

# Relativistic Self-Focusing Of Cosh-Gaussian Laser Beam in Dense Plasma Under Density Transition With Plasma Density Ramp

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**ABSTRACT** - Relativistic self-focusing of cosh-gaussian beam in plasma under density transition has been studied. It has been investigated the spot size of the beam. The periodic change in spot size due to the ponderomotive force of laser beam with plasma density ramp has been showed this work shows that decreasing the diffraction effect, laser becomes more focused and concentrated into the plasma. Solving equation in form of nonlinear diffraction equation and optimizing focusing parameters. The oscillated value of beam width parameter is observed with varying value of the decentered parameter  $b$  and ripple wave number  $d$ , using paraxial ray approximation work are showed more reliable result.

**Key words:** Non linear refraction, Paraxial ray approximation, Plasma density ramp, Ponderomotive force, Self-Focusing,

## I. INTRODUCTION

The relativistic self-focusing, when the ultra-intense high-power short pulse laser beam interact with plasma occurs due to ponderomotive force acting on the electron and mass of the electron relatively oscillate [1-4]. Relative self-focusing of ultra-intense short pulses been very interesting topic for researchers because of its great application in many areas of optical science like laser driven particle accelerator x-rays, plasma-based accelerator and high harmonic generation [5]. In dense plasma refractive index is variable due to effective increment of mass of electron by relativistic motion and ponderomotive force on electron. A large amount of literature is available on plasma interaction with different density properties [6] it has been studied the plasma channel charging and coulomb explosion when ultra-short pulse laser penetrated in plasma [7].

The concentrated laser beam exerts a radial ponderomotive force on electron. It diffracted them outward producing a laser density in the middle which results concentrating the beam. Habibi et al., observed thunderation between the laser pulse and plasma. Considering ramp density profile [8]. It is observed that after going through the various research on self-focusing

It has been observed that due to relativistic self-focusing, ultra-high-power laser penetrated through the plasma, the spot size reduces but the spot size of the laser increases above the focus [9]. Gupta et al [4,10] observed a slowly varying plasma density ramp to produce the self-focusing effect. Liu et al [2,11] showed that ultra-short laser pulse when penetrating into plasma with density ripple in case of third harmonic generation. Hatizi et al [12] observed the penetration of an intense laser beam in plasma with relativistic ponderomotive effect. Sadighi – Bonabi et. al [13]

observe that with specific plasma density rank profile the spot size oscillation of the laser beam become the larger due to relativistic effect and beam gets more focused. Kaar et. al [15] observed the self-focusing the gaussian laser beam with plasma density ripple. Parasar et al [16] observed that second harmonic generation of laser radiation with density ripple in plasma. Self-focusing effect is increased with another method with density ripple in plasma, Different ripples in obtained by using different lasers [17]. Prakash et al studied the self-focusing of the gaussian electro-magnetic beam and multi photon absorption in a radial in homogeneous medium.

Using paraxial approximation, the study state focusing of laser beam in homogeneous non linear medium has been observed. For a relativistic self-focusing, Gaussian laser pulse diffracted within limit of the interaction length to releigh length  $Z_R = \frac{\omega Q r_0^2}{2c}$  where  $r_0$  is the radius of the spot. Outside the focusing point, nonlinear refraction of the

$2c$

laser beam will be weakening and spot size of the beam start increasing. It shows that oscillatory characteristics with the distance of propagation [18]. Guiding the laser beam over several relay length is useful for the area of monoenergetic electron generation, X-ray lasers, Harmonic generation and laser plasma oscillator. To sort out the diffraction we consider a specific slowly varying plasma density ramp.

The aim of the present study is to be analyses, the relativistic self-focusing of cosh Gaussian laser beam in plasma. Considering the sinusoidal density ripple & enhanced the laser plasma parameter for relativizing self-focusing cosh-Gaussian laser beam is preferred because of its important scientific problem. Cosh Gaussian laser beam is highly powerful in compression Gaussian laser beam [19].

