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A Review of Mathematical Models to Predict Sound Absorption Coefficient of Materials

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Abstract: A variety of mathematical models can estimate the acoustic behavior of fibrous and porous materials. These developed mathematical models are used to find the Sound Absorption Coefficient (SAC) of fibers analytically. Even though these models have been developed for polyester fibrous material they can be also applied for natural fibers. Different mathematical models like Delany and Bazley, Garai and Pompoli, Miki Model, Ramis Model, Johnson Allard Model, and Lafarge Allard Model have been discussed in this paper This paper reviews frequently used mathematical models that estimate the Sound Absorption Coefficient (SAC) of these materials.

Keywords — Sound Absorption Coefficient (SAC), Green fibers, Delany-Bazley Model, Flow Resistivity, Wave Propagation Constant.

I. INTRODUCTION

Noise pollution is any undesired or irritating sound that has an impact on the health and wellbeing of humans and other living things. It is an unseen threat resulting in noise-induced health concerns. All age groups, particularly children and elderly persons, can experience these health issues. Noise can have detrimental psychological and physiological consequences on humans, such as producing anger, anguish, hearing loss, elevated blood pressure, increased heart rate, communication disruptions, changes in human behavior, and sleep disturbances [1],[2].

To mitigate the effects of noise pollution, sound-absorbing panels are widely used. These panels either decrease the intensity of sound or reduce the echoes. The asbestos and rockwool are popular sound-insulating materials; but, due to advancement in material technologies, custom-made Glass fiber-based and mineral fiber-based composite materials are trending. Unfortunately, these synthetic materials are not environment-friendly [3].

The natural fibers with at par acoustic properties are the substitute to these harmful synthetic fibers. The natural fibers are also called as green fibers. They have low cost, abundant availability, and eco-friendliness compared with synthetic ones. The research is underway to develop of the acoustic panels to reduce the noise at source and it's propagation across the medium. The acoustic behavior is of interest due to different conditions in fabrication like the orientation of layers in panels, thickness of the panels, fiber volume fraction, and porosity of panels.

Researchers have developed various mathematical models for fibrous and porous materials. In this review article, we review popular models to predict the SAC of these materials.

II. MATHEMATICAL MODELS

A. Delany and Bazley Model

It is one of the most famous mathematical model used for estimating the sound absorption coefficient of natural fibres. Delany and Bazley [4] studied the acoustic properties of fibrous absorbent materials. The main advantage of this model is that it requires only one input parameter which is flow resistivity. Authors curve fitted power-law relation based on ratio of frequency and air flow resistivity using extensive experimental data available for mineral fibrous porous absorbers. The power-law relation for the surface characteristic impedance Z_c, and propagation constant K_c are presented below. Using Z_c and K_c surface impedance Z_s is calculated. Surface impedance value can be directly used for estimating the sound absorption coefficient.

$$Z_C = \rho_0 C_0 \left(1 + 9.08\left(\frac{f}{\sigma}\right)^{-0.754} - 11.9j\left(\frac{f}{\sigma}\right)^{-0.732}\right) \quad (1)$$

$$K_c = \frac{\omega}{C_0}\left(1 + 10.8\left(\frac{f}{\sigma}\right)^{-0.7} - 10.3j\left(\frac{f}{\sigma}\right)^{-0.595}\right) \quad (2)$$

$$Z_s = -jZ_C \cot(K_c d) \quad (3)$$

$$R = \frac{Z_s - \rho_0 C_0}{Z_s + \rho_0 C_0} \quad (4)$$

where, C_0 is velocity of sound in the air, ρ_0 is density of air, K_c is propagation constant, Z_c is characteristic impedance, f is frequency; ω is angular frequency, σ is airflow resistivity, R is sound pressure reflection coefficient, d is thickness of sample, and Z_s is surface impedance,

SAC is calculated as,

$$\alpha = 1 - |R^2| \quad (5)$$

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The main drawback of the this model is that sometimes the value of the real part of the surface impedance becomes negative at low frequencies.

2.2 Garai and Pompoli model

Garai and Pompoli developed a new empirical model for calculating the flow resistivity, acoustic impedance, and SAC for polyester fibrous material. Authors used 38 polyester fiber samples having different densities and different diameter values ranging from 18 to 48µm [5]. They upgraded the constants in the equations stated by Delany and Bazley for characteristic impedance and wave propagation constant. They also modified the equation for flow resistivity provided by Bies and Hansen [6]. The new flow resistivity equation along with Z_c and K_C equations are as follows

$$\sigma = A(\rho_m)^B \quad (6)$$

A and B are free parameters. $A=K_2(d)^{-2}$ and $B= K_1$. Where r is the airflow resistivity (Pas/m^2), ρ_m is the bulk density (kg/m^3). Value of $K_1 = 1.53$ and $K_2= 3.18*10^9$ for fiberglass. The key assumption in formulating the above equation is that the fiber diameter is uniform and a negligible amount of binder is used in the sample preparation stage. Equation 6 with best-fit values of A and B form a simple model for the polyester fibre material called the New Resistivity Model (NMR).

