International Journal for Research in Engineering Application & Management (IJREAM)
ISSN : 2454-9150 Vol-09, Issue-04, July 2023

Mechanical Response of a Thin Rectangular E-Glass /Kevlar FRP Hybrid Composite Plate by Using Piezoelectric Smart Materials

Dr K Dileep Kumar[1]

[1]Assistant Professor, UCEK, JNTUK, Kakinada, A.P, INDIA

ABSTRACT - In this investigation, two piezoelectric patches/actuators are symmetrically surface bonded on composite laminate plates. Electric voltages with the same magnitude and opposite sign are applied to the two symmetric piezoelectric actuators, which results in the bending affect on the laminate. The bending moment is derived by using the theories of elasticity and piezoelectricity. The analytical solution for the displacement and natural frequency of the composite plate subjected to the bending moment is solved by using the plate theory. Piezoelectric patches are surface bonded to a composite laminate plate are also used as vibration actuators. The loads are then employed to develop the vibration response of a simply supported laminate rectangular plate excited by piezoelectric patches subjected to time harmonic voltages.

The analytical solution for the deflection and the vibration amplitude is computed. The numerical results are presented for the static deflection and vibration amplitude of plate. A parametric study is also carried out to depict the influence of the size and location of the piezoelectric actuators on the cross-ply composite laminate plates.

Keywords: Cross-ply laminates, Piezoelectric actuators and Harmonic voltages

I. INTRODUCTION

Piezoelectric materials have advantages of low cost, low weight, small size, good dynamic performance, fast response, long term stability and high energy conversion efficiency. Surface bonding or embedding segmented elements of these materials in a structure would allow their use in controlling the deformation of structural elements.

The structural components made of single-composite material have been studied and used extensively, and most of the published work is also confined to plates with a single material. Hybrid laminates made up of two or more fiber reinforced laminae have certain improvements in the mechanical properties over plates with single material.

During the past several years, the piezoelectric effect was adopted in many engineering applications, and there has been much research on it. Crawley and de Luis [1] studied about a beam with surface bonded and embedded piezoelectric actuators to investigate the load transfer between the actuator and the beam. S.C. Her and Lin [2] studied about a composite laminate with surface bonded piezoelectric actuators to predict the deformed shape. Her and Lin [3] investigated on the vibration response for a cross-ply composite laminate plate excited by piezoelectric actuators with time harmonic electrical loading. Clark, Flemming and Fuller [4] studied experimentally the vibration excitation of a simply supported plate with multiple piezoelectric patches bonded on the surface. Dimitriadis [5] used surface bonded piezoelectric patches on a simply supported plate as vibration actuators to excite selective modes. S.C. Her and Lin [6] studied about a composite laminate with surface bonded piezoelectric actuators to predict the deformed shape.

II. THEORY OF PIEZOELECTRICITY

The root of the word 'piezo' means 'to press'; hence the original meaning of the word piezoelectricity implies electricity generated from pressure. Piezoelectric materials have the ability to provide desired transformation from mechanical to electrical energy and vice versa. The coupling effects between the mechanical and electric properties of piezoelectric materials have drawn attention for their use as sensors, actuators and power generation units. When an electric field is applied to the poled piezoelectric ceramic through electrodes, the piezoelectric material gets strained.

 DOI : 10.35291/2454-9150.2023.0160

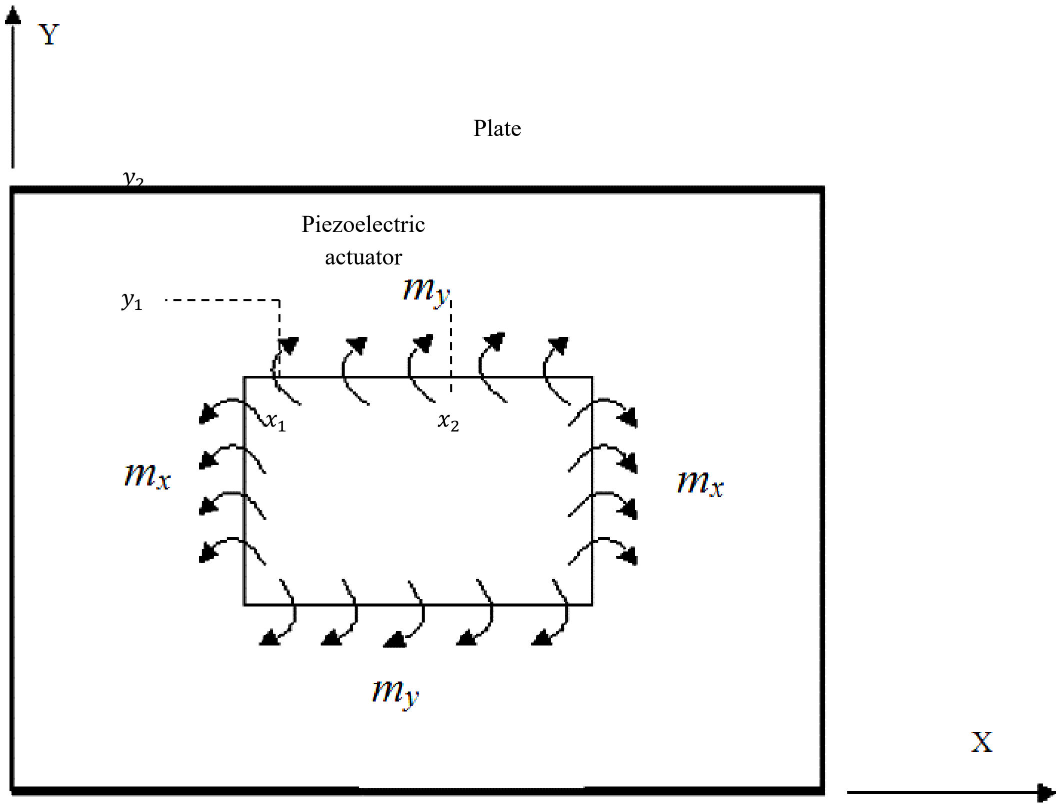

Fig. 1. Bending moment on the isotropic plate induced by the piezoelectric actuator

