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Variable Control Charts based on percentiles of The New Weighted Exponential Distribution

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Abstract: The intention of this article is to evaluate the proposed new weighted exponential distribution's percentiles of mean, median, range, and standard deviation, as well as establish control limits for these parameters. We compare the competence and suitability of the derived control limits to those based on well-known Shewhart control limits by assuming that the variable quality characteristic follows the new weighted exponential distribution. In the simulation phase, the coverage probabilities were computed and compared.

Keywords: most probable, pdf, cdf, equi-tailed, percentiles, NWED

I. INTRODUCTION

In statistical quality control, variable control charts play a vital role in evaluating variation in a process. The two types of variable control charts evaluate variation between the samples and within the samples as well. The process-behavior charts or Shewhart charts, which are based on the premise that the character variate pursues a Gaussian distribution, are also known as variable control charts. The control lines of a process-behavior control chart or Shewhart chart are $E(t_n) \pm 3 * S.E(t_n)$ if $x_1, x_2, \ldots, x_n$ is a list of n measurements on a quality variable of a material and t_n is a specific characteristic according to this study. Regardless the population model, to test the process quality we use Shewhart control limits. These limits involve Shewhart constants which are readily available in any SQC text book. The Shewhart control charts are constructed based on the assumption that the population model tend to normal distribution. This is valid from the central limit theorem which states that as the size n of the sample gets larger, for almost all the distributions, the distribution of sample means approximates a normal distribution, assuming that all the samples are in same size. But, in quality assurance studies majority of the times, the observed information is consistently taken in limited samples and the sample size may not be the same. Furthermore, if evidence was provided that the sampled variable quality characteristic follows a distribution pattern other than the normal distribution, the online step of such a characteristic can be tracked using the corresponding distribution theory. In such situations, the inference of normality should be avoided unless a prior analysis of the goodness of fit test has been performed. Using the central limit theorem, on the other hand, is not recommended since it provides asymptotic normality for a given statistic. As a consequence, when the population distribution is non-normal, an alternate method for the development of control charts is required.

Many authors have attempted to create statistical quality control approaches using skewed distributions. Some of them are "Edgeman(1989) [3]-Inverse Gaussion Distribution, Gonzalez and Viles(2000) [4]-Gamma Distribution, Kantam and Sriram(2001) [5]-Gamma Distribution, Chan and Cui (2003) [2] have developed control chart constants for skewed distributions where the constants are dependent on the coefficient of skewness of the distribution, Kantam *et al*(2006) [6]-Log logistic Distribution, Betul and Yaziki(2006) [1]-Burr Distribution, Subba Rao and Kantam(2008) [18]-Double exponential distribution, Kantam and Rao(2010) [7]-control charts for process variate, Rao and Sarath Babu (2012) [13]-Linear failure rate distribution, Rao and Kantam (2012) [17]-Half logistic distribution, Rao *et al*(2013) [11]-Inverse Rayleigh distribution, Rao *et al*(2014) [14]-Size biased lomax distribution, Rao and Kumar (2015) [15]-Exponential Gamma distribution, Rao *et al*(2016) [16]-Half normal distribution, Rosaiah *et al*(2018) [9]-Gumbel distribution, Rosaiah *et al*(2019) [10]-Exponentiated inverse Kumaraswamy distribution, Sricharani and Rao (2019) [12]-Dagum distribution" and references there in.

In this paper, we made an assumption on process quality variable that it follows a new weighted exponential distribution and constructed control limits for simulated data in similar to the familiar Shewhart control limits. Consider X as a variable drawn randomly from a new weighted

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exponential distribution (NWED) whose distribution function (cdf) is,

$$F\left(x;\alpha,\lambda\right)=1-e^{-(1+\lambda)\alpha x}; \qquad x>0,\alpha>0,\lambda>0.$$

(1.1)

Where, α is a location parameter and λ is a spread parameter.

The probability distribution function (pdf) corresponds to (1.1) is,

$$f\left(x;\alpha,\lambda\right)=\left(1+\lambda\right)\alpha.e^{-(1+\lambda)\alpha x}; \quad x>0,\alpha>0,\lambda>0.$$

(1.2)

Plots of the pdf_s and cdf_s of NWED for selected parameter values are shown in Figures 1 and 2.