$$Z_c = \rho_0 C_0\left(1 + 0.078\left(\frac{\rho_0 f}{\sigma}\right)^{-0.623} - j0.074\left(\frac{\rho_0 f}{\sigma}\right)^{-0.66}\right) \quad (7)$$

$$K_c = \frac{\omega}{c}\left(1 + 0.121\left(\frac{\rho_0 f}{\sigma}\right)^{-0.53} - j0.159\left(\frac{\rho_0 f}{\sigma}\right)^{-0.571}\right) \quad (8)$$

The newly developed model is more accurate for glass wool when compared Delany- Bazley Model. In the absence of detailed information on the material microstructure, noise control engineers can use this model.

B. 2.3 Miki Model

Miki [7] worked on the drawbacks in Delany and Bazley model. As stated earlier, the real part of surface impedance sometimes becomes negative at low frequency for Delany-Bazley model. Miki [7] modified the original model which proved more accurate for porous materials.

$$Z_c = \rho_0 C_0\left(1 + 5.05\left(\frac{10^3 f}{\sigma}\right)^{-0.622} - 8.43j\left(\frac{10^3 f}{\sigma}\right)^{-0.632}\right) \quad (9)$$

$$K_c = \frac{\omega}{C_0}\left(1 + 7.81\left(\frac{10^3 f}{\sigma}\right)^{-0.618} - 11.41j\left(\frac{10^3 f}{\sigma}\right)^{-0.618}\right) \quad (10)$$

2.3 Ramis Model

Ramis and his group [8] developed a methodology to obtain the sound absorption characteristics using empirical equations. The authors focused on the fibers derived from coconut. The newly developed model requires the least number of non-intrinsic physical parameters.

$$\alpha = \left(\frac{2\pi f}{C_0}\right) * \left[C_5 * \left(\frac{\rho * f}{r}\right)^{-C_6}\right] \quad (11)$$

$$\beta = \left(\frac{2\pi f}{C_0}\right) * \left[1 + C_7 * \left(\frac{\rho * f}{r}\right)^{-C_8}\right] \quad (12)$$

$$R = \rho * C_0\left[1 + C_1 * \left(\frac{\rho * f}{r}\right)^{-C_2}\right] \quad (13)$$

$$X = -\rho * C_0\left[C_3 * \left(\frac{\rho * f}{r}\right)^{-C_4}\right] \quad (14)$$

Where α and β are the real and imaginary parts of the propagation constant K_c of the material, R and X are the real and imaginary parts of its characteristic impedance Z_c. Values of constants C_i were calculated by using least square fit method. For coir fibre the constant values are as follows $C_1 = 0.0713$, $C_2 = -0.8749$, $C_3 = -0.1216$, $C_4 = -0.4520$, $C_5 = 0.2129$, $C_6 = -0.4857$, $C_7 = 0.0997$, $C_8 = -0.5988$.

2.4 Johnson Allard Model

Johnson and his associates [9] studied the response of Newtonian fluid-saturated in the pores of porous material by subjecting it to a very infinitesimal oscillatory pressure gradient.. They studied the analytic properties of linear response function within high and low-frequency limits. Authors defined a new parameter called vicious characteristic length Λ, it was defined to integrate the geometry of the pore complexity.

$$\rho = \alpha_\infty \rho_0\left[1 - j\frac{\sigma\phi}{\rho_0 \alpha_\infty \omega}\sqrt{1 + \frac{4j\rho_0 \alpha_\infty{}^2 \omega\eta}{\sigma^2\phi^2\Lambda^2}}\right] \quad (15)$$

$$K_{JA}(\omega) = \gamma P_0\left[\gamma - \frac{(\gamma - 1)}{1 + \frac{8\eta}{j\omega\rho N_{pr}\Lambda'^2}\sqrt{1 + \frac{j\omega\rho N_{pr}\Lambda'^2}{16\eta}}}\right]^{-1} \quad (16)$$

where η represents viscosity of air, c is adiabatic constant, α_∞ is tortuosity, N_{pr} is Prandtl number, ϕ is porosity, P_0 is the atmospheric pressure, Λ is viscous characteristic lengths and Λ` is thermal characteristic lengths and k_0 is thermal permeability. γ is the specific heat of air.