Due to relativistic & ponderomotive effect & considering the non-linearity, high power laser beam is penetrating into plasma & electron gets oscillatory velocity in compression to the velocity of the light and effective dielectric constant of the medium is modified which is the main cause of relativistic self-focusing of the laser beam.

If the frequency of the laser beam is greater than the natural frequency of the electron oscillation the ponderomotive force starts to play which penetrates the electron out of the beam field, from high intensity region to low intensity region and hence the focal electron density gets reduced and laser beam gets highly focused.

Present paper is structured in four sections. In 2<sup>nd</sup> section we have focused intensity profile of cosh-Gaussian laser beam considering paraxial ray approximation. In 3<sup>rd</sup> section, we calculated spot size of the laser beam and we have evaluated the spot size of the laser beam. Numerical results are observed in section 4 and in section 5, we draw the conclusion of the paper.

## II. FORMULATION

If  $n_0$  is the electron density of plasma sinusoidal then,

$$n_0 = n_0(1 + a_2 \cos(qz)) \quad (1)$$

$n_0$  represents the maximum electron density and  $a_2$  represents the depth of modulation with ripple vector  $q$

The expression for angular frequency  $\omega_0$  when a cosh Gaussian beam propagates in  $z$  direction in plasma medium and it is given by

$$E = x'(r, z) \exp[-i(\omega_0 t - k_0 z)] \quad (2)$$

$k_0$  is the wave number where  $k_0 = \{(\omega_0/\sqrt{\epsilon_0})/c\}$  is the propagation constant of the wave,

When  $\epsilon_0$  and  $c$  are the dielectric constant and speed of light

$$(r \cdot 0) = A_{00} \exp\left(\frac{-r^2}{r_0^2}\right) \cosh(\Omega_0 r) \quad (3)$$

This equation represents field distribution of the cosh Gaussian beam at  $z=0$ .  $A_{00}$  is the amplitude  $r_0^2$  of the centre,  $r_0$  is the beam width and  $\Omega_0$  is the parameters related to the hyperbolic cosine function.

When  $Z > 0$ ,

$$(r, z) = A_{00} \exp(b^2) [\exp\{- (r+b)^2\} + \exp\{- (r-b)^2\}] \quad (4)$$

Here  $b = \Omega_0 r$ , normalized model parameter  $f$  is dimensionless beam width parameter of the laser beam in plasma medium.

If we consider axial region, profile gains the form,

$$A_0 = A_{00} f \left[ 1 - \frac{b^2 f r^2}{2} - \frac{b^4 f r^4}{24} \right] \times \left[ 1 + \frac{b^2 f r^2}{4} + \frac{b^4 f r^4}{24} \right] \quad (5)$$

When the ponderomotive force exerts a force on electrons, modified electron density is represented as,

$$F_p = -mc^2 (\gamma - 1), \quad \text{here } \gamma = (1 + a^2)^{1/2} \quad (6)$$

Indicates the relativistic factor growing from the intensity dependence of the electron mass.

$$a = \frac{e|A|}{m\omega_0 c}$$

Represents normalized laser amplitude at  $z > 0$

$$a_0 = m \frac{e \omega_0 A_{00}}{(\mu m^2)} = [I (W cm^{-2})]^{1/2} \quad (7)$$

$a_0$  represents the intensity parameter  $n_0$ ,  $m$  and  $e$  are the modified density, rest mass and charge of electron. Considering Tripathi et al (1980), electron density with proper modification can be represented as

$$n = n_0 (1 + a_2 \cos(qz)) + \frac{4\pi e^2}{m} \frac{E^2}{2(\gamma - 1)} \quad (8)$$

