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Approximate Solution of Non-Linear Equation for Imbibition Phenomena in One Dimensional Fractured Heterogeneous Porous Medium

[1]Dhara T. Patel, Research Scholar, Mehsana Urban Institute of Sciences, Ganpat University, Mehsana, Gujarat, India, Email id: dharar89@yahoo.in

[2]Amit K.Parikh, Principal, Mehsana Urban Institute of Sciences, Ganpat University Mehsana, Gujarat, India

Abstract: In present paper, we discussed the imbibition phenomena in a fractured heterogeneous porous media that took place during the secondary oil retrieval procedure. An approximate solution of arisen differential equation is acquired using Homotopy Perturbation Laplace Transform Method (HPLTM). The solution reveals the saturation of poured water in a porous medium during imbibition process. The statistical outcomes were derived utilizing MATLAB. This method is easy & reliable.

Keywords — **Fractured heterogeneous porous media, Homotopy perturbation method, Imbibition phenomenon, Laplace transformation.**

I. INTRODUCTION

Imbibition processes execute a lead role in oil retrieval procedure from fractured reservoirs. Counter-current spontaneous imbibition is a principal technique of retrieval from tight matrix blocks in naturally fractured reservoirs. Naturally fractured oil reservoirs account for a significant portion of the world's oil resources. Fractures are normally having vast rate of flow volume and low rate of storage volume.

"When a porous medium filled with some fluid, is brought in contact with another fluid which preferentially wets the medium then there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the native fluid from the medium. This phenomenon arising near to the common border due to difference in wetting abilities of the fluids and viscosity of fluids flowing in the medium is called counter current imbibition phenomenon."

Various authors, such as Brownscombe and Dyes (1952), Graham and Richardson(1959), Scheidegger (1960), Blair (1964), Verma (1969), Mehta and verma (1977) have explored the phenomenon of imbibition from various analytical and numerical perspectives.

P. M. Blair [1] has derived numerical solution of the imbibition of water and the counter current flow of oil in porous rocks. J. Bourblaux, F. J. Kalaydjian [2] have talked through experimental study of co-current and counter current flows in natural porous media. M. N. Mehta has described analytically the phenomenon of imbibition in porous media using a singular perturbation method. M. Pooladi-Darvish, A. Firoozabadi [3] have discussed the similarities and differences of co-current and counter current imbibition and pointed out the consequences for practical applications. S.Yadav and M. N. Mehta [4] have derived the mathematical model and similarity solution of counter current imbibition phenomenon in banded porous media. M. S. Joshi, N. B. Desai, M. N. Mehta [5] have discussed an analytical solution of the counter current imbibition phenomenon in homogeneous porous media. K.R.Patel, M.N.Mehta, T.R.Singh [6] have discussed a mathematical model of imbibition phenomenon in heterogeneous porous media during secondary oil recovery process using a similarity transformation. A. K. Parikh, M. N. Mehta, V. H. Pradhan [7] have described the counter current imbibition in vertical downward homogeneous porous media. K.K.Patel, M.N.Mehta, T.R.Singh [8] have discussed a homotopy series solution to a nonlinear partial differential equation arising from a mathematical model of the counter-current imbibition phenomenon in a heterogeneous porous medium. H.S.Patel, R.K.Meher [9] have discussed simulation of counter-current imbibition phenomenon in a double phase flow through fracture porous medium with capillary pressure.

This paper discussed natural phenomenon of counter current imbibition in a fractured porous medium. During secondary oil retrieval procedure, when water is poured in oil formatted fractured porous media at the common border $(x = 0)$ the counter current imbibition takes place. The goal is to determine the saturation of water at any distance x and any time t in fractured porous media.

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The nonlinear partial differential equation has been obtained for the counter current imbibition phenomenon, and solved by Homotopy Perturbation Laplace transform Method using suitable initial condition.

II. STATEMENT OF THE PROBLEM

From the massive natural oil soaked porous media shown in Fig.1, we take a cylindrical piece of fractured heterogeneous porous media. The vertical cross-section of this sample was then sliced into a rectangle as seen in Fig. 2. Three sides are impenetrable, whereas one open end is permeable.