Consider two piezoelectric actuators bonded symmetrically in a cross ply composite laminate plate. The strain in the piezoelectric actuator is expressed in terms of piezoelectric constant d_{31}, actuator thickness t_{pe} and applied voltage V, as follows:

$$(\varepsilon_x)_{pe} = (\varepsilon_y)_{pe} = \varepsilon_{pe} = \frac{d_{31}}{t_{pe}} V \tag{1}$$

The bending moment induced by the piezoelectric actuators can be expressed as follows [2]:

$$m_x = C_1 \varepsilon_{pe} \;\; ; \;\; m_y = C_2 \varepsilon_{pe} \tag{2}$$

$$C_1 = A_1 (D_{11})_P + A_2 (D_{12})_P \tag{3a}$$

$$C_2 = A_1 (D_{12})_P + A_2 (D_{22})_P \tag{3b}$$

$$A_1 = \frac{2(B_{11})_{pe}(1+\vartheta_{pe})}{(D_{11})_p + 2(D_{11})_{pe}} - \frac{(D_{12})_p + 2(D_{12})_{pe}}{(D_{11})_p + 2(D_{11})_{pe}} A_2 \tag{4a}$$

$$A_2 = \frac{2(B_{11})_{pe}(1+\vartheta_{pe})[((D_{11})_p + 2(D_{11})_{pe}) - ((D_{12})_p + 2(D_{12})_{pe})]}{-((D_{12})_p + 2(D_{12})_{pe})^2 + [(D_{11})_p + 2(D_{11})_{pe}][(D_{22})_p + 2(D_{22})_{pe}]} \tag{4b}$$

$$(D_{11})_{pe} = (D_{22})_{pe} = \frac{1}{3} \frac{E_{pe}}{(1-\vartheta_{pe}^2)} ((t+h)^3 - t^3) \tag{5a}$$

$$(D_{12})_{pe} = \frac{1}{3} \frac{\vartheta_{pe} E_{pe}}{(1-\vartheta_{pe}^2)} ((t+h)^3 - t^3) \tag{5b}$$

$$(B_{11})_{pe} = \frac{1}{2} \frac{E_{pe}}{(1-\vartheta_{pe}^2)} ((t+h)^2 - t^2) \tag{5c}$$

$$(D_{11})_p = \frac{1}{3} \sum_{k=1}^{N} \overline{Q}_{11}^k \, (Z_k^3 - Z_{k-1}^3) \tag{6a}$$

 DOI : 10.35291/2454-9150.2023.0160

$$(D_{22})_p = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{22}^{k} (Z_k^3 - Z_{k-1}^3) \quad (6b)$$

$$(D_{12})_p = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{12}^{k} (Z_k^3 - Z_{k-1}^3) \quad (6c)$$

III. GOVERNING EQUATIONS OF COMPOSITE PLATE

When a piezoelectric element is stressed electrically by a voltage, its dimensions change. This activated piezoelectric element will induce bending moment on the plate and it can be expressed in terms of unit step function as follows:

