a process of finding large odd factor of even number and also finding large even factor of even number. In this process we finally reach 1.



For Example  $N=68$

Factors of 68 is 1,2,4,17,34... The large even factor is 34 and large odd factor is 34

We can write  $\frac{3n+1}{2}$  as 2

If  $n$  is odd  $=N \times 3+1 \rightarrow$  (Large odd factor)  $\times 3+1 \rightarrow$  (Large odd factor)  $\times 3+1 \dots 4,2,1$ .

If  $n$  is even = Large even factor  $\rightarrow$  Large even factor  $\rightarrow$  Large even factor at last large odd factor ie.,  $16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

Collatz Conjecture is mixed process for some  $N$  number then factor system also go on as mixed factorization.

Odd factorization converge  $N$  number into  $S_5$  Series

Even factorization converge  $n$  number into  $S_4$  Series

Mixed factorization converge  $n$  number into  $S_1 - S_2$  series. Which is always converge into  $S_4$  and  $S_5$  series.

#### 9. The Last series of Collatz Conjecture

In this series proves that any odd number multiply by 3 and add one and divide by 2 until reach odd number, this process give a odd number which belong to  $S_1 - (S_2+S_3)$  no matter how big odd number you choose to this process. The remaining odd number is always part of  $S_1 - (S_2+S_3)$ . If odd  $N = \infty$  (Infinity) No problem it repeat a part of  $S_1 - (S_2+S_3)$  series in this process if it give number, any term in  $S_6$  series then Collatz Conjecture will stop, lets check the series:

$$S_1 - (S_2 + S_3) \{1, 3, 5, 7, 9, 11, \dots\}$$

In Collatz Conjecture we always multiply 3 by this series (term)

So,

$$S_6 = 3 \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, \dots\}$$

$$S_6 = \{3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, \dots\}$$

$$\text{Now add 1 to each term} = \{4, 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, \dots\} = S_7$$

$$\text{Now divide each term of } S_7 \text{ by } 2 = \{1, 5, 11, 17, 13, 29, 1, 35, \dots\} = S_8$$

(until get odd number)

We get  $S_8$  dividing  $S_7$  by 2 is always part of  $S_1 - (S_2+S_3)$  series if it remain a part of  $S_6$  then Collatz Conjecture will stop.

If we add 1 to  $S_6$  series (each term) and divide by 2, continue the process then the process convert one by one number into  $2^n$  form, this is the clear proof that  $N+1/2$  ( $N=\text{odd}$ ) continue process convert  $N$  (odd) into  $2^n$  form.

#### 10. Reverse Collatz Conjecture

Can we find past number or starting number? Yes we can find the starting number also.

Series  $S_6$  are no previous steps, they generate only next step in Collatz Conjecture so, this series terms or numbers are no reverse step. Only  $S_8$  series are give previous steps and next step.

Example : Find the base number of 23

Step 1 : Divide 23 by 3 (if not divide completely)

Step 2 : Multiply by 2 ( $23 \times 2 = 46$ )

Step 3: Now subtract 1 from 46 ( $46 - 1 = 45$ )

Step 4: Now divide 45 by 3 ( $45 \div 3 = 15$ )

Step 5: 15 is your base number (that mean you came from 15 or  $15 \times 2 \times 2 \times 2 \dots$ )

Finding base number of  $N$  is always  $N$  or  $N \times 2 \times 2 \times 2 \dots$  depending on steps ( $N=\text{odd}$ )

According to these examples we define reverse Collatz Conjecture as

$$f_n = \left\{ \begin{array}{l} \frac{2n-1}{3} \quad \text{If } N \text{ is odd then } 2n \\ \frac{N}{2} \quad \text{If } n \text{ is even then } \frac{N}{2} \\ N \neq S_6 \\ N = S_8 \\ \text{If } N = S_6 \text{ then } \frac{N}{3} \end{array} \right\}$$

Example 3 : Find the base number of 97

97 is not divisible by 3

So  $97 \times 2 = 194 - 1 = 193$

193 is not divisible by 3

So  $194 \times 2 = 388$  (Take the 194 because random divisible by 2)

$= 388 - 1$

$= 387$

$= 129$  (you came to 97 from 129 or  $129 \times 2 \times 2 \times 2 \dots$ )

$$\frac{3n+1}{2} = x \text{ and } \frac{2x-1}{3} = n \text{ then } x = n$$

$$\frac{3n+1}{2} = \frac{2n-1}{3}$$

If we simply adding 1 to each term of  $S_6$  series and divided by 2 randomly and again add 1 if term became greater than 1, and again divide by 2. This process also give the same result like Collatz Conjecture i.e., every number reach 1.