When water is brought into contact with oil-saturated heterogeneous porous matrix at the common border $l = 0$, then wetting fluid water flows naturally inside the medium and counter flow of resident fluid oil out of the medium. This is named as imbibition phenomenon. The shape and size of small fingers are distinct and irregular. For the study, small fingers are supposed to be a rectangle that is given by Fig. 3.

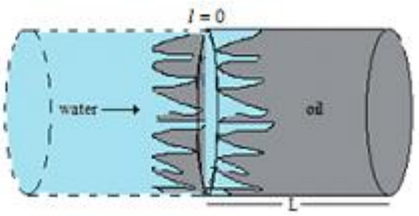

Fig.1. Cylindrical sample

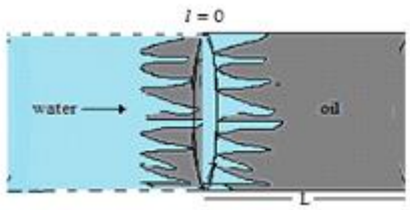

Fig. 2. Vertical cross-section

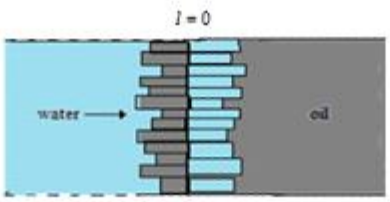

Fig. 3. Rectangle fingers

Inserted water is assumed to be more wetting than that of resident oil. Hence during the imbibition process, the oil will be shifted to a short distance l. Since the water is poured in oil formatted area and water and oil are flowing through porous matrix then it will satisfy Darcy's law[10] for low Reynolds number.

III. MATHEMATICAL FORMULATION OF COUNTER CURRENT IMBIBITION PHENOMENON

A. Fundamental equations:

i) Gauss divergent theorem:

To transform surface integral into volume integral and vise- versa, gauss divergent theorem is useful.

It indicate that, the surface integral of the normal component $\hat{n}$ of a vector function F, taken over a closed surface S is same as the volume integral of the divergence of vector function F taken over a volume enclosed by the closed surface S

$$\oiint_S (F \cdot \hat{n})\, dS = \iiint_V (\nabla \cdot F)\, dV \qquad (1)$$

ii) Mass conservation law**:**

Conservation of mass equation indicates the balance between rate of mass change in an arbitrary volume and inflow of mass through the boundary surface area S.

In integral form, this balance is conveyed as follows

$$\frac{\partial}{\partial t}\iiint_V \rho P\, dV + \oiint_S (\rho F \cdot \hat{n})\, dS = \iiint_V \rho q\, dV \qquad (2)$$

where ρ represent density of fluid, P and q represent porosity and source term of porous medium and F

represent velocity vector of fluid.

The second term of left hand side of the above equation can be converted into a volume integral form by the use of the divergence theorem as

$$\oiint_S (\rho F \cdot \hat{n})\, dS = \iiint_V \nabla \cdot (\rho v)\, dV \qquad (3)$$

where v is a velocity of fluid.

Using above relation, balance equation (2) can be written as

$$\frac{\partial}{\partial t}\iiint_V \rho P\, dV + \iiint_V \nabla \cdot (\rho v)\, dV = \iiint_V \rho q\, dV \qquad (4)$$

The derivative can be shifted under the integral sign because the controlled volume is constant. So for a constant control volume, the integral form of the conservation law will be

$$\iiint_V \frac{\partial}{\partial t}(\rho P)\, dV + \iiint_V \nabla \cdot (\rho v)\, dV = \iiint_V \rho q\, dV \qquad (5)$$

Rearranging

$$\iiint_V \left[\frac{\partial}{\partial t}(\rho P + \nabla \cdot (\rho v) - \rho q)\right] dV = 0 \qquad (6)$$

The above equation always works with any controlled volume. As a result, the integrand has to be zero.

$$\frac{\partial}{\partial t}(\rho P) + \nabla \cdot (\rho v) = \rho q \qquad (7)$$

In the multiphase flow, each phase’s saturation must.be taken into account .As a result equation (7) within each phase α becomes

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$$\frac{\partial}{\partial t}(\rho_\alpha S_\alpha P)+\nabla\cdot(\rho_\alpha v_\alpha)=\rho_\alpha q_\alpha$$

where α is phase index.