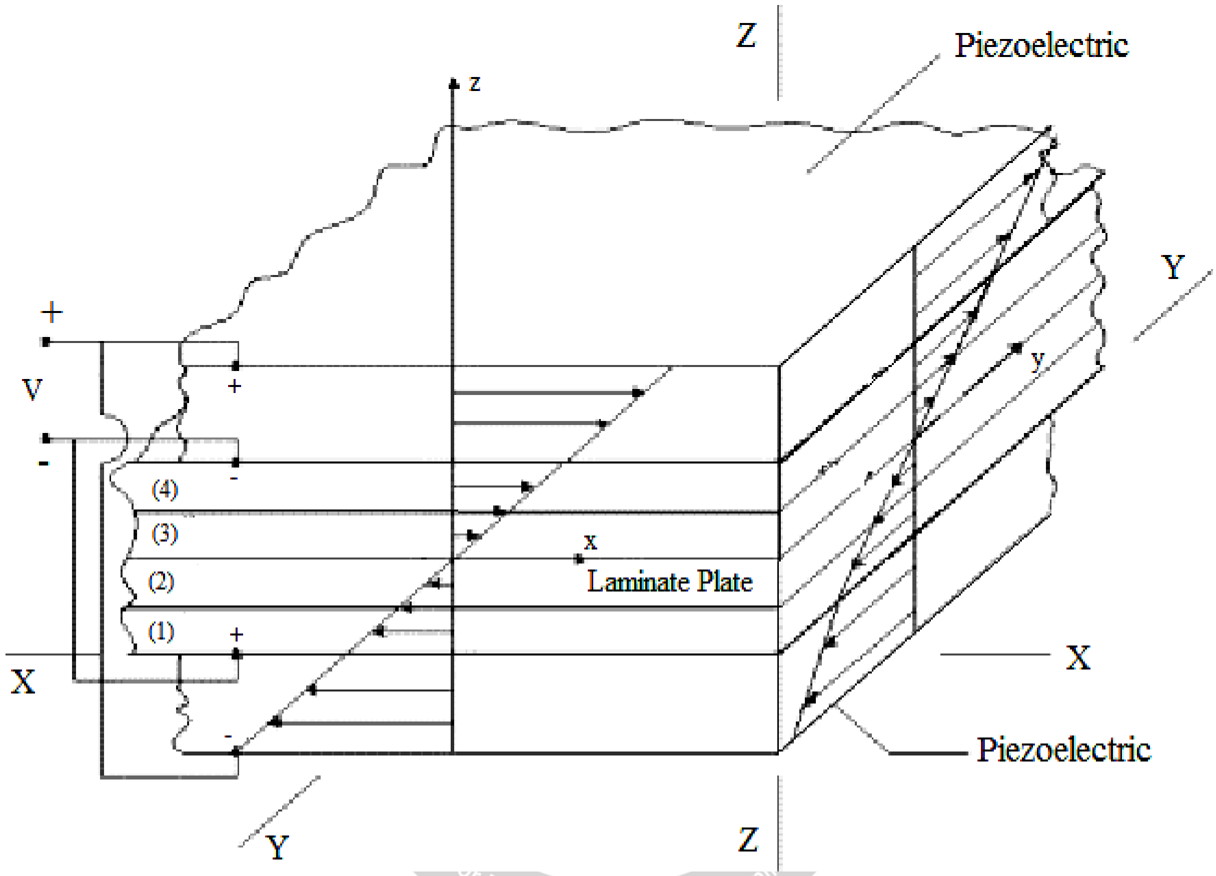

Fig. 2. Interface between plate and actuator

$$m_x = C_1\varepsilon_{pe}[h(x - x_1) - h(x - x_2)][h(y - y_1) - h(y - y_2)] \quad (7a)$$

$$m_y = C_2\varepsilon_{pe}[h(x - x_1) - h(x - x_2)][h(y - y_1) - h(y - y_2)] \quad (7b)$$

By using plate theory, the equilibrium equation for the plate is given by

$$\frac{\partial^2 (M_x + m_x)}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\,\partial y} + \frac{\partial^2 (M_y + m_y)}{\partial y^2} = 0 \quad (8)$$

Where M_x, M_y, M_{xy} are the plate internal moments and m_x, m_y are induced moments by the actuator. The internal moments in the plate can be expressed in terms of displacement w. the Eq. (8) yields to

$$(D_{11})_p \frac{\partial^4 w}{\partial x^4} + 2H_1 \frac{\partial^4 w}{\partial x^2\,\partial y^2} + (D_{22})_p \frac{\partial^4 w}{\partial y^4} = P \quad (9)$$

$$P = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2}$$

$$H_1 = (D_{12})_p + 2(D_{66})_p$$

Where $(D_{11})_p, (D_{22})_p, (D_{66})_p$ are the bending stiffness of the composite plate.

 DOI : 10.35291/2454-9150.2023.0160

For the simply supported rectangular plate, the displacement can be expressed by using Fourier series as follows

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{10}$$

Substituting Eq.(10) and Eq.(7) in Eq.(9), and solving the equation for Fourier coefficient W_{mn} as follows

$$W_{mn} = \frac{P_{mn}}{\gamma_m^4 (D_{11})_p + \gamma_m^2\gamma_n^2 2H_1 + \gamma_n^4 (D_{22})_p} \tag{11}$$

$$P_{mn} = \frac{4}{ab}\left[-\frac{m_y\gamma_m^2 + m_x\gamma_n^2}{\gamma_m\,\gamma_n}(\cos\gamma_m x_1 - \cos\gamma_m x_2)(\cos\gamma_n y_1 - \cos\gamma_n y_2)\right] \tag{12}$$

$$\gamma_m = \frac{m\pi}{a} \quad \text{and} \quad \gamma_n = \frac{n\pi}{b}$$

The equation of motion for the plate can be expressed in terms of displacement ‘A’ and induced moments m_x, m_y as follows

$$(D_{11})_p\frac{\partial^4 A}{\partial x^4} + 2H_1\frac{\partial^4 A}{\partial x^2\,\partial y^2} + (D_{22})_p\frac{\partial^4 A}{\partial y^4} + M\frac{\partial^2 A}{\partial t^2} = F(x,y)\sin pt \tag{13}$$

$$F(x,y) = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2}$$

Where M is the area mass density of the composite plate and p is the excitation frequency. The forcing function F(x, y) can be expressed in terms of Fourier series as follows

$$F(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} F_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{14}$$

$$F_{mn} = \frac{4}{ab}\left[-\frac{m_y\gamma_m^2 + m_x\gamma_n^2}{\gamma_m\,\gamma_n}(\cos\gamma_m x_1 - \cos\gamma_m x_2)(\cos\gamma_n y_1 - \cos\gamma_n y_2)\right] \tag{15}$$

For a simply supported rectangular plate, the displacement ‘A’ can be expressed in terms of Fourier series as

$$A(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin pt \tag{16}$$

The natural frequency of a simply supported plate is

$$\omega_{mn} = \sqrt{\frac{\gamma_m^4 (D_{11})_p + \gamma_m^2\gamma_n^2 2H_1 + \gamma_n^4 (D_{22})_p}{M}} \tag{17}$$

$$A_{mn} = \frac{F_{mn}}{M(\omega_{mn}^2 - p^2)} \tag{18}$$

IV. PARAMETRIC STUDY

The Composite laminate is assumed to have stacking sequence Kevlar/E-glass/E-glass/Kevlar with orientation of 0/90/90/0. The composite material is Kevlar/epoxy, the composite material properties are longitudinal modulus $E_1 = 87$GPa, transverse modulus $E_2 = 5.5$GPa, shear modulus $G_{12} = 2.2$GPa and Poisson’s ratio $\vartheta_{21} = 0.34$ and for E-glass/epoxy, the composite material properties are longitudinal modulus $E_1 = 37.8$GPa, transverse modulus $E_2 = 10.9$GPa, shear modulus $G_{12} = 4.91$GPa and Poisson’s ratio $\vartheta_{21} = 0.29$ The dimensions of the plate are length a = 0.38 m, width b = 0.3m, thickness t_p = 1.5876 mm.

