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PROPOSING THE ALPHA GROUP

Cleber Correa, Institute of Aeronautics and Space, São José dos Campos, SP, Brazil

Thiago de Melo, Institute of Aeronautics and Space, São José dos Campos, SP, Brazil

Diogo Custodio, Institute of Aeronautics and Space, São José dos Campos, SP, Brazil

Abstract: **Does infinity have a specific geometric and topological representation of its own nature, in general? This work seeks to generate a new way of interpreting the intrinsic nature of numbers and their mathematical operations associated with very large quantities, close to infinity. Group theory allows, through a mathematical operation, to relate two elements to a third, which defines a new set and the operation must satisfy some conditions called group axioms: associativity, neutral element, and inverse elements. In this work, the operation is the division. The consequences of this operation represent a maximum deformation between infinite planes leading to the generation of a new numerical structure, as well as a geometric representation in 4 dimensions on a spherical surface in revolution. Results in asymmetrical and mirrored multi-planes. Also, parts of this larger group in R^4 are numerical subgroups, already defined as real and complex numbers. This proposal is based on the in Fraction Rings or Quotient Rings, seeking to use the morphism theory referring to mapping one mathematical structure to another in such a way that it is preserved in the new structure. Because we understand that his view of the possibility of different infinities opens up possibilities for new interpretations and consequences based on group theory, maintaining the one-to-one correspondence between the elements of the two groups and would persevere in its operations in both groups.**

Keywords* — *infinity geometry, group theory, four dimensions, abstract algebra

I. INTRODUCTION

In the history of mathematics, some digits had different times of emergence, and yet they have a relationship with each other ($1 - e^{i\pi} = 0$), knowing that each term is a contribution from different peoples. The discovery of Non-Euclidean geometries dealt a devastating blow to Kantian philosophy, comparable to the effect that the discovery of in-commensurable magnitudes had on Pythagorean conceptions. In Riemann's work, he saw that geometry should not necessarily be about points or lines or space in the ordinary sense, but about collections of n-uples, that are combined according to certain rules, he also proposed a global view of geometry as a study of manifolds of any number of dimensions in any type of space. In the development of human thought in the formation of concepts about the structure, form, and nature of numbers, George Cantor's ideas about transfinite numbers. The sets of infinities had the same magnitude (cardinality), but Cantor conclusively proved that this was not true, as the number in the set of reals was greater than the number of rationals, ([5], [4] and [13]). This idea allows us to think about the existence of different types of infinities or cardinalities. In Felix Klein's works, ([10], [16] and [11]), he systematized the Lie contact transformations ([15], [8] and [1]), establishing and structuring the definition of new mathematical relationships for existences of abstract algebras. In doing so, he helped define group theory relationships. Within this respect a collection of elements is said to form a group with respect to a given operation, if (i) the collection is closed under the operation, (ii) the collection contains an identity element concerning for the operation, (iii) for each element in the collection there is an inverse element concerning for the operation and (iv) the operation is associative. Elements can be numbers (as in arithmetic), points (in geometry), and transformations (in algebra or geometry). The definition of a mathematical operation can be arithmetic (such as addition, multiplication, and or division) or geometric (such as rotation around a point or axis), or any other rule for combining two elements of a set, such as two transformations, of to form a third element of the set. In the literature, group theories with Quaternions or Octonions are already developed, which deal with generalizations of complex planes and are considered hyper-complex planes, ([17] and [18]).

The philosophy in the development of this group follows the definition that a group is a set of elements associated with an operation that combines any two elements to form a third. To qualify as a group, the set and the operation must satisfy some conditions called group axioms: associativity, neutral element, and inverse elements. Such features show the generality of the group theory concept. In many fields of mathematics, it is also referred as morphism to mapping one mathematical structure to another in such a way that the structure is preserved, ([20] and [21]). Much of the terminology of morphisms, as well as the underlying intuition, comes from concrete categories, where objects are simply sets with some additional structure, and morphisms

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are structure-preserving functions. Therefore, this work seeks to develop the basis of the description of a proposal for the existence of a new numerical Alpha Group in R^4, which is based on the transformation of two infinite planes that interact by transforming the division operation between them, creating a third element with morphism and that preserves the operations in both. In this respect, the new proposal adds a new imaginary point to the quaternions. It seeks to describe the possibility of infinity having a specific geometric and topological representation of its own nature, representing a general geometry and topology.

