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Reliability Estimation of Power Lomax Distribution Via Bayesian Approach

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Abstract - Power Lomax distribution is considered. Bayesian method of estimation is employed in order to estimate the reliability function of power Lomax distribution by using non-informative and beta priors. In this paper, the Bayes estimators of the reliability function have been obtained under squared error, precautionary and entropy loss functions.

Keywords: Power Lomax distribution, Reliability, Bayesian method, Non-informative and beta priors, Squared error, precautionary and entropy loss functions.

I. INTRODUCTION

The power Lomax distribution has been proposed by Rady et al. [1]. They studied this distribution for remission times of bladder cancer data. The probability function $f(x;\theta)$ and distribution function $F(x;\theta)$ of power Lomax distribution are respectively given by

$$f(x;\theta)=a\theta\lambda^{\theta}x^{a-1}\left(\lambda+x^{a}\right)^{-(\theta+1)} \quad ; x>0. \tag{1}$$

$$F(x;\theta)=1-\lambda^{\theta}\left(\lambda+x^{a}\right)^{-\theta} \quad ; x>0. \tag{2}$$

Let $R(t)$ denote the reliability function, that is, the probability that a system will survive a specified time t comes out to be

$$R(t)=\lambda^{\theta}\left(\lambda+t^{a}\right)^{-\theta} \quad ; t>0. \tag{3}$$

And the instantaneous failure rate or hazard rate, h(t) is given by

$$h(t)=a\theta t^{a-1}\left(\lambda+t^{a}\right)^{-1}. \tag{4}$$

From equation (1) and (3), we get

$$f(x;R(t))=\frac{ax^{a-1}\left(\lambda+x^{a}\right)}{log\left(1+\left(t^{a}/\lambda\right)\right)}\left[-log\,R(t)\right]\left[R(t)\right]^{\frac{log\left(\lambda+x^{a}\right)}{log\left(1+\left(t^{a}/\lambda\right)\right)}} \;;\; 0<R(t)\leq 1. \tag{5}$$

The joint density function or likelihood function of (5) is given by

$$f\left(\underline{x}\,|\,R(t)\right)=\prod_{i=1}^{n}\left[\frac{ax_i^{a-1}\left(\lambda+x_i^{a}\right)}{log\left(1+\left(t^{a}/\lambda\right)\right)}\right]\left[-log\,R(t)\right]^{n}\left[R(t)\right]^{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^{a}\right)/log\left(1+\left(t^{a}/\lambda\right)\right)\right)} \tag{6}$$

The log likelihood function is given by

$$\begin{aligned}log\,f\left(\underline{x}\,|\,R(t)\right)=&\,log\left(\prod_{i=1}^{n}\left(\frac{ax_i^{a-1}\left(\lambda+x_i^{a}\right)}{log\left(1+\left(t^{a}/\lambda\right)\right)}\right)\right)\\&+n\,log\left[-log\,R(t)\right]+\left(\sum_{i=1}^{n}log\left(\lambda+x_i^{a}\right)/log\left(1+\left(t^{a}/\lambda\right)\right)\right)log\left[R(t)\right]\end{aligned} \tag{7}$$

Differentiating (7) with respect to R (t) and equating to zero, we get the maximum likelihood estimator of R (t) as

$$\hat{R}(t)=exp\left[-n\left\{log\left(1+\left(t^{a}/\lambda\right)\right)\Big/\sum_{i=1}^{n}log\left(\lambda+x_i^{a}\right)\right\}\right]. \tag{8}$$

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II. BAYESIAN METHOD OF ESTIMATION

The Bayesian estimation procedure have been developed generally under squared error loss function

$$L\left(\hat{R}(t),R(t)\right)=\left(\hat{R}(t)-R(t)\right)^2. \quad (9)$$

where $\hat{R}(t)$ is an estimate of $R(t)$. The Bayes estimator under the above loss function, say $\hat{R}(t)_S$, is the posterior mean, i.e.,

$$\hat{R}(t)_S=E\left[R(t)\right]. \quad (10)$$

The squared error loss function is often used also because it does not lead extensive numerical computation but several authors (Zellner [2], Basu & Ebrahimi [3]) have recognized the inappropriateness of using symmetric loss function. Canfield [4] points out that the use of symmetric loss function may be inappropriate in the estimation of reliability function. Norstrom [5] introduced an alternative asymmetric precautionary loss function and also presented a general class of precautionary loss function with quadratic loss function as a special case. A very useful and simple asymmetric precautionary loss function is

$$L\left(\hat{R}(t),R(t)\right)=\frac{\left(\hat{R}(t)-R(t)\right)^2}{\hat{R}(t)} \quad (11)$$