Where  $\epsilon$  is the electric permittivity &

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2 \gamma} \quad (9)$$

$$\omega_p = \omega_0 (n_e/n_0)$$

It shows the electric permittivity where

$$\omega_p^2 = \frac{4\pi e^2 n_0}{m}$$

The dielectric permittivity in general form considering the paraxial approximation can be represented as a series of  $r^2$

$$\epsilon = \epsilon_0 - \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^4}{\omega^4} \quad (10)$$

Using equation (8) and (10) and expanding  $n_0 \gamma$  at  $r=0$  we get,

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} + \frac{\omega_p^4}{\omega^4} - \frac{\omega_p^2}{\omega^2} \frac{a_2 \cos(qz)}{1 + a_2 \cos(qz)} \quad (11)$$

Here  $\gamma_0$  is  $\gamma$  at  $r=0$ , and

$$\phi = \frac{\omega_p^2}{\omega^2} \frac{a_2 \cos(qz)}{1 + a_2 \cos(qz)} + \frac{\omega_p^4}{\omega^4} \frac{a_2^2 \cos^2(qz)}{(1 + a_2 \cos(qz))^2} \quad (12)$$

## III. SELF-FOCUSING

The wave equation in nonlinear form determining the evaluation of electric field in the dense plasma is given by,

$$\nabla^2 E + \frac{\omega^2}{c^2} \epsilon E = 0 \quad (13)$$

Substituting the value of  $E$  from equation (2) in equation (13) and considering WKB approximation, the wave equation takes the form,

$$\frac{\partial^2 A}{\partial z^2} + \nabla_{\perp}^2 A - \frac{\omega^2}{c^2} \epsilon A = 0 \quad (14)$$

Now using an eikonal,

$$A = A_0(r, z) \exp[ik_0 S(r, z)]$$

here  $A_0(r, z)$  and  $S(r, z)$  are real function of space variables

The expression for  $A$  in the equation (14) and collecting the real and imaginary parts we get,

$$2\partial_z \partial_s + (\partial_r \partial_s)^2 = k_1^2 \nabla^2 A_0 - r r_0^2 \epsilon_0 \partial_z^2 A_0 + \partial_r A_0 \partial_r \partial_s + \nabla^2 A_0 (r^1 \partial_r \partial_s + \partial_r^2 z^2 s) = 0 \quad (15)$$

If the field is considered slowly converging or diverging, the term  $\partial_r^2 z^2 A_0$  can be ignored [1].

Expanding eikonal S, assuming the paraxial ray approximation,

$$S(r, z) = s_0(z) + S_2(z) r^2 \quad \text{and} \quad s_2(z) = \frac{1}{2} \frac{d^2 f}{dz^2} \quad (16)$$

Now putting  $A_0$  and  $S(r, z)$  from equation (15) and considering both sides the coefficient of  $r^2$ ,

$$\frac{d^2 f}{dz^2} = \frac{12 - 12b - b}{4} \frac{f(z)}{d(z)} \quad (17)$$

here  $\varphi = \frac{\omega_p^2 a_0^2}{\omega_0^2} + \frac{a_0^2 c^2 [(4-b)^2 + ((b-3)a_0^2/f_2)/(Y_2 a_0)]}{\omega_0^2 Y_2^2 \cos^2 d\xi}$

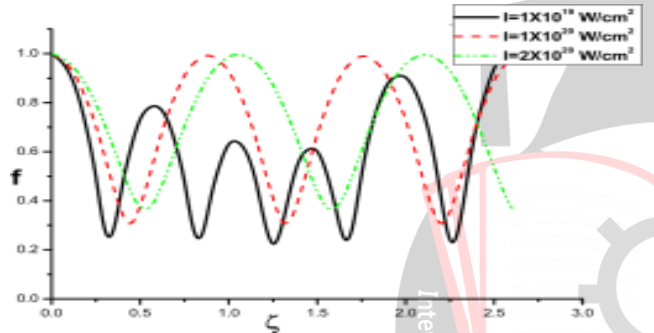


Fig. 1 Beam-width parameter  $f$  plotted against the dimensionless distance of propagation at different values of intensity and for other parameters as  $(m) = 0.5, d = 50, b = 1, (r_0 \omega_0)/c = 60, \omega_p^2/\omega_0^2 = 0.01, \epsilon_2 = 1, \omega_0 = 1.778 \times 10^{14}$  rad/s.