II. RESULTS

Starting from the group theory presents Fraction Rings or Quotient Rings [19], showed that groups with homomorphism can be defined as *f:G -> S*, in which its image is the subgroup of *S* and its Kernel is a subgroup of *G* Given some normal subgroup *N* of *G*, a structure can be defined with a closed set in the operation G|N in which a mapping path *a -> aN* from *G* to *G|N* is a homomorphism with kernel N. By similarity we can show a spatial geometry can be defined with n infinite planes, where each plane represents a real number. These planes range from positive infinity to negative infinity, following by similarity a real axis. The initial basic idea is to follow by similarity what Galois did as the solution to the classical problem of solving algebraic equations by radicals. This used a group of permutations to describe how the various roots of a certain polynomial equation are related to each other. In this case, we will start from the real numbers that will compose a matrix with infinite planes. This can be seen in Table 1. It will define the transformation as the operation performed by dividing one multiplane by another with the same similarity, in which there will be a rotation of one plane by the other by *90* degrees, the rotation can be interpreted as a function of $\pi/2 + n\pi$, for all elements, the operation of a root with the index equal to *2* will be performed. All resulting operations will be defined as multiplication by zero will be as result zero. A region will appear in this new structure that will define a ratio between one plane and the other. This ratio can be interpreted as the transformation ratio between the planes such that when the denominator is getting smaller and closer to zero. It will represent the maximum deformation and define properties in this new group. And when it's zero and this group is called Alpha, with the definition of an imaginary point, geometrically and numerically infinite. It will be a canonical vector of this new space and represented by the Greek letter μ. The results can be seen in Table 2.

The canonical vector that appears from the transformation by the division operation is interpreted as the transformation ratio between the planes, such that, when the denominator is getting smaller and closer to zero, it would reach a ratio that would imply the maximum deformation and define properties in the new alpha group. In this aspect the branch of Ergodic theory [7], the work of Poncelet [14] shows in the famous Traité des Propriétés Projectives des Figures, being a work of synthetic form makes his statements of synthetic geometry as general as possible, Poncelet formulated what he called the principle of continuity or the principles of the permanence of mathematical relationships. In this sense, the principle approached Carnot's ideas, but Poncelet took it further, including the points at infinity that Johannes Kepler and Girard Desargues had suggested ([6]; [3] and [2]). Thus, it could be said that two lines always intersect either at an ordinary point or (in the case of two parallel lines) at a point at infinity called the ideal point. In order to reach Poncelet's generality, he found it necessary to introduce into synthetic geometry not only ideal points but also imaginary points, because only then could he say that a circle and a straight line always intersect. Whiting, in this aspect, the Alpha Group has an imaginary point that appears by the operation of the division of infinite planes, when the deformation is maximum this point will be, in which the asymmetric and mirroring properties arise. In this case, defining a canonical vector of this numerical space in R^4, representing an imaginary point with geometric and topological representation.

↖	↑	↑	↑	↑	↑	↗
←	2	2	2	2	2	→
←	1	1	1	1	1	→
←	0	0	0	0	0	→
←	-1	-1	-1	-1	-1	→
←	-2	-2	-2	-2	-2	→
↙	↓	↓	↓	↓	↓	↘

Table 1 - Numerical planes, ranging from $+\infty$ to $-\infty$.

The numerical structure of the Alpha group is composed by the first term a which is real, the second term with complex bi and the third term with cμ and the fourth term with *d iμ* presenting two imaginary numbers, $AG: a + b\,i + c\,\mu + d\,i\mu$. The third and fourth term have the complex numerical structure multiplied by the number μ of the Alpha group, generating geometric and topological consequences.