2.5 Lafarge Allard Model

Champoux and Allard [10],[11] redefined the model given by Biot [12] to define a new parameter called thermal characteristic length (Λ). Later Lafarge and Allard introduced a term called dynamic compressibility which included this thermal characteristic length.

The dynamic compressibility of Lafarge Allard is K_{LA} is as follows

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$$K_{LA}(\omega) = \gamma P_0 \left[\gamma - \frac{(\gamma - 1)}{1 + \frac{8\eta}{j\omega\rho N_{pr} K_0^{'}} \sqrt{1 + \frac{4j\omega\rho N_{pr} K_0^{'2}}{\eta \phi^2 \Lambda^{'2}}}} \right]^{-1} \quad (17)$$

The propagation constant k_c and characteristic impedance Z_c were defined by using effective density and K_{JA} for the Jonhson-Allard model and K_{LA} for the Lafarge-Allard model as follows:

$$Z_c = \sqrt{\rho K_{JA,LA}} \quad (18)$$

$$K_c = \omega * \sqrt{\frac{\rho}{K_{JA,LA}}} \quad (19)$$

It can be seen that the air condition plays an important role in the calculation of dynamic compressibility.

III. APPLICATION OF MATHEMATICAL MODELS

Table 1 shows some of the researcher who have carried out research on understanding the acoustic nature of natural fibres. Notice that, Delany and Bazley [4] model is popular and have been used to predict the SAC for variety of natural fibers such as coconut coir, corn, grass, sugar cane, jute, sheep wool and date palm. Johnson-Allard and Miki model are comparatively new and have been used for SAC prediction of date palm, Boston ferns and baby tears.

Table 1: Application of Mathematical Models

Author and Year	Natural Fibre	Mathematical Model Used	Inferences
Fouladi et al [13], 2012	Coconut coir Corn Grass sugar cane.	Delany and Bazley	Thickness has a significant influence on the SAC.
Raja et al [14], 2012	Date Palm Fibre	Johnson-Allard Model	Increase in thickness increases SAC As the density of the samples SAC also increases.
Othmani et al [15], 2015	Sugarcane Waste	Delany-Bazley Johnson-Allard Lafarge-Allard	Sugarcane shows a good sound-absorbing nature at low and high frequencies.
Francesco D'Alessandro et al [16], 2015	Boston Ferns and Baby Tears	Miki Model	Designed room using a vertical green wall made of natural fibers.
Bansod et al [17], 2017	Jute Fibre	Delany and Bazley	Micro-perforated panels can help to improve sound absorption.
R del Rey et al [18], 2017	Sheep wool	Delany and Bazley	Sheep wool is a good sound absorber
Taban Ebrahim et al [19], 2019	Date palm empty fruit (DPEFB) fibers	Delany and Bazley	Density affects the SAC significantly.

Fouladi et al. studied the acoustic nature of coconut coir, corn, grass, and sugarcane. Similarly, Othmani studied the sugarcane waste material potential for sound absorption. Raja et al. studied date palm fiber and the findings they found were date palm fiber showed excellent sound absorption tendency. Also, by increasing the thickness of samples the SAC also increases. Francesco D' Alessandro et al. constructed a room whose walls were covered with Boston ferns and Baby Tears. They found that there was more sound damping compared with the condition when walls were not covered with plants. Some more research is carried out by different researchers are shown in table 1 along with their key findings.

IV. CONCLUSION

We studied six mathematical models which predict the sound absorption coefficient of porous and fibrous materials. These models have been evolved to overcome limitations of one another. The first model based on extensive experimental data- Delany and Bazley [4] - need only flow resistivity as an input and hence is popular among the researchers.–Garai and Pompoli [5] improved empirical relation for flow resistivity proposed by Bies and Hensen [6]. Further, they evaluated the constants in Delany and Bazley [4] model which provided better results. -At low acoustic frequencies. At low acoustic frequencies Miki [7] observed that the real part of impedance in the Delany and Bazley model becomes negative. By upgrading the power index and constant values Miki [7] obtained better results. Ramis [8] also improved the accuracy of Delany and Bazley model using curve fitting method. Johnson Allard [9] and Lafarge Allard's [11] models are little complex and require several input parameters compared with the earlier stated ones. They studied the response of Newtonian fluid-saturated in the pores of porous material by subjecting it to the infinitesimal oscillatory pressure gradient. The prediction of this model matched the experiments. The last two models are new, require several inputs and need extensive calculations. All these models are used to predict SAC by different researchers. Most of them were found to be in line when the results were compared with the results obtained by performing experiment on stand equipment called Impedance Tube. Some model's accuracy deviation was very less almost coinciding with the experimental values. Overall, these models are helpful to predict the SAC for fibrous and porous material. There is scope to simplify, unify these models to be applicable for wider range of materials.

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