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Optimizing Operators with Different kinds of Water Footprint of Crop Planting and Trading Techniques in Bhavanisagar River Basin, Erode

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Abstract - This article associate water footprint theory, multi-objective programming model(MOPM)and Goal programming (GP) into a general optimization framework to help seek the optimal crop planting patterns for the agricultural water management(AWM) sys- tem.The modeling framework can not only address the system objectives and tackles uncertainties by using fuzzy sets, but also support the cleaner production of crops by controlling the portion of green, blue and grey water components control, water-food nexus, balance of crop trade benefit and water footprint loss of trading crops. This work is applied to agricultural water planning and management in Bhavanisagar River Basin, Erode. The article results indicated that more mesophytic and xerophytic crops need to be increase over most districts. The optimal crop planting pattern(under γ = 0.75) would reduce blue and grey water in the central and southeastern, and almost all districts, respectively. When the credibility level increased from 0.55 to 0.95,the optimal economic water productivity would decrease with decreasing greu and blue water footprints, increasing trade benefit and decreasing footprint loss. It results that a higher credibility searching a better objectives. Thus, the proposed modelling framework could help obtain a series of crop planting patterns under credibility level is 0.75 and ensure to optimize the water footprint of crop planting and trading underuncertainties.

I. INTRODUCTION

Shortage of water resources, triggered by climate change, population rise, urban- ization and economic development, is becoming one of the human crises over the world. Agriculture accounts for about 70% of total global freshwater withdrawals from watercourses and aquifers to date and some agricultural activities(e.g. improper appli- cation of fertilizers and pesticides) contribute to nonpoint source pollution and water quality deterioration in rivers and lakes, making agricultural water management(AWM) increasingly important. Optimization of crop planting pattern is one of the agriculturalwater-saving management techniques, and has the potential to improve the water productivity of AWM system. However, such a process is fraught with complexities as multiple targets, including increasing crop yields, reducing water consumption andguarantying food security, need to be achieved with limited arable land under the spatial diversities of crop growth and precipitation conditions. Thus, it is necessaryto have a systems analysis approach to identify the optimal crop planting pattern for effective agricultural water management.

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Formerly, a variety of mathematical models have been widely used for optimization ofcrop planting pattern in different watershed cases. For example Mainuddin et al.(1997) used a linear programming model for finding an optimal cropping pattern and ground-water abstraction rule to ensure proper utilization of the available land and groundwater resources in an existing groundwater development project in Thailand. Gorantiwar and Smout(2005) integrated a linear programming model and reservoir water balance model to allocate land area and water resources optimally to different crops in a medium irrigation district situated on the Karha river of Maharashtraa State, India, where the objective is to generate maximum net benefits of agricultural production within the constraints of food requirements, land resources, canal capacity, and water resourcesfrom a reservoir.Karamouz et al.(2010) developed a crop water-benefit function-based nonlinear programming model for the determination of optimal cropping pattern, considering water allocation priorities and surface water and groundwater availability, and applied it to eight irrigation districts in Tehran province, Iran. Zhang et al.(2018) proposed an inexact optimization model to address the uncertain information from the stochastic features of available surface and underground water resources, and indicate the optimal crop planting pattern and irrigation scheme for a sub-basin of the Heihe River Basin, northwest China.