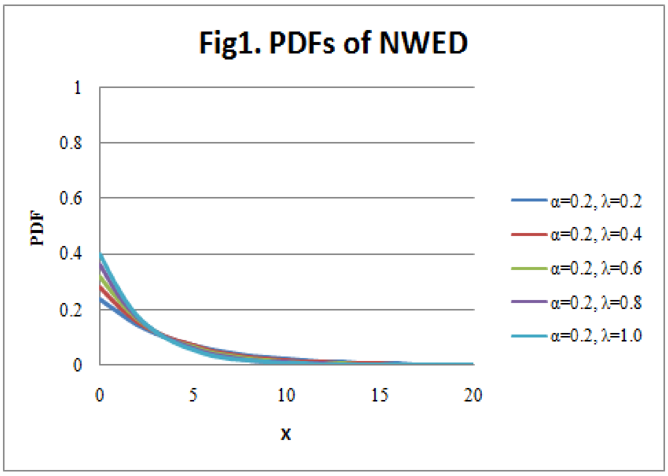

Fig1. PDFs of NWED

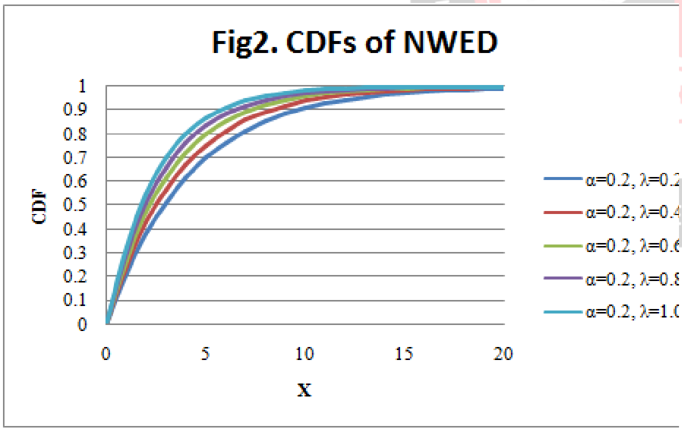

Fig2. CDFs of NWED

Figure 1 shows that the curve of the NWED is very close to that of a standard Exponential distribution with parameter α as $\lambda \to 0$.

The survival (or) reliability function of NWED is given by,

$$\bar{F}\left(x\right)=e^{-(1+\lambda)\alpha x} \quad (1.3)$$

The hazard function of NWED is given by,

$$h\left(x\right)=\left(1+\lambda\right)\alpha \quad (1.4)$$

The distributional properties are:

Mean: $$E\left(x\right)=\left(\frac{1}{\left(1+\alpha\right)\lambda}\right) \quad (1.5)$$

Median: $$Md=\left(\frac{\ln\left(2\right)}{\left(1+\alpha\right)\lambda}\right) \quad (1.6)$$

Variance: $$V\left(x\right)=\left(\frac{1}{\left(1+\alpha\right)\lambda}\right)^2 \quad (1.7)$$

Quantile function: $$Q\left(U\right)=\left(\frac{\ln\left(\frac{1}{1-U}\right)}{\left(1+\alpha\right)\lambda}\right) \quad (1.8)$$

where, U follows uniform distribution $[0,1]$.

The density function of the i^{th} order statistic $X_{(i)}$ is provided by,

$$\alpha_{(i)}=\frac{n!}{\left(i-1\right)!\left(n-i\right)!}\left(1+\lambda\right)\alpha.e^{-(1+\lambda)\alpha x}\left(1-e^{-(1+\lambda)\alpha x}\right)^{i-1}\left(e^{-(1+\lambda)\alpha\right.$$

(1.9)

If $i=1$, the density function of the minimum order statistic $X_{(1)}$ is provided by,

$$\alpha_{(1)}=n\left(1+\lambda\right)\alpha.e^{-(1+\lambda)\alpha x}\left(e^{-(1+\lambda)\alpha x}\right)^{n-1} \quad (1.10)$$

