

Implementation of Wavelet based Thresholding Technique for Denoising an Image

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Abstract - The image naturally corrupted with noise is the most classical problem in the field of image processing. Random noise can be easily removed by utilizing simple threshold methods. De-noising of the natural images distorted by the Gaussian noise using wavelet technique is the very efficient technique because of its ability to capture the energy of a signal in few energy transform value. The wavelet based denoising scheme thresholds the wavelet coefficients arising from the discrete wavelet transform. The goal of denoising algorithm using different thresholding technique is to remove the unwanted noise from the text images and to preserve the edges as much as possible. Noise elimination introduces blur in the images. So image denoising is still a challenging task faced by investigators and several methods are being developed to remove the noise of a corrupted image. In this paper, it is proposed that the threshold value can be estimated by using Gradient Descent algorithm and Givens rotation algorithm for obtaining matrix inversion using QR Decomposition and compare the quality of de-noised image in terms of MSE and PSNR.

Keywords: Image, De-noising, Wavelet transform, Gradient descent, Givens rotation.

I. INTRODUCTION

One of the major problems in the image processing is dealing with the noisy images. This finds applications in image stitching and enhancement etc. There is a lot of development going in this domain because we need this algorithm to be implemented in handheld devices and also to make the implementation simpler and faster to execute. The Additive random noise is added to the original signal like for example $v[n]$ is the noise vector which is added to the original signal $s[n]$ by which it gives $x[n]$. This is the most commonly used noise model. Example for this noise model is Gaussian noise.

$$x[n] = v[n] + s[n]$$

Here, the proposed algorithm gives the effective threshold value. So, that should be multiplied to sub-band coefficients except for low frequency image and apply inverse wavelet transform and finally observed the performance values by using PSNR. This thesis involves two main modules.

- Wavelets transform using normal thresholding technique.
- Wavelets transform using Gradient descent algorithm for finding threshold value and the Givens rotation for the inversion of a matrix.

There are several algorithms available for all the mentioned steps but we choose the most optimum algorithms available. For the image denoising we used the algorithm called as

Wavelet transform. For text extraction we used algorithms separately to find the best among many algorithms existing.

The key point needs the median filter for removing the noise. It should also provide optical sharpening so as that it gives the best correspondence to human vision. Out of the options available to us we select Gaussian noise and median filter. It is because it provides the best optical sharpening and also is very efficient in eliminating low contrast edges. De-noising of the natural images distorted by the Gaussian noise using wavelet technique is the very efficient technique because of its ability to capture the energy of a signal in few energy transform value. The wavelet based denoising scheme thresholds the wavelet coefficients arising from the discrete wavelet transform. De-noising algorithm using wavelet transform consists of three main steps.

- Calculate the Haar wavelet transform for the noisy image.
- Modify the Haar wavelet coefficients by using some thresholding methods.
- Compute the inverse wavelet transform using the modified co-efficients.

One of the most well known methods for the second step is hard thresholding Due to its effectiveness and it is very useful for our application because hard threshold preserve edges. The main idea behind this hard thresholding is the keep or kill rule according to which if the threshold value is larger the input is kept as it is else set it to zero or null. In this paper it is proposed to investigate the suitability of

different thresholding techniques and the performance of the quality of the image is measured using PSNR.

II. IMAGE DENOISING VERSUS IMAGE ENHANCEMENT

Image denoising is not quite the same as Image enhancement. As Gonzalez and Woods [1] clarify, image enhancement is a goal procedure, though image denoising is a abstract procedure. Image denoising is a restoration process, where endeavors are made to recover an image that has been corrupted by utilizing prior knowledge of the degradation process. Image enhancement, then again, involves manipulation of the image characteristics to make it more appealing to the human eye. There is some overlap between the two processes.