The piezoelectric actuator is assumed to be PZT G-1195 with the material properties of Young’s modulus E_{pe} = 63GPa, Poisson’s ratio ϑ_{pe} = 0.3, density ρ_{pe} = 7600Kg/m^3. The piezoelectric constant d_{31} = 1.9×10^{-10}m/V and thickness t_{pe} = 0.15876mm.

The piezoelectric actuator PZT 5H has the material properties of Young’s modulus E_{pe} = 63GPa, Poisson’s ratio ϑ_{pe} = 0.3, density ρ_{pe} = 7600Kg/m^3. The piezoelectric constant d_{31} = 1.9×10^{-10}m/V.

 DOI : 10.35291/2454-9150.2023.0160

The effect of size and location of the actuator are presented to investigate the deformed shape of the plate as shown in the Fig.3, 4 and Fig.5, 6 respectively. The voltage of ±1V is applied to the upper and lower actuators respectively, and this voltage results in bending moment acting on the host plate. The deflection is increasing as the size of actuator increases.

4.1. STATIC ANALYSIS

Table 1. Maximum deflection (mm) of plate obtained from the Eq.(10) for three different sizes of piezoelectric actuator.

Size	Position of patch (m)				Deflection G1195	Deflection PZT 5H
	x_1	x_2	y_1	y_2		
60mm×40mm	0.16	0.22	0.13	0.17	0.001624	0.002349
80mm×60mm	0.15	0.23	0.12	0.18	0.0027978	0.0040469
100mm×80mm	0.14	0.24	0.11	0.19	0.0039768	0.0057525

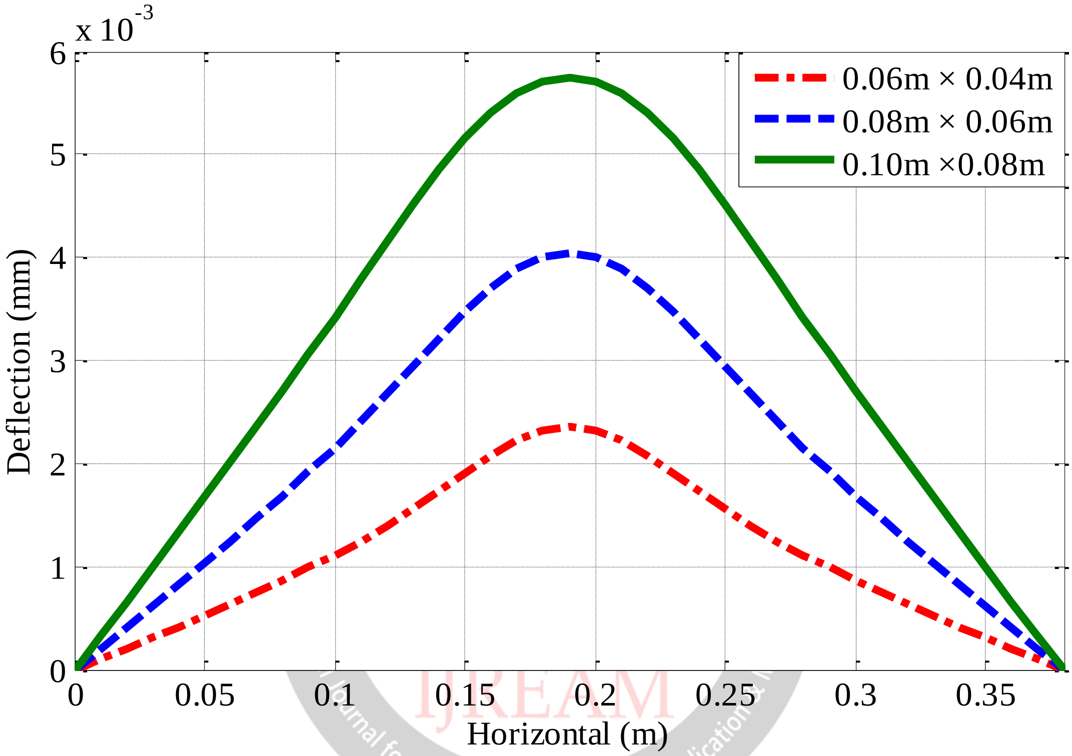

Fig. 3. Deflection of the plate obtained from the Eq.(10) along the horizontal (y=b/2) for three different sizes of actuator(PZT 5H).