For water and oil above equation can be written as

$$\frac{\partial}{\partial t}(\rho_w S_w P)+\nabla\cdot(\rho_w v_w)=\rho_w q_w \quad (8)$$

where P denotes the porosity of porous medium.

$$\frac{\partial}{\partial t}(\rho_o S_o P)+\nabla\cdot(\rho_o v_o)=\rho_o q_o \quad (9)$$

S_w and S_o represent saturation of water and oil respectively. ρ_w and ρ_o represent density of water and oil respectively. q_w and q_o represent source term. v_w and v_o represent volumetric flow rate of water and oil respectively.

According to Darcy's law [10] the velocities of injected water v_w and native oil v_o are expressed as

$$v_w=-K\left(\frac{K_w}{\mu_w}\right)\left(\frac{\partial P_w}{\partial x}\right) \quad (10)$$

$$v_o=-K\left(\frac{K_o}{\mu_o}\right)\left(\frac{\partial P_o}{\partial x}\right) \quad (11)$$

In above equations K_w and K_o are the relative perm abilities which are function of water saturation .The permeability K is variable, the porous medium being fractured heterogeneous. P_w, μ_w are the pressure and viscosity of injected water respectively whereas P_o, μ_o are the pressure and viscosity of oil respectively.

If the compressibility of fluid is neglected then density ρ_w and ρ_o are constant and mass conservation law will becomes

$$P\frac{\partial S_w}{\partial t}+\frac{\partial v_w}{\partial x}=q \quad (12)$$

where P is the variable porosity of the fractured heterogeneous porous medium.

The fluid can flow only due to capillary pressure that is defined as the pressure difference of the flowing fluid across its common border via interconnected capillary. It may be written as

$$P_c=P_o-P_w \quad (13)$$

The linear relation between capillary pressure (P_c) and poured water saturation (S_w) was applied by M.N.Mehta [11] as

$$P_c=-\beta S_w \quad (14)$$

We suppose that the conventional relationship between phase saturation and relative permeability as proposed by A.E.Scheidegger, E.F. Johnson [12]

$$K_w=S_w \quad , K_o=1-\alpha S_w \quad (\alpha=1.11) \quad (15)$$

Also for definiteness, we use the similar function for porosity of the heterogeneous porous matrix applied by kinjal and Mehta[6] as

$$P=P(x)=\frac{1}{a-bx} \quad (16)$$

where a & b are constant

According to Scheidegger A. E [13], in countercurrent imbibition phenomenon, the sum of the velocities of injected water and native oil is zero.

$$v_w+v_o=0 \quad (17)$$

Using equation (10) and (11) into equation (17),

$$\left(\frac{K_w}{\mu_w}K\right)\left(\frac{\partial P_w}{\partial x}\right)+\left(\frac{K_o}{\mu_o}K\right)\left(\frac{\partial P_o}{\partial x}\right)=0 \quad (18)$$

From equations (18) and (13)

$$\left(\frac{K_w}{\mu_w}+\frac{K_o}{\mu_o}\right)\frac{\partial P_w}{\partial x}+\frac{K_o}{\mu_o}\frac{\partial P_c}{\partial x}=0 \quad (19)$$

$$\frac{\partial P_w}{\partial x}=-\frac{\frac{K_o}{\mu_o}\frac{\partial P_c}{\partial x}}{\left(\frac{K_w}{\mu_w}+\frac{K_o}{\mu_o}\right)} \quad (20)$$

Using equation (20) into equation (10), we get

$$v_w=\frac{K_w}{\mu_w}K\left[\frac{\frac{K_o}{\mu_o}\frac{\partial P_c}{\partial x}}{\left(\frac{K_w}{\mu_w}+\frac{K_o}{\mu_o}\right)}\right] \quad (21)$$