III. LITERATURE SURVEY

This literature review envelops in detail all the previous work reviewed during this research. To truly embrace denoising the Kannada text image it is important to understand the earlier works that preceded the chosen algorithms. During the process of looking for documentation on image denoising, a lot of work was found that addresses the early image denoising technique This is a good progress of their importance to this process. A large portion of the early executions created appeared to function admirably under certain restricted image condition. The real challenge for those authors was to achieve denoised image using different thresholding techniques.

This literature review aims to provide with an insight on what have been done, what is currently being studied and where the future work is pointing in the field of Image denoising.

[2] This paper focuses on types of noise present in the image processing and filters for removing noise in an image and analyzed that median filter and Gaussian noise is better to remove the noise due its normal distribution and due to its statistical noise having PDF which is equal to the Gaussian distribution as it is differentiable, Median filter is to sharpen the objects contained in the image.

[3] This paper displays Wavelet technique is very effective because of its ability to capture the energy of a signal in only few energy transform values. It is proposed to investigate the suitability of different wavelet bases, Among all the wavelet bases performs well in image denoising and discrete wavelet transform(DWT) is analyzed, on the performance of image denoising algorithms in terms of PSNR.

[4] DWT is a computerized technique to compute fast wavelet transform. Image denoising using DWT is analyzed and it is easier in operation and implementation. Discrete wavelet transforms can be used for image processing. The various types of DWT are Haar wavelet transform,

Daubechies wavelet transform (order 1,2,4 and 8), Symlets wavelet transform (order 1,2,4 and 8), Coeiflets wavelet transform (order 1 and 2) among which Haar wavelet is the simplest technique and it is based on decomposition principle.

IV. SYSTEM FLOW DIAGRAM

Image denoising is one of the important and essential components of image processing. The initial stage of the work is to capture the image containing text in it from the mobile camera and Gaussian noise is added to it where it is good noise model and then DWT is applied using Haar wavelet to decompose the image. Discrete wavelet transform (DWT) is used instead of Fast Fourier Transform (FFT), to get the sharp edges of denoised image as it is in the original image and to provide the time and frequency localization because in Fourier transform the temporal information is lost in the transformation process. Median filter is used to remove the unwanted noise from the image and thresholding method such as hard thresholding is used to preserve the edges in the image and recomposition is done by using Inverse discrete wavelet transform (IDWT)

V. DISCRETE WAVELET TRANSFORM

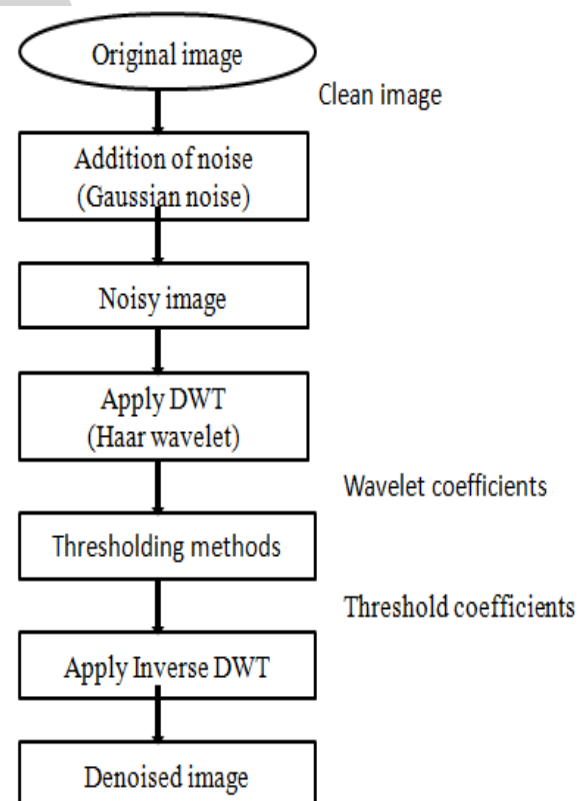
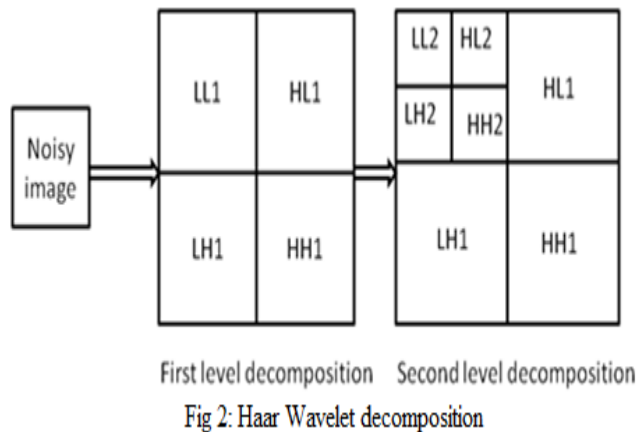


