Optimal Solution for Solving Intuitionistic Fuzzy Assignment Problem by New Labeling Algorithm

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Abstract: **In this paper a new labeling algorithm for finding an optimal solution for an intuitionistic triangular fuzzy optimal assignment problem is proposed. The main feature of this algorithm is that we determine the solution by finding an optimal matching in the corresponding bipartite graph by using a new labeling technique.**

Keywords*: — *Assignment Problem, labeling Algorithm, feasible labeling, bipartite graph, Ranking of Intuitionistic triangular fuzzy numbers.

I. INTRODUCTION

Assignment Problem (AP) is used worldwide in solving real world problems. An assignment problem plays an important role in an assigning of persons to jobs, or classes to rooms, operators to machines, drivers to trucks, trucks to routes, or problems to research teams, etc. The assignment problem is a special type of linear programming problem (LPP) in which our objective is to assign n number of jobs to n number of machines (persons) at a minimum cost. Find solution of assignment problems in various algorithms such as linear programming, Hungarian algorithm, Neural network, Genetic Algorithm, Branch and Bounded Technique etc. The proposed Algorithm, in spite of its unfamiliar and peculiar accessories, is a much faster and more efficient tool to handle the Assignment problem than the Hungarian and genetic algorithm. However, in real life situations, the parameters of assignment problem are imprecise numbers instead of fixed real numbers because time/cost for doing a job by a facility (machine/persion) might vary due to different reasons. The theory of fuzzy set introduced by Zadeh [14] in 1965 has achieved successful applications in various fields. Chi-Jen Lin, Ue-Pyng Wen [3], A Labelling algorithm for the fuzzy assignment problem, fuzzy Sets and system. Srinivas.B and Sankara rao. B [9]" An Optimal Solution for intuitionistic Fuzzy Assignment Problem Using Genetic Algorithm. Hussain, RJ and SenthilKumar [5], P. Algorithm approach for solving intuitionistic fuzzy transportation problem. Kalaiarasi et.al [6] Optimization of fuzzy assignment model with triangular fuzzy numbers. Different kinds of fuzzy assignment problems are solved in the papers [2,4,7,8].

In Section 2, we provide some basic definitions and results which will be used later. Section 3, Mathematical form of intuitionistic fuzzy Assignment problem, In Section 4, we prove some theorems which are used for proposed method and present a practical procedure. The introduced method is illustrated by solving some examples in Section 5 and conclusions are drawn in Section 6.

II. PRELIMINARIES

Fuzzy Set: Fuzzy sets were introduction by Lotfi A. Zadeh in 1965 as an extension of the classical set theory. Fuzzy sets are sets whose elements have degrees of member ship function valued in the real unit interval [0, 1]

Let A be a classical set $\mu_{\tilde{A}}$ be a function from X to [0, 1] A fuzzy set $\tilde{A}$ is defined as a set of ordered pairs $\{(x_i, \mu_{\tilde{A}}(x_i)) : x_i \in A$ and $\mu_{\tilde{A}}(x) \in [0, 1]$ $\mu_{\tilde{A}}(x)$ is called the degree of membership of X in $\tilde{A}$.

Fuzzy Number: A fuzzy set $\tilde{A}$ defined on the universal set of real numbers R is said to be fuzzy number if

(i) $\tilde{A}$ is convex fuzzy set

(ii) $\tilde{A}$ is normalized fuzzy set

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(iii) it's membership function is piecewise continuous

Intuitionistic Fuzzy Number (IFN): An intuitionistic fuzzy set of real line R is an Intuitionistic fuzzy number if the following conditions holds

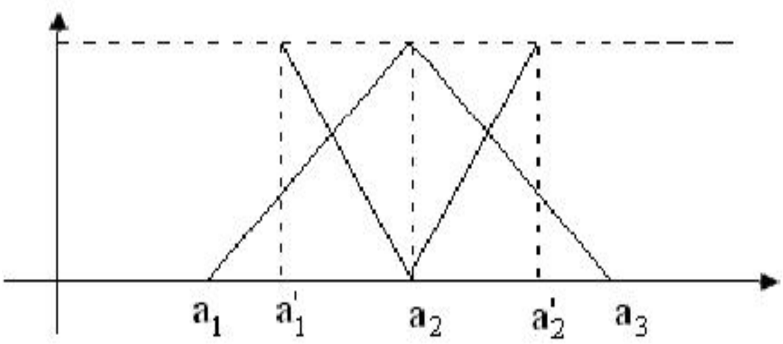
i) There exists m$\in R$such that $\mu_{\tilde{A}^I}$ (m) =1and $\vartheta_{\tilde{A}^I}$ (m) =0

ii) $\mu_{\tilde{A}^I}$ is a continuous function from R → [0, 1] such that $0 \leq \mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \leq 1$ all x ∈X

