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UNITARY BINARY GROUP VARIETIES AND ITS GEOMETRICAL APPLICATIONS

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ABSTRACT: One consider the variety of the unitary binary group in *n* variable and it is shown that, this algebraic variety is rational and it has n^2 parameters. Then it is given a parametrical rational representation of this variety. We have established someone parameter subgroups of the unitary binary group. Besides we showed that these subgroups are geodesics of the pseudo-Riemannian space V_{n^2}.

KEY WORDS: Unitary binary group

I. INTRODUCTION

It is known that the unitary complex group $U(n)$ is formed by the complex matrices A of n-th order which satisfy the relation $AA^* = E_n$, where A* is the adjoint of A and E_n is the identity matrix of nth order. This group is isomorphic with the simplistic orthogonal group in 2n variable formed by the real matrices **B** of 2n-th order, which satisfy the relations.

$$\begin{cases} B\,\bar{B} = E_{2n} \\ B\,I = I\,B \end{cases}$$

Where $\bar{B}$ is the transpose of B, E_{2n} is the identify matrix of $2nth$ order and I is the matrix

$$\begin{Vmatrix} I_2 & 0 & \dots & 0 \\ 0 & I_2 & \dots & 0 \\ . & . & \dots & . \\ . & . & \dots & . \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_2 \end{Vmatrix} \quad \text{Where} \quad I_2 = \begin{Vmatrix} 0 & 1 \\ -1 & 0 \end{Vmatrix}$$

The purpose of this speech is to show that the unitary binary group $\beta(n)$ formed by the matrices A of nth order which elements belong to the algebra of binary numbers. Which satisfy the relation $A\,A' = E_n$, with A* the adjoint of A **isomorphic** with the group of real matrices B of 2nth order which fulfil the relations $B\,I\bar{B} = I, BJ = JB$, where

$$I = \begin{Vmatrix} I_2 & 0 & \dots & 0 \\ 0 & I_2 & \dots & 0 \\ . & . & \dots & . \\ . & . & \dots & . \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_2 \end{Vmatrix} \qquad J = \begin{Vmatrix} J_2 & 0 & \dots & 0 \\ 0 & J_2 & \dots & 0 \\ . & . & \dots & . \\ . & . & \dots & . \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & J_2 \end{Vmatrix}$$

With

$$I_2 = \begin{Vmatrix} 0 & 1 \\ -1 & 0 \end{Vmatrix} \quad J_2 = \begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix}$$

Then we prove that this algebraic variety is rational, finding effectively rational parametrical representation and proving that it depends on n^2 parameters.

Finally we shall give some geometrical application.

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II. THE VARIETY OF THE UNITARY BINARY GROUP

The algebra of binary numbers A is a commutative algebra of order two over the field R of real numbers generated by two elements 1, e that satisfy the relations.

$1^2 = e^2 = 1 \quad 1.e = e.1 = e$

An element $x = \in \boldsymbol{A}$ of the algebrs $\boldsymbol{A}$ is like $x = a + eb, with\ a, b\ \in \boldsymbol{R}$ and the conjugate of $x\ is\ by\ defination\ \bar{x} = a - eb.$

The set U_2 of real matrices of order two of the form $\begin{Vmatrix} a & b \\ b & a \end{Vmatrix}$ is an algebra over the field **R** of real number.

It is known that the function $f : A \rightarrow U_2$defined by $f(a + eb) = \begin{Vmatrix} a & b \\ b & a \end{Vmatrix}$ is an isomorphism of algebras (Rosenfield).

The unitary binary group $\beta(n)$ is formed by the matrices A of nth order with elements form the algebra A, which fulfils the relations $AA^* = E_n$ and the multiplication of matrices is a composition law. ($A^*$is the adjoint of $A\ and\ E_n$ is the idenitifity matrix of nth order).