Fig 1: Flow diagram

A discrete wavelet transform (DWT) is a type of wavelet transform for which the wavelets are discretely sampled. It is an optimum solution for computational time overhead. It is easier in operation and implementation. This provides better spatial and spectral localization for image formation compared to other multi scale representation. DWT is used to reduce the size of an image without compromising on

quality and hence resolution increases. There are various types of DWT techniques among which Haar Wavelet transform is the simplest and which is most suitable for our application. It is a sequence of rescaled “square-shaped” functions which form a wavelet family or basis. After adding a Gaussian noise to a image we will get a Noisy image.

After applying Haar Wavelet Transform to Noisy image it undergoes N level of decomposition. Where as in the first level decomposition the image is split into 4 sub-bands, namely LL, LH, HL, HH sub-bands. The LL sub band is the low frequency component and its sub bands are further split at higher level decompositions. The LH and HL represents the vertical and horizontal details of the image. The HH sub-band represents the diagonal details of the image. As the level of decomposition increases the coefficients of the sub bands becomes smoother and the higher resolution is obtained. We can observe the first level and second level decomposition from the Fig 2:



VI. THRESHOLDING TECHNIQUES

Various methods are being used for removal of noise from the noisy image. Compared to the filter based techniques the thresholding techniques are way better.

Method 1: Hard thresholding

After finding the wavelet transform the wavelet coefficients are obtained. For this wavelet co-efficients the threshold value is to be estimated by using median filter. Here, we are using median filter because it preserve the edges which is most required for our application.

The equation for the Median absolute deviation is represented as

$$\partial(\text{med}) = \frac{\text{Median}\{|c_0|, |c_1|, \dots, |c_{2^m-1}|\}}{0.6745}$$

Here c components are the coefficients. The factor 0.6745 in the denominator rescales the numerator. $\partial(\text{med})$ is a suitable estimator for the standard deviation for Gaussian

white noise. After obtaining median absolute value hard thresholding is applied to the coefficients.

The thresholding method is to be applied by giving obtained threshold value as an input. Most frequently used thresholding method is hard thresholding. It follows the keep or kill rule according to which if the threshold value is larger the input is kept as it is else set it to zero or null as follows

$$\omega(\partial) = \begin{cases} \omega, & |\omega| \geq \partial \\ 0, & |\omega| < \partial \end{cases}$$

Such that ∂ = Median absolute deviation,

ω = Magnitude of the wavelet coefficient

$\omega(\partial)$ = Wavelet coefficient

Method 2: Thresholding using Gradient descent and Matrix inversion using Givens rotation