Traditional optimization techniques have made significant contributions in optimal use of available water and land resources for the maximization of net benefits froncrops; however, these studies often focused on control of the physical water(surface-and ground-water) consumptions of agricultural sectors. Water footprint, firstly introduced by Hoekstra(2003), may provide a new angle to address the extent of water usage with respect to human activities on water resources (Hoekstra and Chapagain, 2006). Within the context of agricultural systems, the water footprint of growing crops (also denoted as virtual water content of crops) refers to the amount of water consumed during the crop growth period, and it has three components including green, blue and grey water. Green water is the soil moisture from precipitation, blue water is the freshwater stored in lakes, rivers and reservoir and groundwater aquifer, and grey water refers to the volume of freshwater that is needed to assimilate pollutant loading in abiding by the water quality standards (Mekonnen and Hoekstra, 2011). Compared with physical water, water footprint is a more useful tool for achieving cleaner production of crops in actual agricultural water management practices, because it can not only improve the efficiencies in the use of fresh water by increasing the green water footprint and reducingthe blue water footprint, but also mitigate the production of wastewater by controlling the grey water footprint. According to the **plant adaptations to habitats** , it is classified into four types: they are **Mesophytes, Xerophytes, Halophytes and Hydrophytes**. Mesophytes are the crops adapted to a habitat with adequate water, Xerophytes are the crops adapted to a dry habitat, Halophytes are the crops adapted to salty habitat and Hydrophytes are the crops adapted to a freshwater habitat. In recent years, some studies coupling the water footprint theory and multi-objective programming model(MOPM) with the objectives covering both benefit-related and water footprint related aspects, have emerged (Mojtabavi et al., 2018, Sedghamiz et al., 2018a, 2018b; Su et al., 2014; Ye et al., 2018). For examples Mekonnen and Hoekstra (2010) assessed the current water footprint and virtual water trade of crops and introduced a Mopm to identify the optimal crop planting patterns for the maximum net benefit of crops and the minimum water footprint of the Qazvin Plain, Iran. Suet al.(2014) subdivided the water footprint of crops into blue and green water and developed a MOPM for supporting water resources allocation in the Shiyang River basin, China; the considered multiple objectives involved agricultural net beneit, water-user fairness, and green-water use proportion. Ye et al. (2018) proposed a MOPM model to optimally allocate physical and virtual water resources for various water users in Beijing, China, in light of the balance between economy and environment. Sedghamiz et al. (2018a) introduced a MOPM model to help distribute water to agricultural and environmental users using virtual water concept; the factors involved in model objectives included wquity, agricultural benefit, green water usage and environmental water shortage. Later on, Sedghamiz et al. (2018b) developed a bi-level programming mode to optimize agricultural water allocation and crop cultivation area in Golestran province, Iran, where a MOPM aiming at maximizing the benefit from selling water to the agricultural sectors and the share of green water in total water footprint was set as the upper level model(i.e. leader), and a single objective programming of maximizing benefits fore the selected crop patterns was designed as the lower level model(i.e. followers).

II. METHODOLOGY

Consider an AWM system at a watershed scale, where the decision makers are facing the task of designing crop planting patterns in various administrative districts. It is expected to use the limited land resources efficiently to maximize the ratio between agricultural output and water footprint(i.e., economic water productivity), and at the same time, address the uncertainties originated from the subjective human judgement.A general optimization framework incorporating water footprint theory. Overall, the objective function considers both the total water footprint of planting crops over the basin and the total benefit of planting crops. The major constraints

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include restrictionsto green, blue and grey components of water footprint, water footprint saving of trading crops, land resources, and the balance between food supply and demand. The uncertainties in the framework are described by fuzzy sets. Fermatean fuzzy multi-objective goal programming (FFMOGP) is a popular area of research in fuzzy decision-making, where multiple objectives are to be optimized simultaneously subject to certain constraints. In recent years, there has been growing interest in incorporating the concept of fermatean fuzzy sets into FFMOGP. This concept has been used to develop more efficient and effective solutions to the FFMOGP problem.

Nomenclature

x_{ij}	Planting area of crop j in district i
y_{ij}	Yield of the crop j in district i
W_p	Water footprint of the crop in the district
TW	Total water footprint of the basin
AW_{Bl}	Current blue water footprint of the crop
AW_{Gr}	Current grey water footprint of the crop
AW_{Gn}	Current green water footprint of the crop
η_{Bl}	Uncertain distribution for Blue water footprint
η_{Gr}	Uncertain distribution for Blue water footprint
η_{Gn}	Uncertain distribution for Blue water footprint
X_n	Minimum value of the n^{th} objective
Y_n	Center value of the n^{th} objective
κ	Uncertain distribution for minimum of the water footprint
μ	Uncertain distribution for maximum of net profit
Φ	delight level of defined multi-objective model
Γ	delight level of defined multi-objective model
γ	Mean credibility of occurence of a fuzzy event

1.1 Mathematical Formulation

1.1.1 Fermatean fuzzy set

A Fermatean fuzzy set $\Re$in is an object hosting those methods,

$$\Re = \langle x, \rho_F(x), \sigma_F(x)\rangle: x \in X$$

where $\rho_F(x) : X \to [0,1]$ and $\sigma_F(x) : X \to [0,1]$ which includes the possibility$0 \leq (\rho_F(x))^3 + (\sigma_F(x))^3 \leq 1$, for all $x \in X$.