If $i=n$, the density function of the maximum order statistic $X_{(n)}$ is provided by,

$$\alpha_{(n)}=n\left(1+\lambda\right)\alpha.e^{-(1+\lambda)\alpha x}\left(1-e^{-(1+\lambda)\alpha x}\right)^{n-1} \quad (1.11)$$

The other distributional properties are extensively explored by Oguntunde *et al*(2016)[8]. NWED is another skewed distribution situation that is given a lot of attention in this study when it comes to developing control charts. The hazard function implies a constant failure rate functions if $\alpha=0.2$ and $\lambda=0.6$, making the NWED ideal for modeling components that wears continuously over time. It is also one of the statistical models that can be used for life checking and research on reliability. As a result, if lifetime data is considered to be of high quality, practitioners will benefit from the creation of control charts for it. Since NWED is a skewed distribution, this paper aims to compare it to other distributions. This paper makes an effort to answer and solve this problem to the best of our ability. The remaining portion of the paper is laid out as follows. The fundamentals of statistics are covered in Section 2 as well as the development of process control charts for mean, median, range, and standard deviation. A comparison of the existing Shewhart control lines to the derived NWED control lines is presented in Section 3. A description and conclusions are found in Section 4.

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II. Control Chart Constants Through Percentiles

2.1. Mean-chart

Consider $x_1, x_2, ..., x_n$ be a n sized arbitrary sample drawn from NWED with $\alpha = 0.2$ and $\lambda = 0.6$. If you target the average population, this is a sample of industrial process data. The statistic $\bar{x}$ determines whether the process average is similar to the desired average when using re-sampling. Statistically speaking, we need to determine the most likely limit value beyond the importance of a fall. The term is most likely used here to refer to a relative definition, which should be considered in the context of aggregation. Since the current method is only applicable to the normal distribution, the determination of the 3σ limit value is considered as the most probable limit value. As we all know, the 3σ normal distribution limit gives 0.9973 probability content. As a result, we need to find the two sample distributions of the sample mean in NWED, with a probability level of 0.9973. We must find an L, U

$$P\left(L \le \bar{x} \le U\right) = 0.9973 \quad (2.1)$$

Where, $\bar{x}$ is the mean of sample size n.

Using the equi-tailed concept, the percentiles of the sampling distribution of $\bar{x}$ are 0.00135 and 0.99865, respectively. By simulating the empirical sampling distribution of $\bar{x}$, we were able to compute its percentiles. Table 1 shows this detail.

Table 1: *Percentiles of Mean in NWED*

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	10.1224	8.4359	7.1125	5.7094	4.6020	0.4987	0.4458	0.3998	0.3758	0.3529
3	8.4559	6.9699	6.0168	4.9388	4.1144	0.5664	0.5141	0.4641	0.4393	0.4002
4	6.7542	5.8438	5.2652	4.3630	3.6551	0.6202	0.5715	0.5151	0.4813	0.4279
5	6.2198	5.3871	4.7866	4.0340	3.5009	0.6516	0.5991	0.5512	0.5188	0.4713
6	6.0049	5.0820	4.5851	3.9095	3.3762	0.6811	0.6293	0.5774	0.5480	0.4937
7	5.2979	4.5646	4.2024	3.6288	3.1799	0.7027	0.6564	0.6119	0.5778	0.5266
8	5.1336	4.4300	4.0545	3.4849	3.0881	0.7245	0.6739	0.6291	0.5952	0.5414
9	4.8645	4.2745	3.8867	3.3865	2.9910	0.7549	0.7032	0.6490	0.6253	0.5785
10	4.5500	3.9553	3.7207	3.2355	2.8649	0.7693	0.7135	0.6613	0.6400	0.5909

The sample mean control limits are calculated in the following way using the percentiles in the above table. Take a look at the $\bar{x}$ distribution,

$$P\left(Z_{0.00135} \le \bar{x} \le Z_{0.99865}\right) = 0.9973 \quad (2.2)$$

But $\bar{x}$ of sampling distribution when $\alpha = 0.2$ and $\lambda = 0.6$ is 3.125 for NWED.