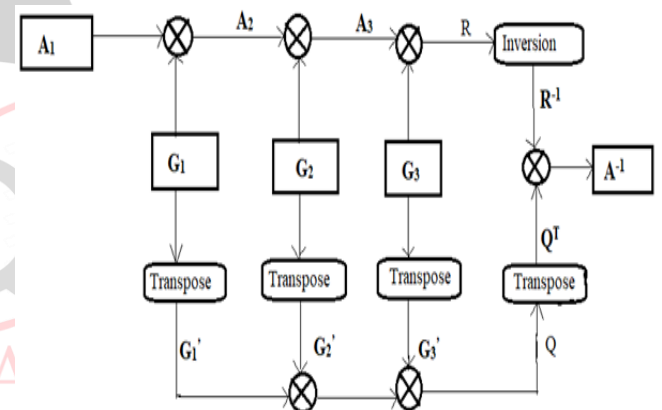


Fig 3: Basic hardware for obtaining matrix inversion using QR decomposition

A. Givens rotation method

Givens rotation method using QR decomposition annihilates off-diagonal matrix elements. They are used to find the inverse of a matrix and often used in solving the symmetric eigenvalue problem, and have received greater attention recently because of they lend themselves in parallel implementation.

Inversion of a matrix A can be performed by firstly decomposing this into an upper triangular form, which can be easily inverted. QR Decomposition is one of such approach to perform this triangularization whereas A is written as the product of two matrices Q and R.

$$A = QR$$

Where Q= unitary matrix

R=upper triangular matrix

The basic hardware for matrix inversion using QRD is as shown in the Fig 3: where as A1 is the input square matrix and G1, G2, G3 are the rotation matrices given by

$$G1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C & S \\ 0 & -S & C \end{bmatrix} \quad G2 = \begin{bmatrix} C & 0 & S \\ 0 & 1 & 0 \\ -S & 0 & C \end{bmatrix}$$

$$G3 = \begin{bmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By doing some mathematical calculation will get orthogonal and upper triangular matrix this is called the triangularization. After obtaining both matrices inverse the upper triangular matrix by using upper triangular inversion technique and transpose the orthogonal matrix the product of this two will get the final inversion matrix of an input.

B. Gradient descent algorithm

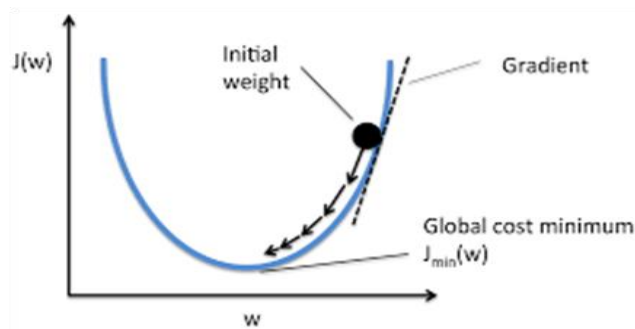


Fig 4: Gradient Descent

Gradient descent is used to find the local minima of a function it is the first order iterative optimization algorithm. We can find local minima or local maxima from the Gradient algorithm to find local maxima one should take steps towards the positive of the gradient which is known as Gradient ascent and for local minima one should take steps towards the negative of the gradient which is known as Gradient descent.

Starting from the top, take the first step downwards in the direction specified by the negative gradient. Next step is to recalculate the negative gradient and take one more step in the specified direction. Continue the process iteratively until we get local minima as shown in the Fig 4

Steps to obtain the local minima of a function

- To obtain the cost function, first we have to calculate the Θ value
 $\Theta = [LL, LH, HL, HH]$ Where Θ is the combination of all the sub-band co-efficients obtained from the wavelet transform. Given the cost function used in the algorithm:
 $J(\Theta) = \Theta' * \Theta$

Here $J(\Theta)$ is the cost function.

- The next step after getting cost function is to inverse the obtained cost function by using Givens rotation method which is computationally more efficient for inversion of a matrix. The inversion matrix is to be multiplied with the error function.

$M = \text{Givens rotation}(J)$

Where $M = \text{Inverse matrix}$

$J = \text{Cost function}$

- Inorder to get the error function the two factors true value and predicted value should obtain. So, the true value can be obtained by the following equation

$$Q = \Theta' * N$$

Where $Q = \text{True value}$

$N = \text{Noisy image co-efficients}$

- The step size is called learning rate. Learning rate should be scalar or a vector, in our process it is a vector. With the low learning rate one can move in the negative gradient descent.