For any FFS $\Re$and $x \in X$

$$\prod(x) = \sqrt[3]{1 - (\rho F(x))^3 - (\sigma F(x))^3}$$

is determined as the indeterminacy extent of to $\Re$.

In the welfare of clearness, we shall note the sign $\Re = (\rho_F, \sigma_F)$ for the FFS

$\Re = \langle x, \rho_F(x), \sigma_F(x)\rangle: x \in X$.

1.1.2 Fermatean Fuzzy Linear Programming Problem

Fermatean fuzzy linear programming is a mathematical optimization technique used to solve decision-making problems involving multiple, conflicting objectives anduncertain data, where the objective functions and constraints are expressed using Fermatean fuzzy sets.

Minimize $A^n(\mathrm{h}) = \sum_{i=1}^{K}\sum_{j=1}^{L}\left[\theta_{ij}^{-1}\left(1-\rho_{ij}\right)\right]$

Subject to constraints

$\sum_{i=1}^{l} x_{ij}y_{ij} \leq \quad K_j^{-1}(\emptyset_i)$, i=1,2,3,….n

$\sum_{j=1}^{k} x_{ij}y_{ij} \leq \quad \mu_i^{-1}(\chi_i)$, i=1,2,3,….m

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for all $x_{ij} \geq 0$, $i = 1, 2, 3, \ldots, n, j$ $1, 2, 3, \ldots, m$.

If the X,Y and Z is the lower value, middle value and upper value for every $A^n(h)$, the fuzzy triangle membership function $\mu_n(A^n(h))$ for multi-objective goal programming problem as follows

$$\mu_n(A^n(h)) = \begin{cases} 0 \ \ if \quad A^n = X_n \\ S^n \ if \ \ X_n < A^n < Z_n \\ R^n \ if \ \ Y_n < A^n < Z_n \\ 1 \ \ if \quad A^n = Z_n \end{cases}$$

where $X_n \neq Y_n \neq Z_n$, n=1,2,3,….m. Otherwise $A^n < X_n, Z_n < A^n$ are omitted

Minimum $\boldsymbol{\phi}$

Subject to contraints

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{BN.ij} \leq (1 - \eta_{BN}) - (1 - 2\gamma).(\tilde{\eta}_{BN} - \eta_{BN})$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{GY.ij} \leq (1 - \eta_{GY}) - (1 - 2\gamma).(\tilde{\eta}_{GY} - \eta_{GY})$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}y_{ij}AW_{GN.ij} \geq TW'_{GN}$$

1.1.3 Fermatean fuzzy score function

Let $\pi_\varphi = \langle \rho\varphi, \sigma\varphi \rangle$ be any FFS, then score function of π_φ characterized by $\pi_\varphi(\varphi)$

1.1.4 Fermatean Fuzzy Goal Programming Problem

$$= \frac{1}{3}(\rho_\varphi^3 - \sigma_\varphi^3).$$

Fermatean fuzzy goal programming is a mathematical optimization technique used to solve decision-making problems involving multiple, conflicting objectives that are expressed using fuzzy goals.

$0 \leq \boldsymbol{\phi} \leq 1.0$, for all $x_{ij} \geq 0, y_{ij} \geq 0, i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, m.$

To formulate $(F2)$ as a goal programming sample, approving (over) and adverse (under)attainment variables have been integrated with the objectives.

1.1.5 Fermatean Fuzzy Multi-Objective Goal Programming Problem

Fermatean fuzzy multi-objective goal programming is an extension of fuzzy goal pro- gramming that allows for the optimization of multiple, conflicting objectives expressed using fuzzy goals, while incorporating fermatean membership functions to account for uncertainty in the goal values.