On substituting the value of v_w from equation (21) into mass conservation equation (12), we get

$$P\left(\frac{\partial S_w}{\partial t}\right)+\frac{\partial}{\partial x}\left[K\frac{\frac{K_w}{\mu_w}\frac{K_o}{\mu_o}}{\frac{K_w}{\mu_w}+\frac{K_o}{\mu_o}}\frac{\partial P_c}{\partial x}\right]=q_w \quad (22)$$

For the investigation flow system involves water and viscous oil, therefore according to Scheidegger A. E[13], We have

$$\frac{\frac{K_w}{\mu_w}\frac{K_o}{\mu_o}}{\frac{K_w}{\mu_w}+\frac{K_o}{\mu_o}}\approx\frac{K_o}{\mu_o}=\frac{(1-\alpha S_w)}{\mu_o} \quad (23)$$

On substituting values from equation (23) and (14) into equation (22), we get

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$$P\left(\frac{\partial S_w}{\partial t}\right)-\frac{\beta}{\mu_o}\frac{\partial}{\partial x}\left[K(1-\alpha S_w)\frac{\partial S_w}{\partial x}\right]=q_w \quad (24)$$

To further simplify equation (24), we consider the standard relation $K \propto P$ given by Z.Chen[14]

$K = K_C P$, where K_C is a constant of proportionality.

Substituting this value in equation (24), we get

$$P\left(\frac{\partial S_w}{\partial t}\right)-\frac{K_C\beta}{\mu_o}\frac{\partial}{\partial x}\left[P(1-\alpha S_w)\frac{\partial S_w}{\partial x}\right]=q_w \quad (25)$$

Which is non-linear partial differential equation for the saturation of injected water during counter-current imbibition phenomena in fractured porous media, where the porous matrix be assumed to be heterogeneous in nature.

Using dimensionless variables

$$T=\frac{K_C\alpha\beta}{L^2\mu_o}t \quad , \quad X=\frac{x}{L} \quad , \; 0\le X\le 1$$

and source term $q_w=\dfrac{1}{a+bx}$, $a,b\ge 1$

We have

$$\frac{\partial S_w}{\partial T}-\frac{1}{P}\frac{\partial P}{\partial X}\left(\frac{1}{\alpha}-S_w\right)\frac{\partial S_w}{\partial X}-\frac{\partial}{\partial X}\left[\left(\frac{1}{\alpha}-S_w\right)\frac{\partial S_w}{\partial X}\right]=\frac{A}{P(a+bXL)} \quad (26)$$

where $A=\dfrac{L^2\mu_o}{K_C\alpha\beta}$

To further simplify, we will take $\dfrac{1}{\alpha}=\lambda$, so we get

$$\frac{\partial S_w}{\partial T}-\frac{1}{P}\frac{\partial P}{\partial X}(\lambda-S_w)\frac{\partial S_w}{\partial X}-\frac{\partial}{\partial X}\left[(\lambda-S_w)\frac{\partial S_w}{\partial X}\right]=\frac{A}{P(a+bXL)} \quad (27)$$

Using equation (16)

$$\frac{\partial S_w}{\partial T}-\frac{bL}{(a-bXL)}(\lambda-S_w)\frac{\partial S_w}{\partial X}-\frac{\partial}{\partial X}\left[(\lambda-S_w)\frac{\partial S_w}{\partial X}\right]=\frac{A}{(a^2-b^2X^2L^2)} \quad (28)$$

$$S_w(X,0)=S_{w_0}e^X \text{ at } T=0 \text{ for } X>0 \quad (29)$$

IV. Solution of Counter Current Imbibition Phenomenon Using Homotopy Perturbation Laplace Transform Method (HPLTM)

We will obtain the approximate solution of (28) subject to initial condition which is given by equation (29), using Laplace Transfrom Homotopy perturbation method as an infinite series.