Triangular Intuitionistic Fuzzy Number (TIFN): $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}$ (x) and non-membership function $\vartheta_{\tilde{A}^I}$ (x)

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & x \leq a_1 \\ \dfrac{x-a_1}{a_2-a_1'} & for\, a_1 \leq x \leq a_2 \\ 1 & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & for\, a_2 \leq x \leq a_3 \\ 0 & for\, x > a_3 \end{cases} \qquad \vartheta_{\tilde{A}^I}(x) = \begin{cases} 1 & x \leq a_1^{1} \\ \dfrac{x-a_1}{a_2-a_1'} & for\, a_1' \leq x \leq a_2 \\ 0 & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & for\, a_2 \leq x \leq a_3' \\ 1 & for\, x > a_3' \end{cases}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\mu_{\tilde{A}^I}(x), V_{\tilde{A}^I}(x) \leq 0.5$ for $\mu_{\tilde{A}^I}(x) = V_{\tilde{A}^I}(x)$ for all x$\in X$



III. MATHEMATICAL FORM OF INTUITIONISTIC FUZZY ASSIGNMENT PROBLEM

Suppose there are n jobs to be performed and n persons are available for doing the jobs. Assume that each person can do each job at a time, depending on their efficiency to do the job. Let $\widetilde{c_{ij}}$ be the effectiveness of the i^{th} person in doing the j^{th} job, where $\widetilde{c_{ij}}$ $'s$ are generalized intuitionistic fuzzy numbers. The objective is to assign all the jobs to the available persons (one job to one person) in a most effective way. Mathematically the problem can be stated as

Mathematically, we can express the problem as follows:

$$\text{(AP)} \quad \text{Minimize} \quad z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij} \otimes \tilde{x}_{ij}$$

Subject to $\sum_{j=1}^{n} \tilde{x}_{ij} \approx 1$ for i=1,2,………n i.e. j^{th} work will be done only by one person.

$\sum_{i=1}^{n} \tilde{x}_{ij} \approx 1$, for j=1, 2,……,n i.e. i^{th} person will do only one work

Where $x_{ij} = \begin{cases} 0 \;\; if\; j^{th} \;\; job\; is\; not\; assign\; to\; i^{th} \;\; mechine \\ 1 \;\; if\; j^{th} \;\; job\; is\;\; assign\; to\; i^{th} \;\; mechine \end{cases}$

Otherwise is the decision variable denoting the assignment of the person i to job j. $\widetilde{c_{ij}}$ is the effectiveness of the i^{th} person in doing the j^{th} job.

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Here we will consider an assignment problem in the following way: Let G be a complete bipartite graph with bipartition $V(G) = X \cup Y$, where $X = \{x_1, x_2, \ldots\ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ have both the same number of elements and, for each i and j, the weight w_{ij} is given to the edge joining x_i to y_j. This corresponds to a situation where there are the same number of workers as there are jobs, each worker can do each job and w_{ij} denotes the effectiveness of worker x_i in job y_j. The optimal assignment problem is then to find a maximum-weight perfect matching in this weighted graph. Such a matching will be called an optimal matching**.**

IV. PROPOSED ALGORITHM

First of all the following definitions and theorems are introduce present an algorithm for finding an optimal matching. It uses a labeling technique.

Definition:

If we label each vertex v of a weighted complete bipartite graph G with a real number $\lambda(v)$, then this labeling is called a $feasible$ $vertex\ labeling$ of G if, for all x_i in X and y_j in Y,

$(x_i) + (y_j) \geq w_{ij}$, the weight of edge $x_i y_j$.