The Bayes estimator of $R(t)$ under precautionary loss function is denoted by $\hat{R}(t)_P$, and is obtained by solving the following equation

$$\hat{R}(t)_P=\left[E\left(R(t)\right)^2\right]^{\frac{1}{2}}. \quad (12)$$

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{R}(t)}{R(t)}$. In this case, Calabria and Pulcini [6] points out that a useful asymmetric loss function is the entropy loss $L(\delta)\propto\left[\delta^p-p\ log_e(\delta)-1\right]$, where $\delta=\hat{R}(t)/R(t)$, and whose minimum occurs at $\hat{R}(t)=R(t)$ when $p>0$, a positive error $\left(\hat{R}(t)>R(t)\right)$ causes more serious consequences than negative error, and vice-versa. For small $|p|$ value, the function is almost symmetric when both $\hat{R}(t)$ *and* $R(t)$ are measured in a logarithmic scale, and approximately

$$L(\delta)\propto\frac{p^2}{2}\left[log_e\hat{R}(t)-log_e R(t)\right]^2.$$

Also, the loss function $L(\delta)$ has been used in Dey et al. [7] and Dey and Liu [8], in the original form having $p=1$. Thus $L(\delta)$ can be written as

$$L(\delta)=b\left[\delta-log_e(\delta)-1\right];\quad b>0. \quad (13)$$

The Bayes estimator of $R(t)$ under entropy loss function is denoted by $\hat{\theta}_E$ and is obtained as

$$\hat{R}(t)_E=\left[E\left(\frac{1}{R(t)}\right)\right]^{-1}. \quad (14)$$

For the situation where we have no prior information about $R(t)$, we may use non-informative prior distribution

$$h_1\left(R(t)\right)=\frac{1}{R(t)\ logR(t)};\quad 0<R(t)\le 1. \quad (15)$$

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The most widely used prior distribution for $R(t)$ is a beta distribution with parameters $\alpha,\beta>0,$ given by

$$h_2\left(R(t)\right)=\frac{1}{B(\alpha,\beta)}\left[R(t)\right]^{\alpha-1}\left[1-R(t)\right]^{\beta-1};\quad 0<R(t)\leq 1. \tag{16}$$

3. Bayes estimators of $R(t)$ under $h_1\left(R(t)\right)$

Under $h_1\left(R(t)\right)$, the posterior distribution is defined by

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{f\left(\underline{x}\,|\,R(t)\right)h_1\left(R(t)\right)}{\int_0^1 f\left(\underline{x}\,|\,R(t)\right)h_1\left(R(t)\right)dR(t)} \tag{17}$$

Substituting the values of $h_1\left(R(t)\right)$ and $f\left(\underline{x}\,|\,R(t)\right)$ from equations (15) and (6) in (17), we get

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{\left[\prod_{i=1}^{n}\left[\frac{ax_i^{a-1}\left(\lambda+x_i^a\right)}{log\left(1+\left(t^a/\lambda\right)\right)}\right]\left[-log\,R(t)\right]^n\left[R(t)\right]^{\frac{\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)}{log\left(1+\left(t^a/\lambda\right)\right)}}\frac{1}{R(t)\;log\,R(t)}\right]}{\int_0^1\left[\prod_{i=1}^{n}\left[\frac{ax_i^{a-1}\left(\lambda+x_i^a\right)}{log\left(1+\left(t^a/\lambda\right)\right)}\right]\left[-log\,R(t)\right]^n\left[R(t)\right]^{\frac{\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)}{log\left(1+\left(t^a/\lambda\right)\right)}}\frac{1}{R(t)\;log\,R(t)}\right]dR(t)}$$

$$=\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)-1}\left[\text{-}log\,R(t)\right]^{n-1}}{\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)-1}\left[\text{-}log\,R(t)\right]^{n-1}dR(t)}$$

or, $$f\left(R(t)\,|\,\underline{x}\right)=\frac{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)}\left[R(t)\right]^{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)-1}\left[\text{-}log\,R(t)\right]^{n-1} \tag{18}$$

Theorem 1. Assuming the squared error loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_S=\left(1+\frac{log\left(1+\left(t^a/\lambda\right)\right)}{\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)}\right)^{-n} \tag{19}$$

Proof. From equation (10), on using (18),

$$\hat{R}(t)_S=E\left[R(t)\right]=\int_0^1 R(t)\,f\left(R(t)\,|\,\underline{x}\right)dR(t)$$

$$=\int_0^1 R(t)\frac{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)}\left[R(t)\right]^{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)-1}\left[\text{-}log\,R(t)\right]^{n-1}dR(t)$$

$$=\frac{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)}\int_0^1\left[R(t)\right]^{\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)}\left[\text{-}log\,R(t)\right]^{n-1}dR(t)$$

$$=\frac{\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)}\frac{\Gamma(n)}{\left(\left(\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)/log\left(1+\left(t^a/\lambda\right)\right)\right)+1\right)^n}$$

or, $$\hat{R}(t)_S=\left(1+\frac{log\left(1+\left(t^a/\lambda\right)\right)}{\sum_{i=1}^{n}log\left(\lambda+x_i^a\right)}\right)^{-n}.$$