From equation (2.2) over the course of repeated sampling, for the mean of i^{th} subgroup, we are able to have

$$P\left(Z_{0.00135} \cdot \frac{\bar{\bar{x}}}{3.125} \le \bar{x}_i \le Z_{0.99865} \cdot \frac{\bar{\bar{x}}}{3.125}\right) = 0.9973 \quad (2.3)$$

This can be expressed as:

$$P\left(A^{*}_{2p} . \bar{\bar{x}} \le \bar{x}_i \le A^{**}_{2p} . \bar{\bar{x}}\right) = 0.9973 \quad (2.4)$$

Where, $\bar{\bar{x}}$ is grand mean, $\bar{x}_i$ is i^{th} subgroup mean, $A^{*}_{2p} = \frac{Z_{0.00135}}{3.125}$, $A^{**}_{2p} = \frac{Z_{0.99865}}{3.125}$. Thus, A^{*}_{2P} and A^{**}_{2P} are percentile constants. Table 2 shows the percentile constants for the $\bar{x}$ chart for NWED.

Table 2: Percentile Constants of Mean chart.

n	A^{*}_{2P}	A^{**}_{2P}
2	0.1753	5.0282
3	0.2009	4.2447
4	0.2133	3.3676
5	0.2379	3.1400
6	0.2528	3.0747
7	0.2642	2.6579
8	0.2737	2.5949
9	0.2939	2.4710
10	0.2988	2.3005

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2.2. Median-chart.

We must find two sampling distribution limits of sample median in NWED that have a probability content of 0.9973. Symbolically, we must locate L, U in a manner that $P\left(L \le m \le U\right) = 0.9973$ (2.5)

Where, m is the sample median of size n. Table 3 shows the percentiles discovered by simulation.

Table 3: *Percentiles of Median in NWED*

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	10.1224	8.4359	7.1125	5.7094	4.6020	0.4987	0.4458	0.3998	0.3758	0.3529
3	8.4192	6.3327	5.4504	4.0038	2.9059	0.4684	0.4260	0.3893	0.3700	0.3466
4	6.0579	4.9053	4.2368	3.2116	2.5132	0.5423	0.4924	0.4453	0.4207	0.3970
5	5.4972	4.3851	3.7097	2.6410	1.8603	0.5236	0.4760	0.4323	0.4118	0.3838
6	4.9894	3.6335	3.1559	2.3781	1.8493	0.5679	0.5253	0.4772	0.4477	0.4181
7	4.1172	3.3842	2.7356	1.8569	1.2365	0.5613	0.5152	0.4679	0.4458	0.4115
8	3.5811	2.8979	2.4114	1.8454	1.4244	0.6014	0.5603	0.5079	0.4806	0.4480
9	3.5471	2.6732	2.1811	1.4688	1.2098	0.5968	0.5497	0.5106	0.4855	0.4432
10	2.8548	2.2435	1.9305	1.4849	1.2145	0.6214	0.5781	0.5338	0.5062	0.4710

The median control limits are determined in the following way using the percentiles in the above table. Take a look at the m distribution

$$P\left(Z_{0.00135} \le m \le Z_{0.99865}\right) = 0.9973 \quad (2.6)$$

But median of sampling distribution when $\alpha = 0.2$ and $\lambda = 0.6$ is 2.1661 for NWED.

From equation(2.6) over repeated sampling, for the i^{th} subgroup median we can have

$$P\left(Z_{0.00135} \cdot \frac{\overline{m}}{2.1661} \le m_i \le Z_{0.99865} \cdot \frac{\overline{m}}{2.1661}\right) = 0.9973 \quad (2.7)$$

This can be written as

$$P\left(\mathrm{A}^{*}_{7p} . \overline{m} \le m_i \le \mathrm{A}^{**}_{7p} . \overline{m}\right) = 0.9973 \quad (2.8)$$

Where, $\overline{m}$ is mean of subgroup medians. Thus $\mathrm{A}^{*}_{7p} = \frac{Z_{0.00135}}{2.1661}$ and $\mathrm{A}^{**}_{7p} = \frac{Z_{0.99865}}{2.1661}$ are the percentile constants of median chart and are given in table 4.