Thus a feasible vertex labeling is a labeling of the vertices of G such that the sum of the labels of the two ends of an edge is never less than the weight of the edge. We can always produce a feasible vertex labeling λ for G by defining

$\lambda\,(x_i) = \max\{w_{ij},: 1 \leq j \leq n\}$ if $x_i \in X$

$\lambda\,(y_j) = 0$, if $y_j \in Y$

In other words, we just label 0 to all vertices in Y and, for any $xi \in X(x_i)$ is the maximum of all the weights of the edges incident with x_i

Definition: Given a feasible vertex labeling λ for G and we let E_λ denote the set of those edges $x_i y_j$ for which $(x_i) + (y_j) = w_{ij}$, i.e., $E_\lambda = \{x_i y_j : \lambda(x_i) + \lambda(y_j) = w_{ij}\}$, where $x_i y_j$ denotes the unique edge from $x_i$to y_j. Then the spanning sub graph of G with edge set E_λ is called the equality sub graph corresponding to λ and is denoted by G_λ.

The proposed algorithm is based on the following theorem.

Theorem: Let λ be a feasible vertex labeling for the weighted complete bipartite graph G. If the equality sub graph G_λ has a perfect matching M^* then M^* is an optimal matching for G.

Proof: Let M^* be a perfect matching for the subgraph G_λ. Since G_λ is a spanning subgraph of G M^* is also perfect matching for G. Now the weight of M^*, $w(M^*)$, is the sum of the weights of all its degrees, i.e.,

$w(M^*) = \sum_{e \in M^*} \mathrm{w(e)}$, and since each edge e in M^* is in the equality subgraph G_λ we have, for such an e , $w(e) = \lambda(x) + \lambda(y)$ where x, y are two end vertices of e. Moreover, since M^* is a perfect matching of G, its set of edges involves all the vertices of (once and only once).

Thus $(M^*) = \sum_{e \in M^*} \mathrm{w(e)} = \sum_{i=1}^{n} (\lambda(x_i) + \lambda(y_j))$ . . . . (i)

On the other hand, if M is any perfect matching of G, then

$w(M) = \sum_{e \in \mathrm{M}} \mathrm{w(e)} \leq \sum_{i=1}^{n} (\lambda(x_i) + \lambda(y_j))$ . . .. (ii)

(again since M is perfect it involves all the vertices of G once and only once but, for any edge e in M

we can only say $w(e) \leq \lambda(x) + \lambda(y)$, instead of $w(e) = \lambda(x) + \lambda(y)$).

Combining (i) and (ii) we get $(M^*) \geq (M)$. Since M was an arbitrary perfect matching of G, it follows that M^* is an optimal matching for G, as required.

We use the above theorem to develop an algorithm for finding an optimal matching in a weighted complete bipartite graph.

Algorithm

Let $x_1, x_2, \ldots\ , x_n$ be the n persons and $y_1, y_2, \ldots, y_n$ be the n jobs to be performed. Also let $\widetilde{c_{ij}}$ be the effectiveness of the i^{th} person in doing the j^{th} job, where $\widetilde{c_{ij}}$ $'s$ are generalized intuitionistic fuzzy numbers. We perform the following steps for solving the optimal assignment problem.

Step 1: From the given assignment problem construct a weighted complete bipartite graph G

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with bipartition $(G) = X \cup Y$, where $X = \{x_1, x_2, \dots, x_n\}$ is the set of persons,

$Y = \{y_1, y_2, \dots, y_n\}$ is the set of jobs to be done and the weight w_{ij} of the edge $x_i y_j$ is

$\Re(\widetilde{c_{ij}})$. $\Re(\widetilde{c_{ij}})$ can be calculated by using a ranking method.

Step 2: Start with an arbitrary feasible vertex labeling λ for G, determine the equality subgraph G_λ and choose an arbitrary matching M in G_λ.

Step 3: If X is M-saturated, then M is a perfect matching in G_λ as $|X| = |Y|$. Then by the given theorem M is a perfect matching for G. In this case, stop. Otherwise, let x_0 be an M-unsaturated vertex in G_λ. Set $S = \{x_0\}$, $T = \emptyset$.

Step 4: If inG_λ, $NG_\lambda(S) \neq T$ go to Step 5. Otherwise $NG_\lambda(S) = T$. Compute the defect d_λ of λ, which is defined by

$d_\lambda = \max\{\lambda(x) + \lambda(y) - w(xy): x \in S, y \notin T\}$.