Figure 1 shows the Alpha Group generates a general numerical group in which the real and complex numbers are specific for each case, which can be seen in quadrants II and III, representing the real planes. In quadrants, I and IV,

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↖	↑	↑	↑	↑	↑	↗
←	i	$\sqrt{2}i$	$\sqrt{2}\mu$	$\sqrt{2}$	1	→
←	$\sqrt{2}i/2$	i	μ	1	$\sqrt{2}/2$	→
←	o	o	o	o	o	→
←	$\sqrt{2}/2$	1	$i\mu$	i	$\sqrt{2}i/2$	→
←	1	$\sqrt{2}$	$\sqrt{2}i\mu$	$\sqrt{2}i$	i	→
↙	↓	↓	↓	↓	↓	↘

Table 2 - Result of mathematical operations between the planes that result in the surface that generates the Alpha group.

it represents the complex planes. Furthermore, it would also represent geometric properties for infinite planes in R4. The new group makes it possible to break with the structure of the Cartesian axes, and the main aspect is, therefore, to allow giving meaning and geometric representation to infinity in R^4. Such a structure also represents an infinite set with infinite order. The planes and surfaces are equipotential and present their volumes in a revolution in four dimensions, more general but similar to a type of Hopf fibration torus in R^3.

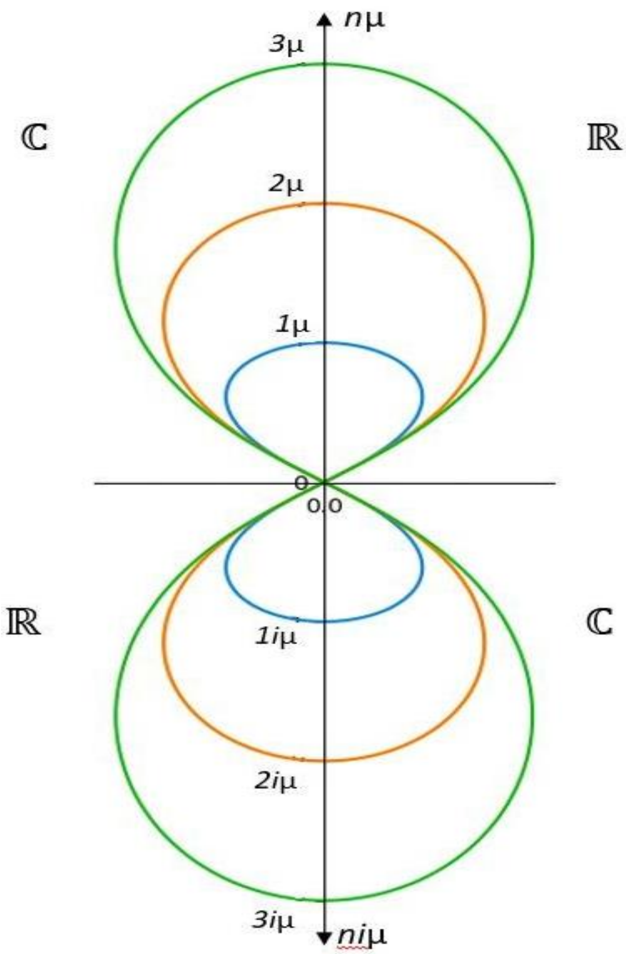

Figure 1 - The Alpha Group geometric space in R^4, Poincaré cut.

III. CONCLUSION

The Alpha Group generates a general numerical group in which real and complex numbers are case-specific. In addition, it would also represent geometric properties associated with infinite planes in a revolution in R^4. The new group allows modifying with the structure of the hypercomplex, making a new interpretation, creating a new spatial geometry and topology, and the main aspect is, therefore, it allows giving a meaning and geometric representation to infinity in R^4. Such a structure also represents an infinite set with infinite order. This resulting group presents morphism with one-to-one correspondence between the elements of the two groups and preserves the operations in both. Therefore, it opens the possibility of developing new research in the analysis and the consequences of its limits and geometric and topological applications in the mathematical and physical sciences.

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