Table 4: *Percentile Constants of Median chart*

n	A^{*}_{7P}	A^{**}_{7P}
2	0.1629	4.6731
3	0.1600	3.8868
4	0.1833	2.7967
5	0.1772	2.5379
6	0.1930	2.3034
7	0.1900	1.9008
8	0.2068	1.6533
9	0.2046	1.6376
10	0.2174	1.3180

2.3. R-chart

In NWED, we must find two sampling distribution sample range limits with a probability content of 0.9973. We must symbolically place L and U in such a way that $P\left(L \le R \le U\right) = 0.9973$ (2.9)

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where, R is the sample range of size n. The percentiles discovered by simulation are shown in Table 5.

Table 5: *Percentiles of Range in NWED*

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	17.1603	13.8253	11.4034	8.6463	6.6313	0.0392	0.0916	0.0078	0.0040	0.0015
3	19.0499	15.5295	13.0925	10.4110	8.3303	0.1877	0.1267	0.0800	0.0533	0.0280
4	19.8896	16.9107	14.5485	11.5053	9.3612	0.3294	0.2588	0.1843	0.1440	0.0876
5	20.4410	17.2470	15.1729	12.2040	10.2067	0.4447	0.3609	0.2745	0.2241	0.1519
6	20.6521	17.8227	15.9946	12.8092	10.7383	0.5391	0.4561	0.3738	0.3174	0.2443
7	21.0641	18.3910	16.4794	13.5350	11.2981	0.6213	0.5390	0.4488	0.3976	0.3142
8	21.5317	18.5142	16.7579	13.7757	11.6490	0.6820	0.6028	0.5141	0.4471	0.3576
9	21.9174	19.0499	17.0796	14.1892	12.0253	0.7452	0.6529	0.5597	0.5068	0.4228
10	22.2275	19.2366	17.3187	14.5794	12.3440	0.7988	0.7074	0.6108	0.5641	0.4801

The sample range's control limits are calculated in the following way, using the percentiles from the above table. Take a look at the R distribution

$$P\left(Z_{0.00135} \le R \le Z_{0.99865}\right) = 0.9973 \quad (2.10)$$

From equation (2.10), for the i^{th} subgroup range we can have

$$P\left(Z_{0.00135}\cdot\frac{\bar{R}}{\alpha_{(n)}-\alpha_{(1)}} \le R_i \le Z_{0.99865}\cdot\frac{\bar{R}}{\alpha_{(n)}-\alpha_{(1)}}\right) = 0.9973 \quad (2.11)$$

This can be expressed as:

$$P\left(\mathrm{D}^{*}_{3p}.\bar{R} \le R_i \le \mathrm{D}^{**}_{4p}.\bar{R}\right) = 0.9973 \quad (2.12)$$

where, $\bar{R}$ is mean of ranges, R_i is i^{th} subgroup range. Thus $\mathrm{D}^{*}_{3p} = \dfrac{Z_{0.00135}}{\alpha_{(n)}-\alpha_{(1)}}$, $\mathrm{D}^{**}_{4p} = \dfrac{Z_{0.99865}}{\alpha_{(n)}-\alpha_{(1)}}$ are the percentile constants of R chart for NWED process data and are given in table 6.

Table 6: *Percentile Constants of Range chart*

n	D^{*}_{3P}	D^{**}_{4P}
2	0.0027	184.3956
3	0.0611	209.2084
4	0.1730	43.8671
5	0.2256	47.9693
6	0.3245	37.3675
7	0.3718	56.5452
8	0.4158	39.7781
9	0.4551	60.6146
10	0.5033	41.8627

2.4. σ -chart.

We would find two sampling distribution limits of sample standard deviation in NWED that have a probability content of 0.9973. Symbolically, we must locate L and U in such a way that

$$P\left(L \le s \le U\right) = 0.9973 \quad (2.13)$$

where, s denotes the standard deviation of an n sized sample. Table 7 shows the percentiles discovered by simulation.