Then, using the defect, compute the new vertex labeling λ' as follows:

$\lambda'(v) = (v) + d_\lambda$ if $v \in S$

$= d_\lambda - (v) +$ if $v \in T$

$= (v)$ if $v \notin S \cup T$

Replace λ by λ' and G_λ by $G_{\lambda'}$ and then rename $G_{\lambda'}$ as G_λ.

(At this stage, in $G_{\lambda'}$ we will have $NG_{\lambda'}(S) \neq T$ as at the first line in this step.)

Step 5: Choose a vertex y in $NG_\lambda(S)$ which is not in T. Then there arise two cases:

(i) If y is M-saturated, with $zy \in M$, replace S by $S \cup \{z\}$ and T by $T \cup \{y\}$ and return to Step 4.

(ii) If y is M-unsaturated, let P be the M-augmenting path from x_0 to y and replace M by the transfer along P of M. Then return to Step 3.

Remarks: (i) If the problem is of minimization type then find the rank of each element of the chosen fuzzy matrix $[\widetilde{c_{ij}}]$ by using a ranking procedure and determine the element with the highest rank. Subtract each element of the cost matrix from this element. Then the problem with the modified matrix is a maximization problem.

(ii) Sometimes technical, legal or other restrictions do not permit the assignment of a particular facility to a particular job. In such cases also we use the algorithm by assigning a very small intuitionistic fuzzy cost to the cells which do not permit the assignment so that the activity will be automatically excluded from the optimal solution.

V. NUMERICAL EXAMPLE: (INTUITIONISTIC TRIANGULAR FUZZY NUMBER)

Consider an intuitionistic fuzzy optimal assignment problem with 4 persons and 4 jobs, where in the effectiveness matrix $[\widetilde{c_{ij}}]$ the rows represent the 4 persons A, B, C,D and columns represent the 4 jobs J_1, J_2, J_3 and J_4.The entries of the effectiveness matrix $[\widetilde{c_{ij}}]$ are triangular intuitionistic fuzzy numbers. The objective is to assign all the jobs to the available persons (one job to one person) in a most effective way

person	Jobs			
	J_1	J_2	J_3	J_4
A	(2,4,7;1,4,8)	(1,3,4;0,3,5)	(2,3,6;1,3,7)	(3,5,8;2,5,13)
B	(3,6,10;2,6,11)	(2,3,5;1,3,6)	(3,7,10;2,7,12)	(6,7,11;3,7,12)
C	(1,3,4;0,3,5)	(2,3,5;1,3,6)	(3,6,10;2,6,11)	(2,3,6;1,3,7)
D	(3,7,10;2,7,12)	(2,3,4;-1,3,5)	(2,3,5;1,3,6)	(3,6,10;2,6,11)

Solution: The above intuitionistic fuzzy assignment problem can be formulated in the following mathematical form

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Mim(2,4,7;1,4,8)x_{11}+(1,3,4;0,3,5)x_{12}+(2,3,6;1,3,7)x_{13}+(3,5,8;2,5,13) x_{14}+(3,6,10;2,6,11)x_{21} +(2,3,5;1,3,6) x_{22}+(3,7,10;2,7,12) x_{23}+(6,7,11;3,7,12) x_{24}+(1,3,4;0,3,5)x_{31} +(2,3,5;1,3,6)x_{32}+(3,6,10;2,6,11)x_{33}+(2,3,6;1,3,7) x_{34}+ (3,7,10;2,7,12)x_{41}+ (2,3,4;-1,3,5)x_{42}+ (2,3,5;1,3,6)x_{43} + (3,6,10;2,6,11)x_{44}]

Subject to

$x_{11}+x_{12}+x_{13}+x_{14}=1$ $x_{11}+x_{21}+x_{31}+x_{41}=1$ $x_{21}+x_{22}+x_{23}+x_{24}=1$

$x_{12}+x_{22}+x_{32}+x_{42}=1$ $x_{31}+x_{32}+x_{33}+x_{34}=1$ $x_{13}+x_{23}+x_{33}+x_{43}=1$