Minimum $\boldsymbol{\phi}$Maximum Γ

subject to contraints

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{BN.ij} \leq (1 - \eta_{BN}) - (1 - 2\gamma).(\tilde{\eta}_{BN} - \eta_{BN})$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{GY.ij} \leq (1 - \eta_{GY}) - (1 - 2\gamma).(\tilde{\eta}_{GY} - \eta_{GY})$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}y_{ij}AW_{GN.ij} \geq TW'_{GN}$$

$0 \leq \Phi \leq 0.1$and $0 \leq \Gamma \leq 0.1$ for all $x_{ij} \geq 0, y_{ij} \geq 0, i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, m$. To formulate $(F2)$ as a goal programming sample, approving (over) and adverse (under)attainment variables have been integrated with the objectives.

1.1.6 Fermatean Fuzzy Splintery function

Let $C = \langle \alpha i, \gamma i, \beta i, \delta i \rangle$ be any FFS, then Splintery function of C characterized by

$C_i(I)$ and is described as

$$C_i(I) = \frac{I+2J+2K+L+(1-2M)(J+K)}{6}$$

where I is $\alpha + 2$, J is γ 1, K is β 2, L is $\delta + 2$ and $\alpha, \gamma, \beta, \delta$ are the membership values of the fermatean fuzzy number at the points 0, $M/2$, $(1+M)/2$ and 1 respectively. Here C

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is the Splintery function and the stability of membership degree is obtainedby $IJKL$, where IJ is the stability of maximum and minimum, KL is the direction of maximum and minimum stability. The variables α, β are maximum and minimum membership degrees, and γ, δ are the maximum and minimum non-membership degrees.

1.2 Solution proposed method

This article solved a Fermatean fuzzy multi-objective programming problem in two methods. This method has one fermatean fuzzy multi-objective programming problem and another method is a fermatean fuzzy multi-objective goal programming.

1.2.1 Fermatean fuzzy multi-objective goal programming problem

Step 1: Choose the fixed values with undetermined parameters and define the decision variables, objective functions and fuzzy constraints.

Step 2: Converting the developed Multi-objective Programming problem into Fermatean fuzzy set.

Step 3: Obtain Fermatean fuzzy numbers single objective programming problem from Fermatean fuzzy numbers multi-objective programming problem by using Splintery function.

Step 4: Develop Fermatean fuzzy multi-objective goal programming problem fromstep 3.

1.2.2 Fermatean fuzzy goal programming problem

Step 1: Choose the fixed values with undetermined parameters and define thedecision variables, objective function and fuzzy constraints.

Step 2: Converting the developed Goal Programming into Fermatean fuzzy set.

Step 3: Obtain Fermatean fuzzy number goal programming model from Fermatean fuzzy numbers linear programming problem by using Splintery function.

Step 4: Solve the simple goal programming problem and obtain the optimumsolution using the solver.

Step 5: Transforming minimum or maximum water footprint from step 4 intominimum water footprint.

Step 6: Calculating the triangle membership function using minimum waterfootprint.

Minimum ΦMaximum Γ

subject to contraints

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{BN.ij} \le (1-\eta_{BN}) - (1-2\gamma).(\tilde{\eta}_{BN} - \eta_{BN})$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{GY.ij} \le (1-\eta_{GY}) - (1-2\gamma).(\tilde{\eta}_{GY} - \eta_{GY})$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}y_{ij}AW_{GN.ij} \ge TW'_{GN}$$ $0 \le \Phi \le 0.1$and $0 \le \Gamma \le 0.1$ for all $x_{ij} \ge 0, y_{ij} \ge 0, i = 1,2,3,\dots,n, j = 1,2,3,\dots,m.$

Step 7: Stop

Min $Z = \Phi$Max $Y = \Gamma$

subject to constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}y_{ij}.W_P \le TW.($$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{BN.ij} + d_1^- - d_1^+ \le (1-\eta_{BN}) - (1-2\gamma).(\tilde{\eta}_{BN} - \eta_{BN})$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{GY.ij} + d_2^- - d_2^+ \le (1-\eta_{GY}) - (1-2\gamma).(\tilde{\eta}_{GY} - \eta_{GY})$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}y_{ij}AW_{GN.ij} + d_3^- - d_3^+ \ge TW'_{GN}$$

$0 \le \Phi \le 0.1$and $0 \le \Gamma \le 0.1$ for all $x_{ij} \ge 0, y_{ij} \ge 0, i = 1,2,3,\dots,n, j = 1,2,3,\dots,m.$

Step 5: Solve the simple Multi-objective Programming Problem in step 5, and weobtain the optimum solution using the solver.