 DOI : 10.35291/2454-9150.2023.0160

ISSN : 2454-9150 Vol-09, Issue-04, July 2023

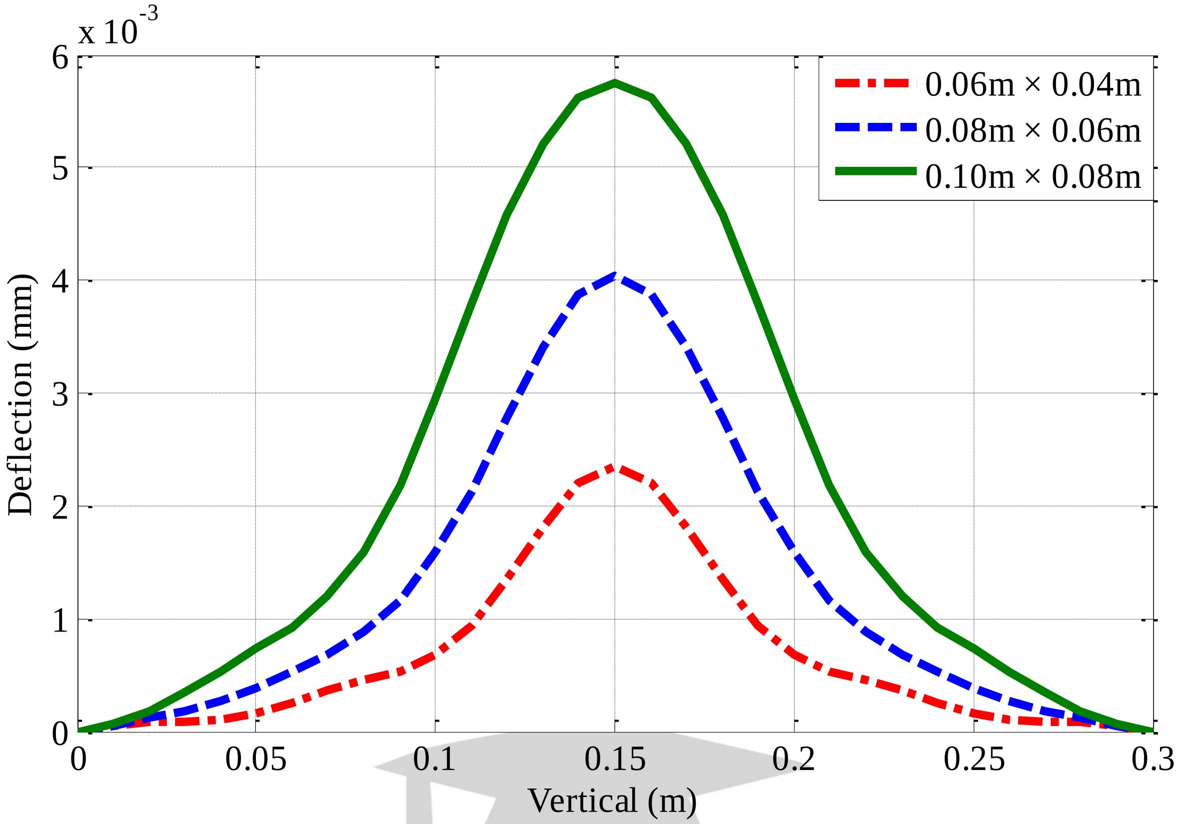

Fig. 4. Deflection of the plate obtained from the Eq.(10) along the vertical (x=a/2) for three different sizes of actuator(PZT 5H).

The piezoelectric actuator with the dimension 60mm × 40mm is bonded at three different locations on the composite plate. The maximum deflections induced in the plate at three locations of actuator are listed in Table 2.

Table 2. Maximum deflection (mm) of plate obtained from the Eq.(10) at three different locations of piezoelectric actuator.

PZT Location	Position of patch (m)				Deflection	Deflection
	x_1	x_2	y_1	y_2	G1195	PZT 5H
Central	0.16	0.22	0.13	0.17	0.001624	0.002349
Right	0.31	0.37	0.13	0.17	0.000704	0.001111
Top	0.16	0.22	0.23	0.27	0.001566	0.002266