$x_{41}+x_{42}+x_{43}+x_{44}=1$ $x_{14}+x_{24}+x_{34}+x_{44}=1$ where x_{ij} ∈[0,1]

$$R(\grave{A}^{I}) = \frac{1}{3}\left[\frac{(a_3' - a_1')(a_2 - 2a_3' - 2a_1') + (a_3 - a_1)(a_1 + a_2 + a_3) + 3(a_3'^{2} - a_1'^{2})}{a_3' - a_1' + a_3 - a_1}\right]$$

Calculated $R(\tilde{C}_{11})$ =5.11≈ (5)$R(\tilde{C}_{12})$ = 3.04 $R(\tilde{C}_{13})$ = 6.16 $R(\tilde{C}_{14})$ =8.08

$R(\tilde{C}_{21})$ =7.08$R(\tilde{C}_{22})$ = 4.16$R(\tilde{C}_{23})$ = 9.12 $R(\tilde{C}_{24})$ = 10.01

$R(\tilde{C}_{31})$ = 3.04 $R(\tilde{C}_{32})$ =4.16 $R(\tilde{C}_{33})$ =7.08 $R(\tilde{C}_{34})$ = 6.16

$R(\tilde{C}_{41})$ = 9.12 $R(\tilde{C}_{42})$ = 2.24 $R(\tilde{C}_{43})$ = 11.04 $R(\tilde{C}_{44})$ = 7.08

person	Jobs			
	J_1	J_2	J_3	J_4
A	5.16	3.04	6.16	8.08
B	7.08	4.16	9.12	10.01
C	3.04	4.16	7.08	6.16
D	9.12	2.24	11.04	7.08

Let G be the weighted complete bipartite graph with bipartition $(G) = X \cup Y$,

Where $X = \{A,B, C,D\} = \{x_1, x_2, x_3, x_4\}$, $Y = \{1,2,3,4\} = \{y_1, y_2, y_3, y_4\}$ and the weight $w_{ij} = \Re(\tilde{C}_{ij})$) is given to the edge joining x_i to y_j .

Here $G = k_{44}$. So G has a perfect matching.

We start with a feasible vertex labeling λ for G which is given by $\lambda(x_i) = \max\{ w_{ij}: 1 \le j \le n\}$ if $x_i \in X$ $\lambda(y_j) = 0$, if $y_j \in Y$ That is,

$\lambda(x_1) = 8.08$, $\lambda(x_2) = 10.01$, $\lambda(x_3) = 7.08$, $\lambda(x_4) = 11.04$

$\lambda(\{ y_1) = 0$, $\lambda(y_2) = 0$, $\lambda(y_3) = 0$, $\lambda(y_4) = 0$

Then $E_\lambda = \{x_i y_j : \lambda(x_i) + \lambda(y_j) = w_{ij}\} = \{x_1y_4, x_2y_4, x_3y_3, x_4y_3\}$

The corresponding equality sub graph G_λ is

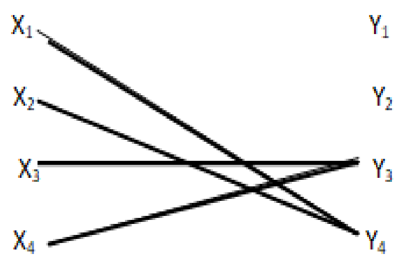

Figure 4.2

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Consider the matching $M = \{ x_1y_4 , x_3y_3, x_2y_4 \}$ in G_λ . Here X is M-unsaturated.
Consider the M-unsaturated vertex x_1 in G_λ. Set $S = \{x_2\}$, $T = \emptyset$. In G_λ , NG_λ $(S) = \{y_4\} \neq T$.
Choose the vertex y_4 in $NG\lambda\,(S)$ which is not in T. Here y_4 is M-saturated with $x_2y_4 \in M$.
So replace S by $S \cup \{x_1\} = \{x_1, x_2\}$ and T by $T \cup \{y_4\} = \{y_4\}$. Then NG_λ $(S) = T$. Since NG_λ $(S) = T$, we compute the defect d_λ of λ, which is given by

$d_\lambda = \max\{\lambda(x) + \lambda(y) - w(xy): x \in S, y \notin T\} = \max \{2.92, 5.04, 1.92, 2.93, 5.85, 0.89\} = 0.89$