Applying the Homotopy Perturbation Method, Dr.J.H.He [15] constructed the function $\phi(X,T;p):\Omega\times[0,1]\to R$, which can be satisfied as follows:

$$(1-p)\left[\frac{\partial S_w}{\partial T}-\frac{\partial u_0}{\partial T}\right]+p\left[\begin{array}{l}\dfrac{\partial S_w}{\partial T}-\dfrac{bL}{(a-bXL)}(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\\ -\dfrac{\partial}{\partial X}\left[(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\right]-\dfrac{A}{(a^2-b^2X^2L^2)}\end{array}\right]=0 \quad (30)$$

$$\frac{\partial S_w}{\partial T}=\frac{\partial v_0}{\partial T}+p\left[\begin{array}{l}-\dfrac{\partial u_0}{\partial T}+\dfrac{bL}{(a-bXL)}(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\\ +\dfrac{\partial}{\partial X}\left[(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\right]+\dfrac{A}{(a^2-b^2X^2L^2)}\end{array}\right]$$

Where $v_0(X,T)=S_{w_0}e^X$ (which can be chosen freely), which satisfy the initial condition.

$$s\,L\{S_w(X,T)\}-S_w(X,0)=L\left\{\frac{\partial u_0}{\partial T}+p\left[\begin{array}{l}-\dfrac{\partial u_0}{\partial T}+\dfrac{bL}{(a-bXL)}(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\\ +\dfrac{\partial}{\partial X}\left[(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\right]+\dfrac{A}{(a^2-b^2X^2L^2)}\end{array}\right]\right\} \quad (31)$$

$$L\{S_w(X,T)\}-\frac{S_w(X,0)}{s}=\frac{1}{s}L\left\{\frac{\partial u_0}{\partial T}+p\left[\begin{array}{l}-\dfrac{\partial u_0}{\partial T}+\dfrac{bL}{(a-bXL)}(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\\ +\dfrac{\partial}{\partial X}\left[(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\right]+\dfrac{A}{(a^2-b^2X^2L^2)}\end{array}\right]\right\} \quad (32)$$

Substituting all the given value and applying inverse Laplace transform on both sides, we get

$$S_w(X,T)=S_{w_0}e^X+L^{-1}\left(\frac{1}{s}L\left\{p\left[\begin{array}{l}-\dfrac{\partial u_0}{\partial T}+\dfrac{bL}{(a-bXL)}(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\\ +\dfrac{\partial}{\partial X}\left[(\lambda-S_w)\dfrac{\partial S_w}{\partial X}\right]+\dfrac{A}{(a^2-b^2X^2L^2)}\end{array}\right]\right\}\right) \quad (33)$$

Now according to homotopy perturbation method

$$S_w(X,T)=S_0+pS_1+p^2S_2+\dots \quad (34)$$

Using equation (34), equation (33) can be written as

$$S_0+pS_1+p^2S_2+\dots=S_{w_0}e^X+L^{-1}\left(\frac{1}{s}L\left\{p\left[\begin{array}{l}-\dfrac{\partial u_0}{\partial T}\\ +\dfrac{bL}{(a-bXL)}\left(\lambda-(S_0+pS_1+\dots)\right)\dfrac{\partial(S_0+pS_1+\dots)}{\partial X}\\ +\dfrac{\partial}{\partial X}\left(\left(\lambda-(S_0+pS_1+\dots)\right)\dfrac{\partial(S_0+pS_1+\dots)}{\partial X}\right)\\ +\dfrac{A}{(a^2-b^2X^2L^2)}\end{array}\right]\right\}\right)$$

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On expansion of the above equation and comparing the coefficient of various power of p, we get

$$p^0 : S_0(X,T) = S_{w_0} e^X$$

$$p^1 : S_1(X,T) = L^{-1}\left[\frac{1}{s} L\left\{ \begin{array}{l} -\frac{\partial u_0}{\partial T} + \lambda \frac{\partial^2 S_0}{\partial X^2} - S_0 \frac{\partial^2 S_0}{\partial X^2} - \left(\frac{\partial S_0}{\partial X}\right)^2 \\ + \frac{bL\lambda}{(a-bXL)} \frac{\partial S_0}{\partial X} - \frac{bL}{(a-bXL)} S_0 \frac{\partial S_0}{\partial X} \end{array} \right\}\right]$$

$$= \left(\begin{array}{l} \lambda S_{w_0} e^X - 2S_{w_0}^2 e^{2X} + \frac{A}{(a^2 - b^2 X^2 L^2)} \\ + \frac{bLS_{w_0}^2 e^{2X}}{bLX - a} - \frac{bL\lambda S_{w_0} e^X}{bLX - a} \end{array} \right) T$$