Given the learning rate:

$$h = [0.25, 0.25, 0.25, 0.0625]$$

- The predicted value can be obtained by the multiplication of the slope function with the learning rate

$$K = \sigma^{2N}$$

$$R = K * h$$

Where $K = \text{Slope function}$

$\sigma = \text{Sigma (Noise value)}$

$N = 128 \times 128$ (Size of the sub-band)

$h = \text{learning rate; } R = \text{predicted value}$

- Now, from the true and predicted value the error function is to be calculated.

$$C = Q - R$$

Where $Q = \text{True value}$

$R = \text{Predicted value}$

$C = \text{Error factor}$

- The next and final step is to find the shrinkage factor value which is the multiplication of the error function and the inverse matrix.

$$a = \sum C * M$$

Where a is the local minima of a function

- The minimum threshold value "a" to be multiplied with all the sub-band co-efficients LH,HL,HH except LL because of its low frequency resolution it is left as it is as shown below

$$LL = LL$$

$$LH = LH * a$$

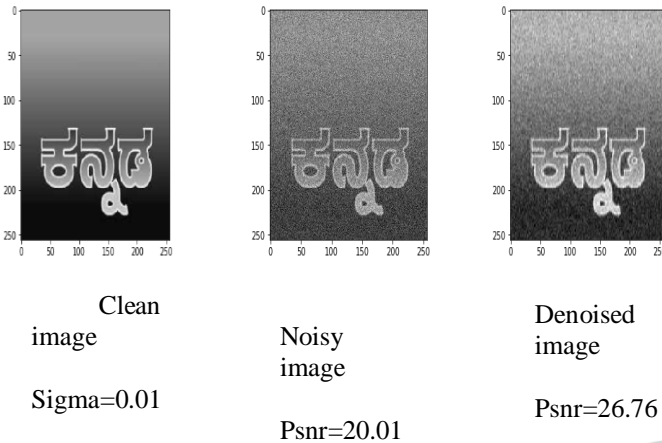
$$HL = HL * a$$

$$HH = HH * a$$

I. Inverse Discrete Wavelet transform

Apply Inverse DWT (IDWT) on the sub-bands LL, LH, HL and HH to obtain the de-noised image and also to reconstruct the original image.

VII. RESULTS AND DISCUSSION



| Sigma | Input MSE1 | Output MSE1 | Input Psnr | Output Psnr |
|-------|------------|-------------|------------|-------------|
| 0.01 | 648.5603 | 136.9915 | 20.0113 | 26.7639 |
| 0.02 | 670.4002 | 158.9554 | 19.8675 | 26.1181 |
| 0.03 | 701.6325 | 191.8278 | 19.6697 | 25.3017 |
| 0.04 | 748.5628 | 234.7209 | 19.3885 | 24.4253 |

Denoised and original images are compared by computing the image quality of the denoising technique. The results are expressed as the values of the Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR). MSE and PSNR values are computed using the following equations respectively,

Mean Square Error (MSE)

$$MSE = \frac{1}{HW} \sum_{i=1}^H \sum_{j=1}^W [X(i, j) - Y(i, j)]^2$$

Peak Signal-to-Noise-Ratio (PSNR)

$$PSNR = 10 \log_{10} \frac{255^2}{MSE} (dB)$$

VIII. CONCLUSION AND FUTURE WORK

In this paper a new method of image denoising is proposed. The proposed method employs Wavelet transform using different types of thresholding techniques. Wavelet transform using gradient descent as the thresholding method

achieves better PSNR values compared to that of applying hard threshold. The quality of the images produced using the proposed de-noising method on the images corrupted with Gaussian noise was compared in terms of PSNR against classic methods for DWT and hard threshold. According to the experimental results, the proposed method presents best values of PSNR for the de-noised images.

In the future, the proposed method can be modified by using more accurate estimation function and other kinds of threshold methods in trying to improve its ability at image de-noising process.

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