Let SO(2n) be the group formed by the real matrices B of 2nth order, which satisfy the relationship $BI\tilde{B} = I, BJ = JB,$ where $\tilde{B}$ is the transpose of B and I, J are the matrices of 2nth order that satisfy the *IJ* = *-JI*.

$$I = \begin{Vmatrix} I_2 & 0 & \dots & 0 \\ 0 & I_2 & \dots & 0 \\ . & . & \dots & . \\ . & . & \dots & . \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & I_2 \end{Vmatrix} \qquad J = \begin{Vmatrix} J_2 & 0 & \dots & 0 \\ 0 & J_2 & \dots & 0 \\ . & . & \dots & . \\ . & . & \dots & . \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & J_2 \end{Vmatrix}$$

With

$$I_2 = \begin{Vmatrix} 0 & 1 \\ -1 & 0 \end{Vmatrix} \quad J_2 = \begin{Vmatrix} 0 & 1 \\ 1 & 0 \end{Vmatrix}$$

Theorem1. *The groups* $\boldsymbol{\beta(n)}$ *and* $SO(2n)$ *are isomorphic.*

We have shown by the computation of the number of independent relation that the number of parameters of the variety of the unitary binary group in n^2. The next result will give a parametrical rational representation of the unitary binary group.

Theorem2.

$If\ X\ is\ a\ symmetric\ matrix\ and\ XJ = -JX,$

$then\ the\ variety\ of\ the\ unitary\ binary\ group\ V_{n^2}\ \ formed\ by\ the\ matrices\ A\ with$

$$\begin{cases} AI\tilde{A} = I \\ AJ = J\,A \end{cases}$$

$$admits\ a\ rational\ parametrial\ represenattion\ of\ the\ form$$

$$A = (I + X)^{-1}\,(I - X) in\ the\ neighbourhood\ getermined\ by\ the\ condition$$

$$|I + X| \neq 0.$$

III. SOME GEOMETRICAL APPLICATIONS.

The vector space $\mathbf{R}_{2n\ x\ 2n}$ of real matrices of 2nth order endowed with the pseudo-scalar product (indefinite bilinear symmetric form),

$$< P, Q >= -\frac{1}{2} Tr(IPI\tilde{Q})$$

Where $Tr(A)$ represents the trace of matrix A,is a pseudo-Euclidian space.

The metric of this space is

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$$ds^2 = -\frac{1}{2}T_r(IdPId\tilde{P})$$

Where *dP* is differential of the matrix *P*.

The path is B: R→ V_{n^2} of pseudo − Riemannian space V_{n^2} defined by $B(t) = e^{tx}$, where

X is an antisymmetric matrix of 2nth order ($\tilde{X}$ = -*X*) which satisfied the relations.

$$\begin{cases} XI = IX \\ XJ = JX \end{cases}$$

Where *I* and *J* are matrices introduced above, is an one parameters subgroup of the unitary binary group $\beta(n)$

For proving this result we showed that the equalities are fulfilled.

$$\begin{cases} B(t)\tilde{B}(t) = I \\ B(t)J = JB(t) \end{cases}$$

Theorem 3. *The one parameter subgroups of the unitary binary group are geodesics of the pseudo-Riemannian space* V_n^2 .

For showing that the path $B(t) = e^{tX}, t \in R,$ is geodesic in the space $\beta(n),$ we proved that the

Vector $B''(t)$is orthogonal to every vector $B'(t)$which satisies the equality
$B'(t)I\tilde{B}(t) + B(t)I\tilde{B}'(t) = 0.$

Some one parameter subgroups of the orthogonal group in n variables *O*(*n*) where established by K.Teleman showing that these subgroups are geodesics of the Riemannian space *O(n).*

E. Grecu has established some one parameter subgroups of the semi-pseudo-orthogonal group $O^{r,t}(p)$ in p variables and of the semi-orthogonal group $O^r(p)$, providing that these subgroups are geodesics of the semi- Riemannian spaces $O^{r,t}(p)$ respectively $O^r(p)$.

A more detailed work will be found in [2].

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