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Theorem 2. Assuming the precautionary loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_P = \left[1 + \frac{2\log\left(1+\left(t^a/\lambda\right)\right)}{\sum_{i=1}^{n}\log\left(\lambda + x_i^a\right)}\right]^{-\frac{n}{2}}. \tag{20}$$

Proof. From equation (12), on using (18),

$$\hat{R}(t)_P = \left[E\left(R(t)\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\int_0^1 \left(R(t)\right)^2 f\left(R(t \mid \underline{x})\right) dR(t)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^1 \left(R(t)\right)^2 \frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)} \left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)-1} \left[-\log R(t)\right]^{n-1} dR(t)\right]^{\frac{1}{2}}$$

$$= \left[\frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)} \int_0^1 \left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+1} \left[-\log R(t)\right]^{n-1} dR(t)\right]^{\frac{1}{2}}$$

$$= \left[\frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)} \frac{\Gamma(n)}{\left(\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+2\right)^n}\right]^{\frac{1}{2}}$$

$$= \left[\frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\left(\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+2\right)^n}\right]^{\frac{1}{2}}$$

or,
$$\hat{R}(t)_P = \left[1 + \frac{2\log\left(1+\left(t^a/\lambda\right)\right)}{\sum_{i=1}^{n}\log\left(\lambda + x_i^a\right)}\right]^{-\frac{n}{2}}.$$

Theorem 3. Assuming the entropy loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_E = \left[1 - \frac{\log\left(1+\left(t^a/\lambda\right)\right)}{\sum_{i=1}^{n}\log\left(\lambda + x_i^a\right)}\right]^{n} \tag{21}$$

Proof. From equation (14), on using (18),

$$\hat{R}(t)_E = \left[E\left(\frac{1}{R(t)}\right)\right]^{-1}$$

$$= \left[\int_0^1 \frac{1}{R(t)} f\left(R(t \mid \underline{x})\right) dR(t)\right]^{-1}$$

$$= \left[\int_0^1 \frac{1}{R(t)} \frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)} \left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)-1} \left[-\log R(t)\right]^{n-1} dR(t)\right]^{-1}$$

$$= \left[\frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)} \int_0^1 \left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)-2} \left[-\log R(t)\right]^{n-1} dR(t)\right]^{-1}$$

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$$=\left[\frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\Gamma(n)}\ \frac{\Gamma(n)}{\left(\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)-1\right)^n}\right]^{-1}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)^n}{\left(\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)-1\right)^n}\right]^{-1}$$

or, $$\hat{R}(t)_E=\left[1-\frac{\log\left(1+\left(t^a/\lambda\right)\right)}{\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)}\right]^n .$$

4. Bayes estimators of $R(t)$ under $h_2\left(R(t)\right)$

Under $h_2\left(R(t)\right)$, the posterior distribution is defined by

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{f\left(\underline{x}\,|\,R(t)\right)h_2\left(R(t)\right)}{\int_0^1 f\left(\underline{x}\,|\,R(t)\right)h_2\left(R(t)\right)dR(t)} \tag{22}$$

Substituting the values of $h_2\left(R(t)\right)$ and $f\left(\underline{x}\,|\,R(t)\right)$ from equations (16) and (6) in (22), we get

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{\left[\prod_{i=1}^{n}\left[\frac{a x_i^{a-1}\left(\lambda+x_i^a\right)}{\log\left(1+\left(t^a/\lambda\right)\right)}\right]\left[-\log R(t)\right]^n\left[R(t)\right]^{\frac{\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)}{\log\left(1+\left(t^a/\lambda\right)\right)}}\times\frac{1}{B(\alpha,\beta)}\left[R(t)\right]^{\alpha-1}\left[1-R(t)\right]^{\beta-1}\right]}{\int_0^1\left[\prod_{i=1}^{n}\left[\frac{a x_i^{a-1}\left(\lambda+x_i^a\right)}{\log\left(1+\left(t^a/\lambda\right)\right)}\right]\left[-\log R(t)\right]^n\left[R(t)\right]^{\frac{\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)}{\log\left(1+\left(t^a/\lambda\right)\right)}}\times\frac{1}{B(\alpha,\beta)}\left[R(t)\right]^{\alpha-1}\left[1-R(t)\right]^{\beta-1}\right]dR(t)}$$

$$=\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha-1}\left[-\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha-1}\left[-\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}dR(t)}$$

or,

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha-1}\left[-\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}\right]} \tag{23}$$

Theorem 4. Assuming the squared error loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_S=\left[\frac{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+1+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}}\right] \tag{24}$$