Then the new feasible vertex labeling λ' for G is given by $\lambda'(v) = (v) + d_\lambda$ if $v \in S$

$= d_\lambda - (v)$ if $v \in T$

$= (v)$ if $v \notin S \cup T$

That is, $\lambda'(x_1) = 13.93$, $\lambda'(x_2) = 15.86$, $\lambda'(x_3) = 12.93$, $\lambda'(x_4) = 16.89$
$\lambda'(y_1) = 0$, $\lambda'(y_2) = 0$, $\lambda'(y_3) = 0$, $\lambda'(y_4) = 5.85$
Then $E_{\lambda'} = \{x_iy_j : \lambda'(x_i) + \lambda'(y_j) = w_{ij}\} = \{x_1y_4 , x_2y_2, x_3y_3, x_2y_4\}$
The corresponding equality sub graph $G_{\lambda'}$ is,

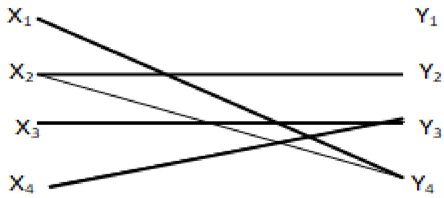

Figure - 4.3

Consider the matching $M' = \{x_1y_4 , x_2y_2, x_3y_3\}$ in $G_{\lambda'}$. Here X is M' - unsaturated.

Consider the M' - unsaturated vertex x_2 in y_2. Set $S = \{x_2\}$, $T = \emptyset$. In $G_{\lambda'}$, $NG_{\lambda'}$ $(S) = \{ y_2 \} \neq T$. Choose the vertex y_2 in $NG_{\lambda'}$ (S) which is not in T. Here y_2 is M' -saturated with $x_2y_2 \in M'$.

So replace S by $S \cup \{x_1\} = \{x_1, x_2\}$ and T by $T \cup \{y_2\} = \{y_2\}$.

Then $NG_{\lambda'}$ $(S) = \{y_2, y_4\} \neq T$. Since $NG_{\lambda'}$ $(S) \neq T$, choose a vertex y_4 in $NG_{\lambda'}$ (S) which is not in T.

Here y_4 is M' -saturated with $x_2y_2 \in M'$.

Replace S by $S \cup \{x_2\} = \{x_1, x_2, x_3\}$ and T by $T \cup \{y_4\} = \{y_2, y_4\}$

Now $NG_{\lambda'}$ $(S) = \{y_2, y_4\} = T$. Since $NG_{\lambda'}(S) = T$, we compute the defect $d_{\lambda'}$ of λ' which is given by

$d_{\lambda'} = \max \{\lambda'(x) + \lambda'(y) - w(xy): x \in S, y \notin T\}$

$= \max \{8.77, 7.77, 8.28, 6.74, 9.89, 5.85\} = 9.89$

Then the new feasible vertex labeling λ'' is

$\lambda''(x_1) = 23.82$, $\lambda''(x_2) = 25.75$, $\lambda''(x_3) = 22.82$, $\lambda''(x_4) = 26.78$

$\lambda''(y_1) = 0$, $\lambda''(y_2) = 9.89$, $\lambda''(y_3) = 0$, $\lambda''(y_4) = 4.04$

So $E_{\lambda''} = \{x_iy_j : \lambda''(x_i) + \lambda''(y_j) = w_{ij}\} = \{x_1y_4 , x_2y_2, x_3y_1, x_2y_4, x_3y_3 , x_2y_4\}$. The corresponding equality subgraph $G_{\lambda''}$ is

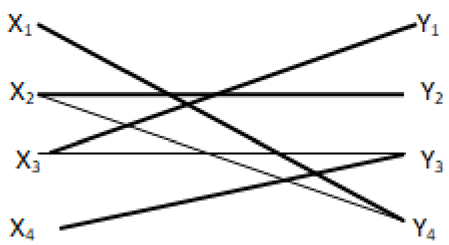

Figure - 4.4

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Consider the matching $M'' = \{x_3y_3, x_2y_2, x_3y_1\}$ in$G_{\lambda''}$. Here X is M'' -unsaturated.

Consider the M'' -unsaturated vertex $x_3$in $G_{\lambda''}$. Set $S = \{x_3\}$, $T = \emptyset$.