DOI : 10.35291/2454-9150.2023.0160

ISSN : 2454-9150 Vol-09, Issue-04, July 2023

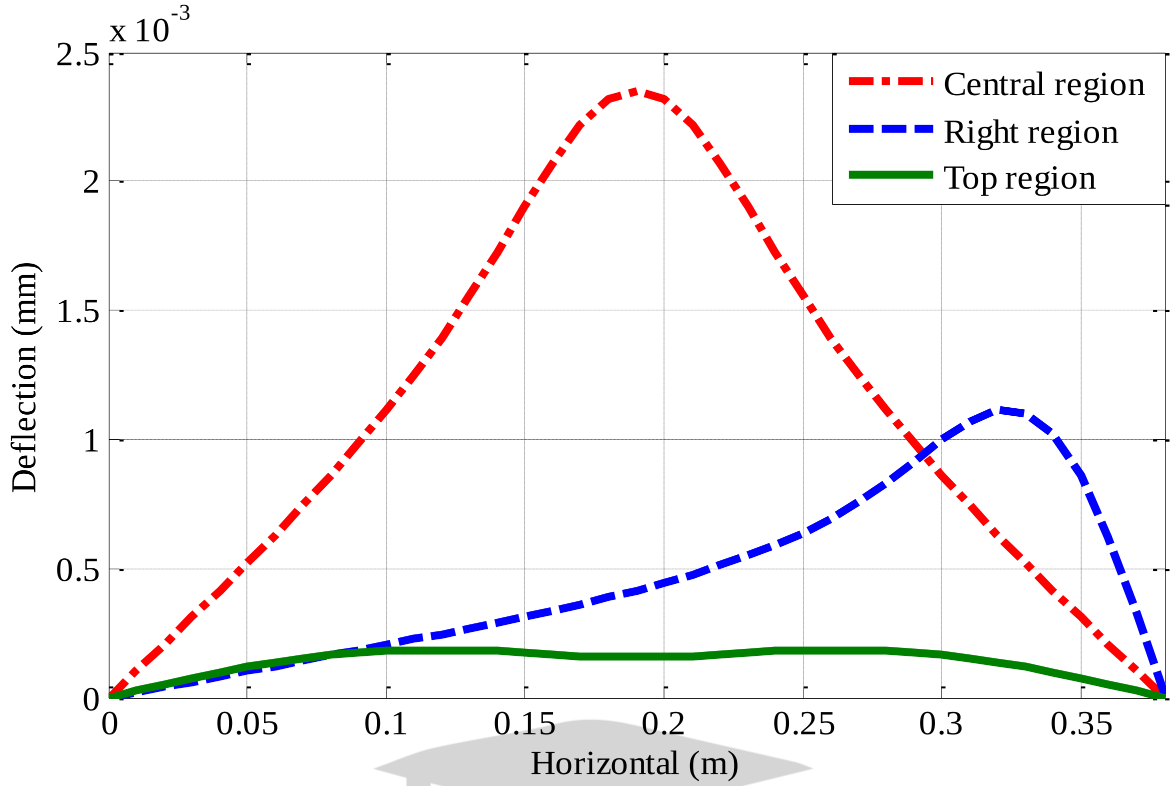

Fig. 5. Deflection of the plate obtained from the Eq.(10) along the horizontal (y=b/2) for three different locations of actuator.

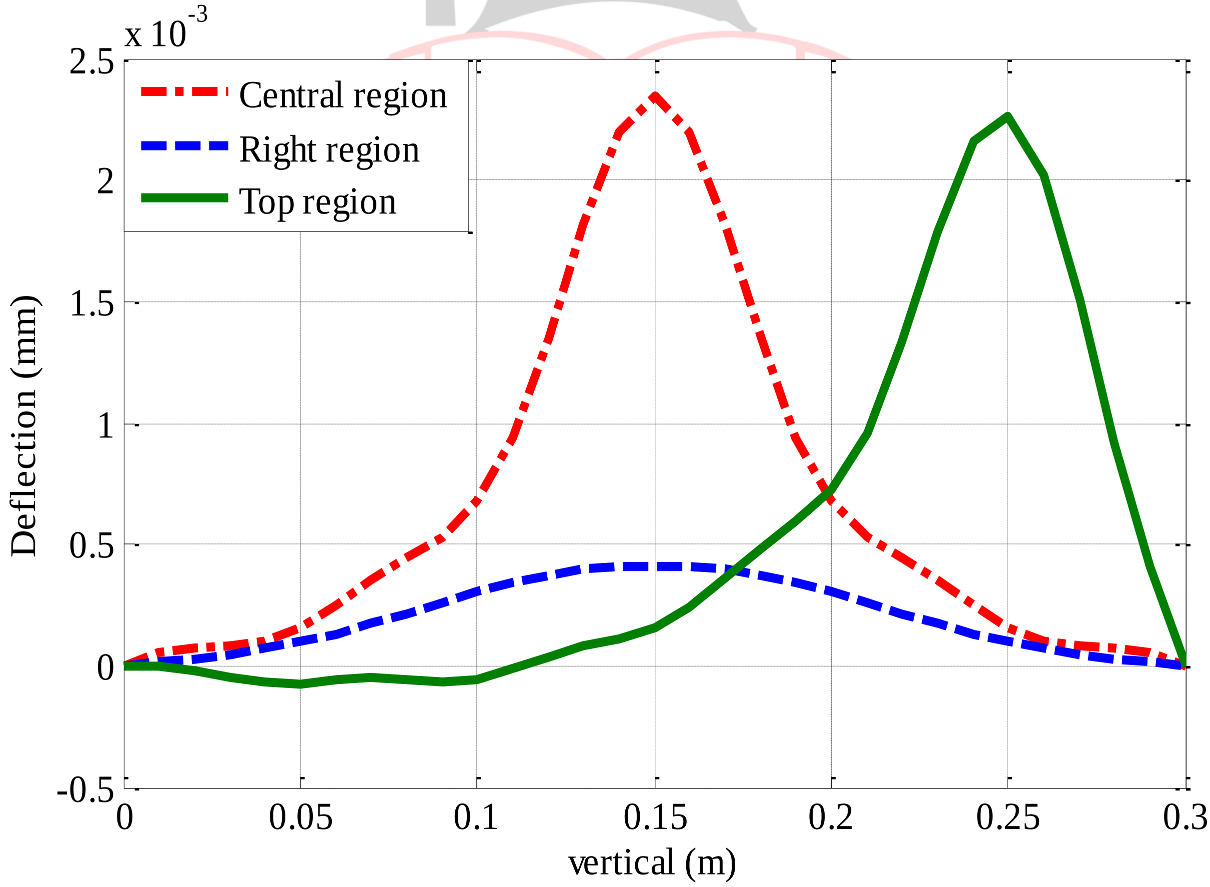

Fig. 6. Deflection of the plate obtained from the Eq.(10) along the vertical (x=a/2) for three different locations of actuator.

4.2. DYNAMIC ANALYSIS

Table 3. Natural frequencies of Graphite/ Epoxy laminate plate obtained by Eq.(17)

 DOI : 10.35291/2454-9150.2023.0160

		n		
		1	2	3
m	1	273.6412	581.5468	1170.6301
	2	901.2686	1094.565	1560.2461
	3	1974.5497	2118.6673	2462.7712

The excitation frequency can be calculated from the Eq.(17) to excite a mode selectively. Three different excitation frequencies (270, 430 and 580 rads/s) were tested. The vibration profile of the plate excited by the piezoelectric actuator with excitation frequencies 270, 430 and 580 rads/s are shown in Fig 5, 6, 7 respectively. The frequencies 270 and 580 rads/s are close to resonant frequencies of mode (1,1) and mode (1,2).The frequency 430 rads/s is far from resonant frequencies of modes (1,1) and (1,2). The maximum amplitude excited by three different sizes of actuators embedded on the center of the plate at three different excitation frequencies are listed in the Table 4. The positions of the patches in Table 5 are corresponding to Table 1.