#### IV. RESULTS AND DISCUSSION

Equation (17) is in form of nonlinear Differential equation of second order. It represents the beam width parameter of cosh-gaussian laser beam taking sinusoidal density Ripple. the relativistic self-focusing of the Beam for initial Beam width is calculated. considering the relative magnitude of nonlinear diffraction term in equation (17). since the function  $f$  of normalised distance of propagation with dimension less beam width parameter cannot solve analytically hence we get the solution of the equation using Ranga kutta method. the parameter is taken as follows

$$r_0 \omega_0 = 60, \quad \omega_p^2/\omega_0^2 = 0.01, \quad n_0 = n_0(1 + \alpha_2 \cos qz) = 10^{17} \text{ cm}^{-3}$$

$$\omega_0 = 1.78 \times 10^{14} \text{ rad/s}, \quad \alpha_2 = 1$$

Using these parameters, we observed the focusing of cosh-Gaussian laser beam with density Ripple.

Fig. 1 shows that for a different value of intensity how the beam width parameter  $f$  varies with normalized distance of

propagation. we have taken the constant value of the wavelength  $\lambda(\mu\text{m})=0.5$ , ripple wave number  $d=50, b=1$ . It is investigated from the figure that when intensity increases the relativistic self-focusing is also increases. Increasing the intensity of the laser beam, the nonlinear term neglected and the result is differential part dominates over the nonlinear terms. The mechanism that reduces the nonlinear term concentrated on Axis plasma density due to increased ponderomotive force and it reduces the the focusing Force to the laser beam. we use the laser relativistic optimum value of intensity  $I=1.21 \times 10^{18} \text{ W/cm}^2$ . For getting stronger self-focusing and stronger self- focusing length.

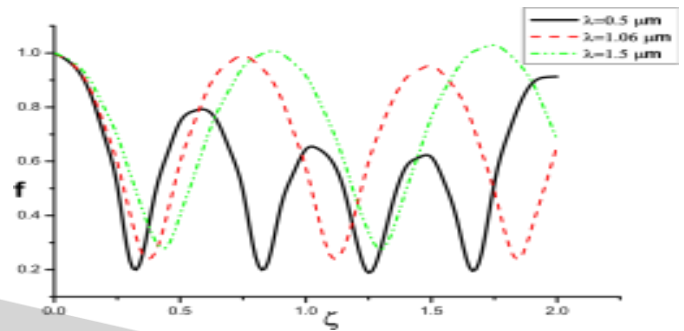


Fig. 2. Beam-width parameter  $f$  plotted against the dimensionless distance of propagation at different values of  $(m)$  and for other parameters as  $I (W \text{ cm}^{-2}) = 10^{19}, d = 50, b = 1, (r_0 \omega_0)/c = 60, \omega_p^2/\omega_0^2 = 0.01, \epsilon_2 = 1$

Figure 2, it shows the dependence of of beam width parameter  $f$  with normalized distance of propagation. for constant value of  $I = 1.21 \times 10^{18} \text{ W/cm}^2$  and rest parameters of same value as in figure1. we observed the stronger self-focusing at  $\lambda = 0.5 (\mu\text{m})$  at optimized intensity as observed from fig.1. For getting stronger self-focusing we choose particular wavelength. We have observed from fig.2 that periodic change of  $f$  on  $\xi$  for  $\lambda = 1.06$  and  $\lambda = 1.5 (\mu\text{m})$

So, for  $\lambda = 1.06$  and  $\lambda = 1.5 (\mu\text{m})$  weak self-focusing is seen with comparison of  $\lambda = 0.5 \mu\text{m}$ . It is seen that for the density profile with higher slope we observed a better focusing.