In $G_{\lambda''}$, $NG_{\lambda''}.\ (S) = \{y_2\} \neq T$.

Choose the vertex y_4 in $N\ G_{\lambda''}\ (S)$ which is not in T.

Here y_2 is M'' -saturated with$x_2y_2 \in M''$.

So replace S by $S \cup \{x_4\} = \{x_1, x_2\}$ and T by $T \cup \{y_2\} = \{y_2\}$.

Then $N\ G_{\lambda''}\ (S) = \{y_2, y_4\} \neq T$. $N\ G_{\lambda''}.\ (S) \neq T$, choose a vertex y_2 in$N\ G_{\lambda''}\ (S)$ which is not in T.

Here y_2 is M'' -saturated with $x_2y_2 \in M''$.

Replace S by $S \cup \{x_3\} = \{x_1, x_2, x_3\}$ and T by $T \cup \{y_2\} = \{y_2, y_4\}$.

Now $NG\lambda''\ (S) = \{y_1,\ y_2, y_4\} \neq T$.

Choose the vertex y_1 in $G_{\lambda''}'\ (S)$ which is not in T.

Here y_1 is M'' -saturated with $x_4y_1 \in M''$. So replace S by $S \cup \{x_4\} = \{x_1, x_2, x_3, x_4\}$ and T by $T \cup \{y_1\} = \{y_1, y_2, y_4\}$. Then $N\ G_{\lambda''}(S) = \{y_1, y_2, y_4\} = T$. Since $N\ G_{\lambda''}.\ (S) = T$,

we compute the defect $d_{\lambda''}$ of λ'', which is given by $d_{\lambda''} = \max\{\lambda''(x) + \lambda''(y) - w(xy)\colon x \in S,\ y \notin T\} = \max\{17.66, 16.63, 15.74, 15.74\} = 17.66$

Then the new feasible vertex labeling λ''' is

$\lambda'''(x_1) = 41.48$, $\lambda'''(x_2) = 43.41$, $\lambda'''(x_3) = 40.48$, $\lambda'''(x_4) = 44.44$

$\lambda'''(y_1) = 17.66$, $\lambda'''(y_2) = 7.77$, $\lambda'''(y_3) = 0$, $\lambda'''(y_4) = 13.62$

So $E_{\lambda'''} = \{x_iy_j\ : \lambda'''(xi) + \lambda'''(y_j) = w_{ij}\} = \{x_1y_4, x_2y_2, x_3y_1, x_2y_4, x_3y_3, x_2y_4, , x_1y_3\}$.

The corresponding equality sub graph λ''' is

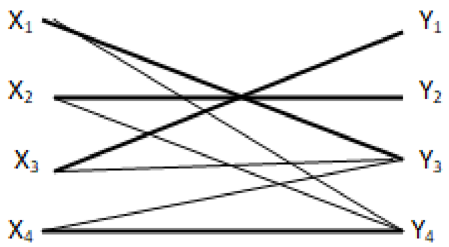

Figure - 4.5

Consider the matching $M''' = \{x_2y_2, x_3y_1, x_1y_3, x_4y_4\}$ in $G_{\lambda'''}$

Here X is M''' -saturated and remaining matching is x_4y_4

Hence M''' is an optimal matching for $G_{\lambda'''}$ and hence for G.

So the optimal assignment is as follows:

$X_1 \rightarrow Y_3$, $X_2 \rightarrow Y_2$, $X_3 \rightarrow Y_1$, $X_4 \rightarrow Y_4$

x_1y_3=6.16, x_2y_2=4.16, x_3y_1=3.04, x_4y_4=7.08

Optimal solution=**20.44**

VI. CONCLUSIONS

In this chapter, we can apply new Labeling algorithm for solving an Intuitionistic fuzzy assignment problem. The assignment cost has been considered as imprecise numbers described by triangle intuitionistic fuzzy numbers by transform the FIFAP into its equivalent crisp AP using the ranking procedure of Varghese and Kuriakose or Robust ranking and applying New Labeling

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algorithm. The problem is to find an optimal assignment so that the total cost of performing all jobs is minimum or the total profit is maximum. Finally, we deduce that the proposed method provides the better optimum to Instuitionstic fuzzy assignment problem and it can serve an important tool in decision making problem.

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