Table 5. Maximum vibration amplitude (x10^{-3} mm) excited by three different sizes of actuators embedded on the center of the plate with different excitation frequencies.

Size	G 1195				PZT 5H		
	270	430	580		270	430	580
60mm×40mm	0.030759	0.000215	0.000306		0.044497	0.000312	0.000443
80mm×60mm	0.060395	0.000433	0.000516		0.087369	0.000628	0.000746
100mm×80mm	0.098189	0.000745	0.000657		0.14204	0.001079	0.000950

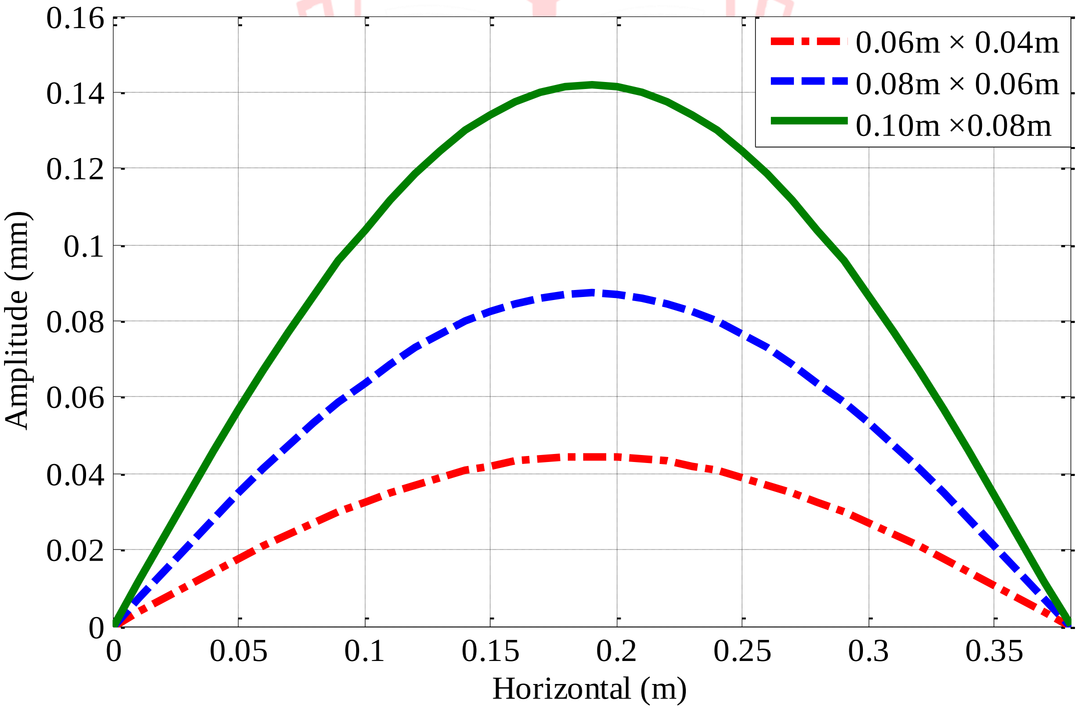

Fig. 7. Vibration amplitude of the plate obtained from the Eq.(16) along the horizontal (y=b/2) for three different sizes of actuator on the center of the plate for excitation frequency 270rad/s.

 DOI : 10.35291/2454-9150.2023.0160

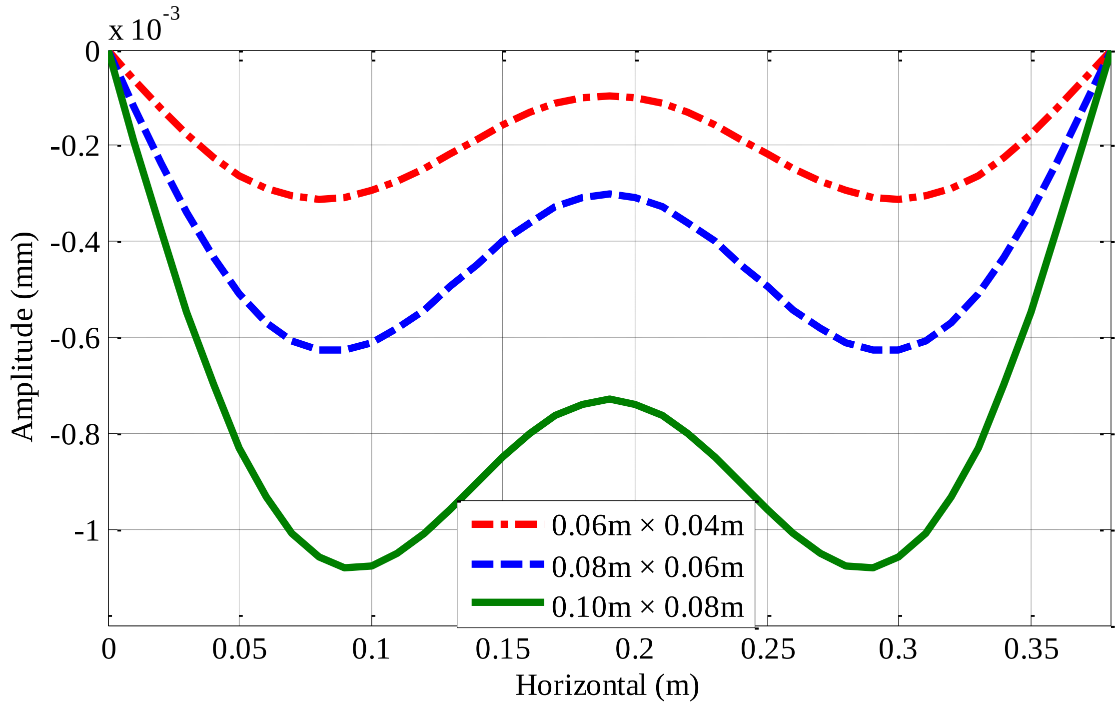

Fig. 8. Vibration amplitude of the plate obtained from the Eq.(16) along the horizontal (y=b/2) for three different sizes of actuator on the center of the plate for excitation frequency 430rad/s.