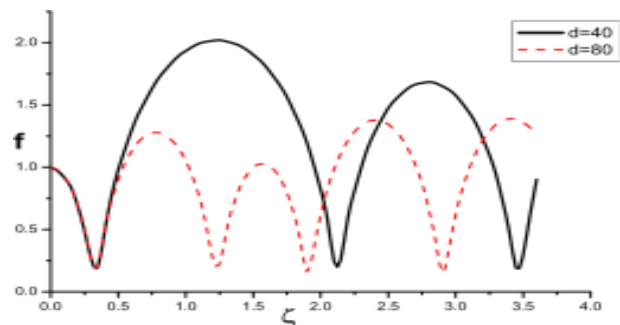


Fig. 3.

Beam-width parameter  $f$  plotted against the dimensionless distance of propagation at different values of ripple wave number  $d = 40$  and  $80$ , with other parameters as  $I = 10^{19} \text{ W cm}^{-2}, m = 0.5, b = 1, (r_0 \omega_0)/c = 60, \omega_p^2/\omega_0^2 = 0.01, \epsilon_2 = 1$ .

In fig.1 and fig.2 you we have observed the value of intensity of laser  $I = 1.2 \times 10^{19} \text{ w/cm}^2$  and  $\lambda = 0.5 \mu\text{m}$ , using the

modified parameter, we have investigated the oscillation value of beamwidth parameters  $f$  with normalized distance of propagation  $\xi$  with different values of ripple wave number  $d$  and parameter  $b$  which is shown in fig.3 and fig4. it is also observed from the graph that reducing the focusing with decrease in  $d$ , from the  $d=80$  to  $d=40$ . Hence increased in self-focusing length and curvature of the wavefront of cosh gaussian beam concentrated more in the density reason in comprising to gaussian laser beam.

In Figure 4 it is shown that the variation of beam width parameter  $f$  with distance of propagation  $\xi$  for various value of decentered parameter  $b=0.5$  and  $0.61$  with  $d=40$  and  $\alpha_2 = 1$ ,  $\lambda = 0.5$ ,  $\omega_p^2 = 0.01\omega_0$

and rest parameters as in figure3. It is found at all curves display oscillatory self-focusing, when the value of decentered parameter is increases from  $b = 0.5$  to  $b=1$ , stronger focusing in result

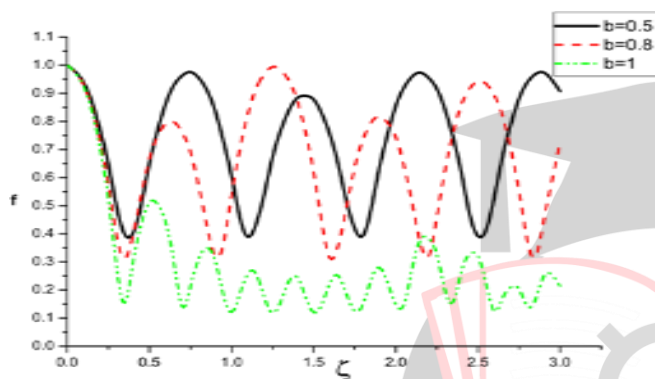


Fig. 4

Beam-width parameter  $f$  plotted against the dimensionless distance of propagation at different values of  $b$  and other parameters as  $I = 1019$   $W/cm^2$ ,  $d = 0.5$   $m$ ,  $b = 1$ ,  $(\omega_0/c) = 60$ ,  $\omega_p^2/\omega_0^2 = 0.01$ ,  $\alpha_2 = 1$ ,  $d = 40$ .

It is observed that for the value of  $B$  equals to 1, result strong self-focusing. we can also observe that the selffocusing effect is decreases due to diffraction effect.