Proof. From equation (10), on using (23),

$$\hat{R}(t)_S=E\left[R(t)\right]$$

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$$=\int_0^1 R(t)\, f\left(R(t)\,|\,\underline{x}\right) dR(t)$$

$$=\int_0^1 R(t)\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha-1}\left[\text{-}\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}\right]}dR(t)$$

$$=\frac{\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha}\left[\text{-}\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}dR(t)}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}\right]}$$

or, $$\hat{R}(t)_S=\left[\frac{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+1+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}}\right].$$

Theorem 5. Assuming the precautionary loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_P=\left[\frac{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+2+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}}\right]^{\frac{1}{2}} \tag{25}$$

Proof. From equation (12), on using (23),

$$\hat{R}(t)_P=\left[E\left(R(t)\right)^2\right]^{\frac{1}{2}}$$

$$=\left[\int_0^1\left(R(t)\right)^2 f\left(R(t\,|\,\underline{x})\right)dR(t)\right]^{\frac{1}{2}}$$

$$=\left[\int_0^1\left(R(t)\right)^2\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha-1}\left[\text{-}\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}\right]}dR(t)\right]^{\frac{1}{2}}$$

$$=\left[\frac{\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+1}\left[\text{-}\log R(t)\right]^n\left[1-R(t)\right]^{\beta-1}dR(t)}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}\right]}\right]^{\frac{1}{2}}$$

or, $$\hat{R}(t)_P=\left[\frac{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+2+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}}\right]^{\frac{1}{2}}.$$

Theorem 6. Assuming the entropy loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_E=\left[\frac{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\log\left(\lambda+x_i^a\right)/\log\left(1+\left(t^a/\lambda\right)\right)\right)+\alpha-1+k\right)^{n+1}}\right] \tag{26}$$

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Proof. From equation (14), on using (23),

$$\hat{R}(t)_E = \left[E\left(\frac{1}{R(t)} \right) \right]^{-1}$$

$$= \left[\int_0^1 \frac{1}{R(t)} f\left(R(t \mid \underline{x}) \right) dR(t) \right]^{-1}$$

$$= \left[\int_0^1 \frac{1}{R(t)} \frac{[R(t)]^{\left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1+\left(t^a/\lambda\right)\right) \right) + \alpha - 1} \left[-log\, R(t) \right]^n \left[1 - R(t) \right]^{\beta-1}}{\Gamma(n+1) \left[\sum_{k=0}^{\beta-1} (-1)^k \binom{\beta-1}{k} \left(1 \Big/ \left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1 + \left(t^a/\lambda\right)\right) \right) + \alpha + k \right)^{n+1} \right]} dR(t) \right]^{-1}$$

$$= \left[\frac{\int_0^1 [R(t)]^{\left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1+\left(t^a/\lambda\right)\right) \right) + \alpha - 2} \left[-log\, R(t) \right]^n \left[1 - R(t) \right]^{\beta-1} dR(t)}{\Gamma(n+1) \left[\sum_{k=0}^{\beta-1} (-1)^k \binom{\beta-1}{k} \left(1 \Big/ \left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1 + \left(t^a/\lambda\right)\right) \right) + \alpha + k \right)^{n+1} \right]} \right]^{-1}$$

$$= \left[\frac{\sum_{k=0}^{\beta-1} (-1)^k \binom{\beta-1}{k} \left(1 \Big/ \left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1 + \left(t^a/\lambda\right)\right) \right) + \alpha - 1 + k \right)^{n+1}}{\sum_{k=0}^{\beta-1} (-1)^k \binom{\beta-1}{k} \left(1 \Big/ \left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1 + \left(t^a/\lambda\right)\right) \right) + \alpha + k \right)^{n+1}} \right]^{-1}$$

or,

$$\hat{R}(t)_E = \left[\frac{\sum_{k=0}^{\beta-1} (-1)^k \binom{\beta-1}{k} \left(1 \Big/ \left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1 + \left(t^a/\lambda\right)\right) \right) + \alpha + k \right)^{n+1}}{\sum_{k=0}^{\beta-1} (-1)^k \binom{\beta-1}{k} \left(1 \Big/ \left(\sum_{i=1}^{n} log\left(\lambda + x_i^a\right) / log\left(1 + \left(t^a/\lambda\right)\right) \right) + \alpha - 1 + k \right)^{n+1}} \right].$$

III. CONCLUSION

We have obtained a number of Bayes estimators of reliability function $R(t)$ of power Lomax distribution. In equations (19), (20), and (21), we have obtained the Bayes estimators by using non-informative prior and in equations (24), (25), and (26), under beta prior. From the above said equation, it is clear that the Bayes estimators of $R(t)$ depend upon the parameters of the prior distribution. In this case the risk function and corresponding Bayes risks do not exist.

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