Step 6: Calculating the triangle membership function using goals.

Minimum ΦMaximum Γ

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Subject to contraints

$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{BN.ij} \leq (1-\eta_{BN}) - (1-2\gamma).(\tilde{\eta}_{BN} - \eta_{BN})$

$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij}y_{ij}AW_{GY.ij} \leq (1-\eta_{GY}) - (1-2\gamma).(\tilde{\eta}_{GY} - \eta_{GY})$

$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}y_{ij}AW_{GN.ij} \geq TW'_{GN}$ $0 \leq \Phi \leq 0.1$ and $0 \leq \Gamma \leq 0.1$ for all $x_{ij} \geq 0, y_{ij} \geq 0, i = 1,2,3,\dots,n, j = 1,2,3,\dots,m$.

Step 7: Stop

III. RESULT AND DISCUSSION

To attest the viability of the offered model, consider a new example has been constructed that is the combination of the crop yields and water footprint. Here water footprint and Yield in the need are also thought uncertain. In every case, the decision-making uses the same to define all the parameters, using the Fermatean Fuzzy Multi-Objective Goal Programming with Splintery function.

1.3 Heuristic Example

Solve Fermatean Fuzzy Goal Programming problem and Fermatean Fuzzy Multi- Objective Goal Programming problem using the Splintery function.

Consider the certain areas of the Bhavanisagar River Basin, there the farmers weregrowing four types of crops that is mesophyte, hallophyte, hydrophte and xerophyte. The farmer wants to optimize the production of these crops while minimizing three types of water footprint: Blue, Green and Grey. Blue water footprint refers to the volume of surface and ground water used for irrigation, Green water footprint refers to the volume of rainwater used for crop growth, and Grey water footprint refers to the volume of water required to dilute pollutants to acceptable. The following table shows the necessary data for substitution:

Crop type	**Yield** tons/ hectare	**Blue WF** cubic meters/tons	**Green WF** cubic meters/tons	**Grey WF** cubic meters/tons	**Labor** hours/ hectare
Mesophyte	7.5	1500	2400	600	300
Hallophyt	12	1800	3000	900	450
Xerophyte	9	1200	2100	750	360
Hydrophyte	15	2400	3600	1200	600

0.2,0.7,0.5,0.9 are the weights used for this Fermatean fuzzy Goal Programming problem. The Fermatean fuzzy numbers are denoted by α, γ, β and δ. Assuming the Fermatean fuzzy numbers and formulate the given data into two models. By using the solver we can obtain the optimum values.

1.3.1 Fermatean Fuzzy Goal Programming Problem Formulation of Fermatean Fuzzy Goal Programming Problem

$$\begin{aligned} Minimize\ W = & |Bl_1 - 1500| + |Bl_2 - 1800| + |Bl_3 - 1200| + |Bl_4 - 2400| + \\ & |Gn_1 - 2400| + |Gn_2 - 3000| + |Gn_3 - 2100| + |Gn_4 - 3600| + \\ & |Gr_1 - 600| + |Gr_2 - 900| + |Gr_3 - 750| + |Gr_4 - 1200| + \\ & + 7.5 \times x_1 + 12 \times x_2 + 9 \times x_3 + 15 \times x_4 \end{aligned}$$

subject to constraints

$$x_1 + x_2 + x_3 + x_4 \leq 4000$$

$$300 \times x_1 + 450 \times x_2 + 360 \times x_3 + 600 \times x_4 \leq 200000$$