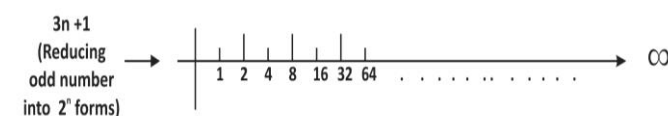
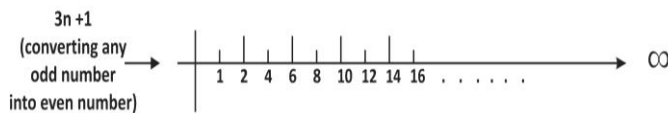
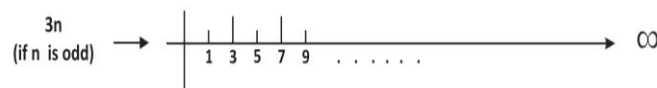
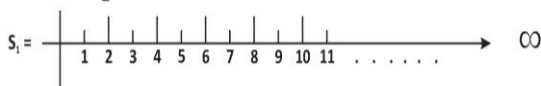
If  $N \neq S_3 = \{1, 5, 21, 85, 341, \dots\}$

Then Collatz Conjecture Predict like

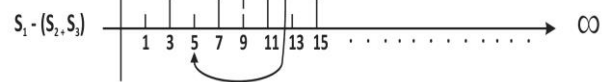
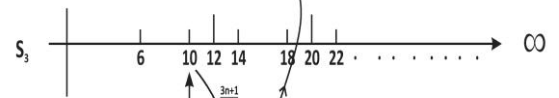
$$\dots 3 \left( 3 \left( 3 \left( 3 \left( \frac{3n+1}{2} + 1 \right) + 1 \right) + 1 \right) + 1 \right) + 1 \dots$$

Collatz Conjecture in Number line

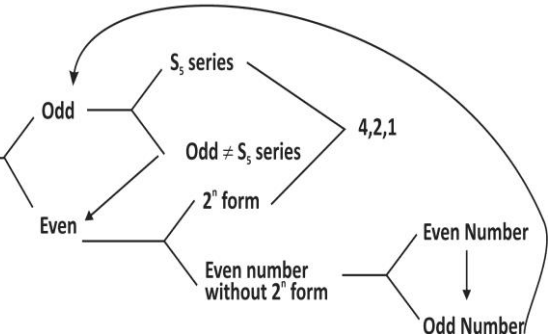
$3n+1$  or  $\frac{3n+1}{2}$  is in number line



For example:  $N=3$  is in number line



Collatz Conjecture cycle



## II. CONCLUSIONS

Here I presented some imagination and ideas of Collatz Conjecture which proves logically as well practically. That Collatz Conjecture is true for all  $N$  integers. The odd formation of Conjecture, we presented clearly proof that every natural numbers goes to 5 is a significant theoretical proof. And also various type of series logically and Mathematically proved that every natural numbers converge into 4,2,1. Described about simple imagination of past - present - future numbers  $A \in B \in C$  Converge into 4,2,1. Try to argue that odd numbers are more convenience to examine than even number. This method save our time and effort also. I presented reverse of Collatz Conjecture where we check, Collatz Conjecture is true here I explore some fundamental properties of odd numbers and its unique series, in Collatz Conjecture increasing rate is always smaller than decreasing rate  $1.6666$  to  $1.5 < 2$ . We can also



find the total steps of any  $N$  integer through the odd formation data or typical way of odd number, below every  $N$  odd integer have its limited steps. Interesting fact is I recognized when working on Collatz Conjecture is a pair of set number which is  $35 \rightarrow 53$ ,  $359 \rightarrow 539$ ,  $395 \rightarrow 593$ . If you got 35 your next step will be 53. Adding 1 to  $S_6$  (each term) and divide by 2. The continue process leads one by one term to reach 4,2,1. This is one of the main theoretical proof. If any  $N > 3$  in this system, then increasing rate is higher than decreasing rate.

## REFERENCES

- [1] T. Oliviera Silva, "Maximum Excursion and stopping time record -holders for the  $3x + 1$  problem ; Computational results". Mathematics of Computation, Vol-68, No, 225, PP 371-384, 1999.
- [2] L. Colussi, "The Convergence Classes of Collatz Function" Theoretical computer Science, Vol, 412, No. 39, PP. 5409 -5419, 2011.
- [3] P.C. Hew, "Working in Benory Protects the Repetends of  $1/34$  : Comment on Collussis The Convergence Classes of Collatz Function "Theoretical Computes Science, Vol, 618 PP. 135-141, 2016
- [4] R.K. Grey, "Dont try to solve these problems" Computer and Mathematics with Applying Vol. 90, No. 1 PP. 35-41, 1983.
- [5] G.T. Leovens and M. Vermeulen, " $3x + 1$  search programs, "Computer and Mathematics with applications, an International Journal, Vol. 24, No. 11, PP. 79-99, 1992.
- [6] R.E. Crandall, "On the ' $3x + 1$ ' Problem", Mathematics of Computation, Vol. 32, No. 144, PP. 1281 -1292, 1978.
- [7] W. Ren, S. Li, R. Xiaa, and W. Bi, "Collatz Conjecture for  $2100000 - 1$  is true algorithms for verifying extremely large numbers, "In proceeding of the IEEE VIC2018, PP. 411-416, Guangzhou, China, October 2018.
- [8] I. Krosikarand J.C. Logorias, "Bounds for the  $3x + 1$  problem using difference inequalities, "Acta Arithmeticu, Vol. 109, No. 3, PP. 237 -258, 2003.
- [9] Youtube "NumberPhile " Prof. David Eisenbud.