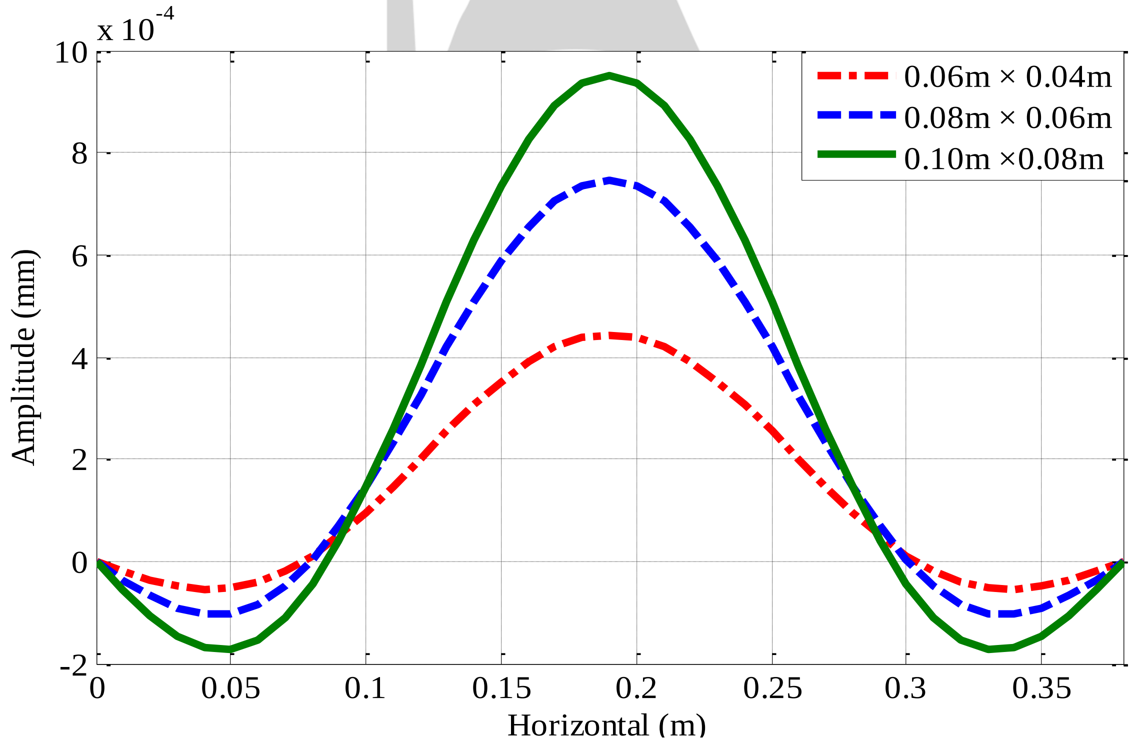

Fig. 9. Vibration amplitude of the plate obtained from the Eq.(16) along the horizontal (y=b/2) for three different sizes of actuator on the center of the plate for excitation frequency 580rad/s.

V. CONCLUSION

Piezoelectric actuators are used for shape control in the structures. By using theory of elasticity and plate theory, the deflection and amplitude of vibration of a simply supported plate subjected to electrical voltages are obtained. The effect of deflection by varying size and location of actuators on the plate are presented through parametric study. The response of plate at various excitation frequencies is also presented. The finite element solution for the deflection and amplitude for the composite plate will be the subject of future work.

 DOI : 10.35291/2454-9150.2023.0160

REFERENCES

[1] E. Crawley and J.de Luis, Use of piezoelectric actuators as elements of intelligent structures, AIAA Journal. 25 (1987) 1373-1385.

[2] Shiuh-Chuan Her and Chi-Sheng Lin, Deflection of cross-ply composite laminates induced by piezoelectric actuators. Sensors,10 (2010) 719-733.

[3] Shiuh-Chuan Her and Chi-Sheng Lin, Vibration analysis of composite laminate plate excited by piezoelectric actuators. Sensors, 13 (2013) 2997-3013.

[4] R.L. Clark, M.R. Flemming and C.R. Fuller, Piezoelectric actuators for distributed vibration excitation of thin plates: A comparison between theory and experiment, ASME Journal. 115 (1993) 332-339.

[5] E.K. Dimitriadis, C.R. Fuller and C.A. Rogers, Piezoelectric actuators for distributed vibration excitation of thin plates. ASME Journal. 113 (1991) 100-107.

[6] Shiuh-Chuan Her and Chi-Sheng Lin, Deflection of cross-ply composite laminates induced by piezoelectric actuators. Sensors,10 (2010) 719-733.

[7] H.S. Tzou and H.Q. Fu, A study on segmentation of distributed piezoelectric sensors and actuators, ASME Journal. 38 (1992) 239-246.

 DOI : 10.35291/2454-9150.2023.0160