Table 7: *Percentiles of Standard Deviation in NWED*

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	8.5801	6.9126	5.7017	4.3232	3.3157	0.0196	0.0098	0.0039	0.0020	0.0008

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3	9.0695	7.1137	6.1452	4.8797	3.9081	0.0759	0.0546	0.0365	0.0271	0.0137
4	8.4848	6.8710	6.0247	4.8608	3.9407	0.1297	0.1000	0.0731	0.0588	0.0400
5	8.2370	6.7509	5.8609	4.8732	3.9621	0.1612	0.1310	0.1029	0.0863	0.0575
6	7.9674	6.6951	5.8930	4.9713	4.1353	0.1928	0.1622	0.1275	0.1068	0.0819
7	7.1379	6.3462	5.6742	4.4920	3.9511	0.2083	0.1787	0.1504	0.1276	0.1063
8	7.2090	6.2343	5.6070	4.7051	3.9760	0.2282	0.2013	0.1714	0.1493	0.1209
9	7.2138	6.0690	5.5177	4.5954	3.9098	0.2464	0.2142	0.1875	0.1658	0.1364
10	6.7540	5.8264	5.2779	4.4744	3.8730	0.2593	0.2273	0.1969	0.1834	0.1519

The sample standard deviation control limits are determined in the following way using the percentiles from the above table. Consider the distribution of s

$$P\left(Z_{0.00135} \le s \le Z_{0.99865}\right) = 0.9973 \quad (2.14)$$

But standard deviation of sampling distribution when $\alpha = 0.2$ and $\lambda = 0.6$ is 3.125 for NWED. From equation (2.14), for the i^{th} subgroup standard deviation we can have

$$P\left(Z_{0.00135}\cdot\frac{\bar{s}}{3.125} \le s_i \le Z_{0.99865}\cdot\frac{\bar{s}}{3.125}\right) = 0.9973 \quad (2.15)$$

This can be written as

$$P\left(B^{*}_{3p}.\bar{s} \le s_i \le B^{**}_{4p}.\bar{s}\right) = 0.9973 \quad (2.16)$$

where, $\bar{s}$ is mean of standard deviations, s_i is i^{th} subgroup standard deviation . Thus $B^{*}_{3p} = \frac{Z_{0.00135}}{3.125}$ and $B^{**}_{4p} = \frac{Z_{0.99865}}{3.125}$ are the constants of standard deviation chart for NWPD process data given in table 8.

Table 8: *Percentile Constants of SD chart*

n	B^{*}_{3P}	B^{**}_{4P}
2	0.0002	2.7456
3	0.0044	2.9022
4	0.0128	2.7151
5	0.0184	2.6358
6	0.0262	2.5496
7	0.0340	2.2841
8	0.0387	2.3069
9	0.0437	2.3085
10	0.0486	2.1613

III. Comparative Study

The population represented by NWED is used to plot the average, median, range and standard deviation constants of the indicators previously described in Section 2 in the control chart. To use it for data, you need to ensure that the data is correctly compliant with NWED. As a result, the strength of the control limit can be estimated by comparing the use of the control limit with the Shewhart limit of the actual NWED data. With this in mind, we simulate the random subset of size n = 2(1)10 in NWED and use the constants in Section 2 to calculate the control limits of the average, median, range, and standard deviation throughout the process. To create this comparative study, the number of readings within each control limit is used to calculate the probability of NWED coverage. Similar to NWED coverage probability, Shewhart coverage probability is calculated by counting the number of readings in certain individual control ranges by using Shewhart constants in quality tables 9, 10, 11, and 12 that indicate coverage probability. For the two scenarios, NWED and Shewhart's real limitations.