$x_1, x_2, x_3, x_4 \geq 0$

where x_1 - No. of. acres allocated mesophyte planted

x_2 - No. of. acres allocated hallophyte plantedx_3 - No. of. acres allocated xerophyte planted x_4 - No. of. acres allocated hydrophyte plantedBl denotes the blue water footprint

Gn denotes the green water footprint

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Gr denotes the grey water footprint

By using the solver we obtain the values that x_1 is 711.11 acres of mesophytic crops, x_2 is 1804.67 acres of hallophytic crops, x_3 is 495.56 acres of xerophytic crops, x_4 is 1988.89 acres of hydrophytic crops are planted with the usage of Blue water footprintis 1, 321, 777m^3, Green water footprint is 2, 292, 444m^3 and Grey water footprint is398, 666m^3.

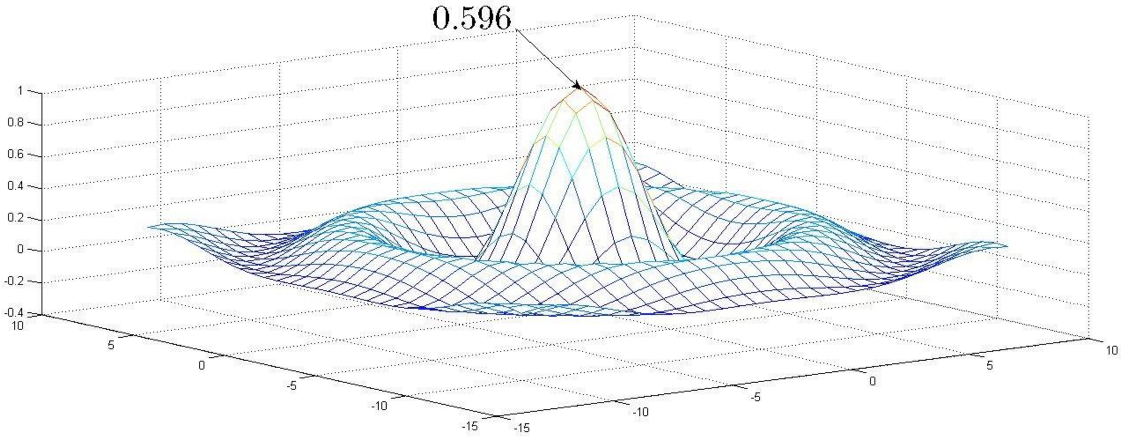

Figure 1: Fermatean Fuzzy Goal Programming Problem in Sine Direction

From this optimum values, we can increase the amount of corn and rice need tobe planted and decrease the amount of wheat and soyabean need to be planted which balances the yield and water footprint of each crop.

1.3.2 Fermatean Fuzzy Multi-Objective Goal Programming Problem

Formulation of Fermatean Fuzzy Multi-Objective Goal ProgrammingProblem

$$MinimizeW = |Bl_1 - 1500| + |Bl_2 - 1800| + |Bl_3 - 1200| + |Bl_4 - 2400| +$$

$$|Gn_1 - 2400| + |Gn_2 - 3000| + |Gn_3 - 2100| + |Gn_4 - 3600| +$$

$$|Gr_1 - 600| + |Gr_2 - 900| + |Gr_3 - 750| + |Gr_4 - 1200|$$

$$MaximumY = 7.5 \times x_1 + 12 \times x_2 + 9 \times x_3 + 15 \times x_4$$

subject to constraints

$$x_1 + x_2 + x_3 + x_4 \leq 4000$$

$$300 \times x_1 + 450 \times x_2 + 360 \times x_3 + 600 \times x_4 \leq 200000$$

$x_1, x_2, x_3, x_4 \geq 0$

where x_1 - No. of. acres allocated mesophyte planted

x_2 - No. of. acres allocated hallophyte plantedx_3 - No. of. acres allocated xerophyte planted x_4 - No. of. acres allocated hydrophyte planted*Bl* denotes the blue water footprint

Gn denotes the green water footprint

Gr denotes the grey water footprint

By using the solver we obtain the values that x_1 is 710.57 acres of mesophytic crops, x_2 is 1804.88 acres of hallophytic crops, x_3 is 494.33 acres of xerophytic crops and x_4 is1988.22 acres of hydrophytic crops are planted with the usage of Blue water footprintis 1, 321, 813.54m^3, Green water footprint is 2, 292, 414.04m^3 and Grey water footprint is 398, 768.79m^3.