Substituting all these values into equation (34), and taking $p = 1$, we get

$$S_w = S_0 + S_1 + S_2 + \ldots$$

$$= S_{w_0} e^X + \lambda S_{w_0} e^X T - 2S_{w_0}^2 e^{2X} T + \frac{A}{(a^2 - b^2 X^2 L^2)} T + \frac{bLS_{w_0}^2 e^{2X} T}{bLX - a} - \frac{bL\lambda S_{w_0} e^X T}{bLX - a} + \ldots \quad (35)$$

This is the required solution of governing equation (28) of the counter-current imbibition phenomenon which represents the saturation of injected water occupied by the schematic fingers of average length at any distance X and for any fixed time T.

V. RESULTS AND DISCUSSION:

The solution (35) represents the saturation of poured water in form of the infinite series of X and T in the secondary oil retrieval procedure. We have trimmed the series, so it will offer an approximate solution of imbibition phenomenon. Table.1 reveals water saturation. The following constant values are taken from the standard literature for numerical calculation as follows.

$L = 1m, S_{w_0} = 0.1, \alpha = 1.11, \lambda = 0.9, A = 2.6, a = 5, b = 1$

We picked a length X range of 0 to 1 based on dimensionless variable and time T of 0 to 0.5 as imbibition phenomenon occurs for a short time. For greater values of time T, the saturation of water S_w will become constant. Here it is observed that saturation of water S_w is rising as length X rises. It is also rising for time T. Statistical and pictorial presentation of solution (35) has been derived by MATLAB. Fig.4. exhibits the visual representation of S_w Vs. X for fixed time T = 0, 0.1, 0.2, 0.3, 0.4, 0.5.

Table 1: Saturation of injected water (S_w) for different X for fixed time T

	T=0	T=0.1	T=0.2	T=0.3	T=0.4	T=0.5
X=0	0.2000	0.2232	0.2464	0.2696	0.2928	0.3160
X=0.1	0.2211	0.2636	0.3061	0.3487	0.3912	0.4337
X=0.2	0.2443	0.3040	0.3636	0.4233	0.4829	0.5426
X=0.3	0.2700	0.3445	0.4191	0.4936	0.5681	0.6426
X=0.4	0.2984	0.3855	0.4725	0.5595	0.6466	0.7336
X=0.5	0.3298	0.4269	0.5240	0.6211	0.7182	0.8153

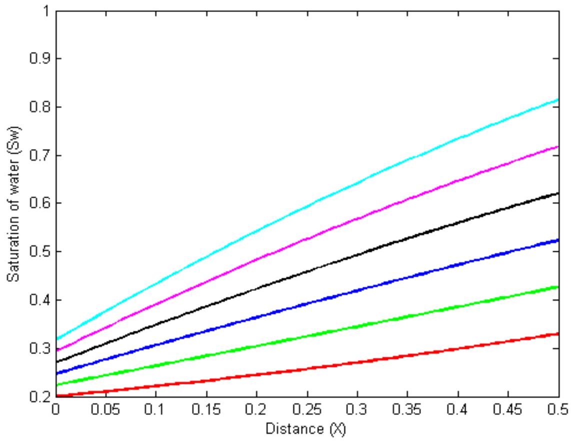

Figure (4): Graph of Saturation of water (S_w) Vs. Distance X for a fixed time T

VI. CONCLUSION

The governing non-linear equation representing counter current imbibition phenomenon in fractured porous media where matrix block is heterogeneous is solved using Homotopy Perturbation Laplace Transform method. It was observed that, in imbibition phenomena the infinite series represent the saturation of water increases with time and increases with space variable in fractured porous media. Oil moves towards the oil production well as water saturation rises, and oil can be regained by this method using the secondary oil retrieval process. At last, this is a reliable technique which one can simply use without having much experience.

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