## V. CONCLUSION

These observations represent an analysis of the characteristics of relativistic self-focusing of laser beam propagates in plasma considering the paraxial approximation. The changing in refractive index of the medium due to Relativistic laser plasma interaction and the role of ponderomotive force on the relativistic self-focusing of laser beam has been considered. in relativistic self-focusing the periodic density Ripple is observed using different intensity and wavelength of the laser beam. we have derived an equation of diffraction divergence which is responsible for self-focusing, relativistic self-focusing and ponderomotive nonlinearities. considering proper laser and plasma parameters the effect of ripple wave number and the characteristics of cosh- gaussian beam, beam width parameter has been investigated. It is observed that reduction in relativistic self-focusing of The beam width increasing the value of laser wavelength and intensity of laser beam.

It is also observed that the dependence of self-focusing on decentered parameter relativistic self-focusing of cosh gaussian laser beam in plasma depends on the intensity and wavelength of the used laser beam, decentered parameters and ripple wave number. it is also observed the effect of intensity on relativistic self-focusing parameter. this investigation may be useful for various applications like laser induced.

## REFERENCES

- [1] V.K.Tripathi, J. Prasar, "Two dimensional effect in a tunned ionized plasma, Phys. Plasma 4 ,3040-3042, 1997. [2] T. Taguchi, C.S.Liu, "Plasma channel charging by an intense short pulse laser and ion coulomb explosion" phys. Plasma 12, 2005.
- [3] C.L.Chang, Wang, S.Y.Chen, "Spatially localised self focusing of electrons in a self modulated laser wave field accelerated by using laser induced transient density ramp". Phys. Rev. LETT 94 , 2005 115003.
- [4] A Sharma, M.P, D.N Gupta, Verma and M.S.Shodha, J Opt. Soc. Am, B(22) ,1968, 2005.
- [5] H.Hora, "Laser Plasma and Nuclear Energy" (Plenum, New York, 1975) p.47. [6] L Hwang, A.K.Sharma, D.N.Gupta, "Plasma density ramp for relativistic self focusing of intense laser , Opt. Soc. AM B (24) 1156-1159, 2007.
- [7] B.Zang, H. Ma. " Propagation properties of cosh- gaussian beams" Opt. Commun.164, 1999.
- [8] P.Sprangle, A.Ting, and C.M.Tang, Phys .Rev. L ett, 59, 702, 1987.
- [9] X.F.Wang, R.Fedosejevs, and G.D.Tsakiris, Opt. Commun.146,363, 1998.
- [10] M.R.Siegrist, Opt. Commun.16,402, 1976.
- [11] F,Osman, R.Castillo, and H. Hora, J. Plasma Phys.61,263, 1999.
- [12] B.Hafizi, A.Ting, R.F.Habbar, Phy.Rev,E 62 4120, 2000.
- [13] G.Laval, Sadighi- Bonabi, P.Mora, A.Heron, Phys.Plasma 2,2807, 1995.
- [14] V.Malka, F.Amiranoff, C.Courtois, K Krushelnick A.E Dangor, P. Mora, J Fura, M.Salavati, J.C Adam, Phys.Plasma 7 3009, 2000.
- [15] R.Schlickeiser, S. Kaar " On the Origin of cosmological magnetic field generation by the Weiber instability" Astrophys. J.599 L 57, 2007.
- [16] J. Prasar, "Two dimensional effect in a tunned ionized plasma, Phys. Plasma 4 ,3040-3042 , 1997.
- [17] P.P.Kronberg " Galaxies and the magnetisation of integratic space" Phys.Plasmas, 10, 1985, 2003.
- [18] M E Dieckmann , "simulation study of the filamentation of counter streaming beam of the electron and positron in plasma" Plasma Phys. Contron fusion, 51, 065015, 1999.
- [19] B.J. Green, P.Mulser " Pondromotive forces in the interaction of laser radiation with a plasma" Phys. Rev.37,319, 2002.