Table 9: *Probabilities of Coverage for a Mean-chart*

	Shewart Limits			NWED Percentile Limits		
n	$\bar{\bar{x}} - A_2\bar{R}$	$\bar{\bar{x}} + A_2\bar{R}$	Coverage probability	$A^{*}_{2P} \times \bar{x}$	$A^{**}_{2P} \times \bar{x}$	Coverage probability
2	0	4.4734	0.9458	0.1753	5.0282	**0.9612**
3	0	4.0013	0.9448	0.2990	4.2447	**0.9539**
4	0	3.7133	0.9521	0.2133	3.3676	0.9335

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5	0	3.6134	0.9566	0.2379	3.1400	0.9261
6	0	3.5620	0.9611	0.2528	3.0747	0.9263
7	0	3.3812	0.9631	0.2642	2.6579	0.8988
8	0	3.3505	0.9684	0.2737	2.5949	0.8980
9	0	3.3103	0.9710	0.2939	2.4710	0.8845
10	0	3.2339	0.9749	0.2988	2.3005	0.8655

Table 10: *Probabilities of Coverage for a Median-chart*

	Shewart limits			NWED Percentile limits		
n	$\bar{m} - A_7\bar{R}$	$\bar{m} + A_7\bar{R}$	Coverage probability	$A^{*}_{7P} \times \bar{m}$	$A^{**}_{7P} \times \bar{m}$	Coverage probability
2	0	4.4734	0.9458	0.2529	7.2541	**0.9905**
3	0	3.6426	0.9680	0.1769	4.4965	**0.9792**
4	0	3.4482	0.9792	0.2007	3.0621	0.9704
5	0	3.3108	0.9856	0.1740	2.4927	0.9708
6	0	3.3417	0.9917	0.1903	2.2705	0.9712
7	0	3.1881	0.9938	0.1773	1.7738	0.9726
8	0	3.1984	0.9974	0.1931	1.5436	0.9590
9	0	3.1889	0.9970	0.1880	1.5048	0.9763
10	0	3.1642	0.9994	0.1988	1.2050	0.9475

Table 11: *Probabilities of Coverage for a Range-chart*

	Shewart limits			NWED Percentile limits		
n	$D_3\bar{R}$	$D_4\bar{R}$	Coverage probability	$D^{*}_{3P} \times \bar{R}$	$D^{**}_{4P} \times \bar{R}$	Coverage probability
2	0	5.0762	0.9136	0.0041	286.5137	**0.9950**
3	0	6.0636	0.8902	0.1438	492.6438	**0.9665**
4	0	6.7533	0.8819	0.5119	129.8202	0.8513
5	0	7.2918	0.8690	0.7778	165.3836	0.7501
6	0	7.8064	0.8611	1.2640	145.5650	0.7283
7	0.3281	8.3078	0.8655	1.6054	244.1620	0.7304
8	0.6326	8.6707	0.8330	1.9341	185.0357	0.7465
9	0.9166	9.0473	0.7572	2.2672	301.9819	0.7489
10	1.1789	9.3948	0.7619	2.6608	221.3238	0.7400

Table 12: *Probabilities of Coverage for a SD-chart*

	Shewart limits			NWED Percentile limits		
n	$B_3\bar{s}$	$B_4\bar{s}$	Coverage probability	$B^{*}_{3P} \times \bar{s}$	$B^{**}_{4P} \times \bar{s}$	Coverage probability
2	0	2.5380	0.9136	0.0001	2.1329	0.8865
3	0	2.7207	0.8896	0.0046	3.0748	**0.9128**
4	0	2.7412	0.8784	0.0154	3.2845	**0.9176**
5	0	2.8183	0.8689	0.0248	3.5560	**0.9318**
6	0.0440	2.8928	0.8642	0.0384	3.7439	**0.9297**
7	0.1752	2.7954	0.8346	0.0505	3.3927	**0.9165**
8	0.2895	2.8403	0.7409	0.0605	3.6100	**0.9290**
9	0.3870	2.8515	0.7176	0.0707	3.7381	**0.9405**
10	0.4679	2.8275	0.7116	0.0800	3.5612	**0.9311**

IV. Summary & Conclusions

In statistical quality control problems the Shewhart constants are used irrespective of the data follows a normal distribution or else. If the given data does not follow normal distribution, the applications of Shewhart constants for any statistical quality control problems are at stake.

The above tables show that when the Shewhart limits are used in a mechanical way and if the data follows a true NWED, the decision of process variation coefficient for mean, median, range and standard deviation charts will be reduced. So, in order to overcome such a problem, the control constants given at various sample sizes in the above tables are more preferable to Shewhart constants in statistical quality control.

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