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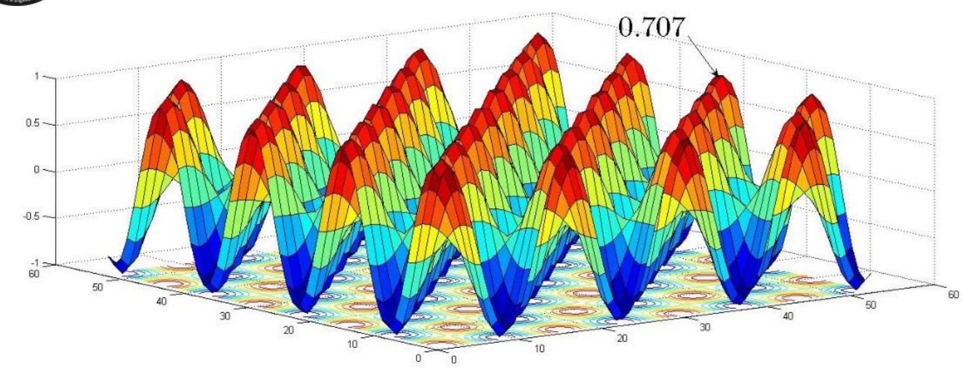


Figure 2: Fermatean Fuzzy Multi-Objective Goal Programming Problem in Sine Direction

From this optimum values, we can increase the amount of hallophyte and hydrophyteneed to be planted and decrease the amount of mesophyte and xerophyte need to be planted which balances the yield and water footprint of each crop.

The percentage of triangle membership and non-membership are calculated by usingthe formula:

- Percentage of triangle membership = (Optimum value − Lower Bound)/(UpperBound −Lower Bound) ×100%

  Percentage of non-triangle membership = 100% − Percentage of triangle member-ship

A fuzzy membership is given for both the fermatean fuzzy goal programming problem and fermatean fuzzy multi-objective goal programming problem. Thus, whenconsidering according to the fuzzy membership one can either choose the fermatean fuzzygoal programming problem or the fermatean fuzzy multi-objective goal programmingproblem

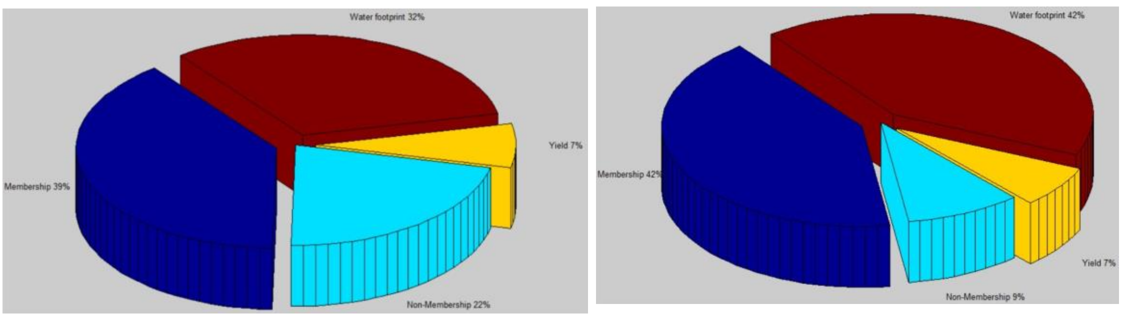
IV. COMPARATIVE STUDY

We consider three types of water footprint and converting into fuzzy numbers. We use the Splintery function to convert it into a crisp value. We solve that crisp value with the Fermatean Fuzzy Goal Programming Problem concept and by the Fermatean FuzzyMulti-Objective Goal Programming Problem concept where we analyze the solutions.

Method	**Fermatean Fuzzy Goal Programming**	**Fermatean Fuzzy Multi-Objective Goal Programming**
Minimize total water footprint	4400	4265.60
Maximize Yield	11.44	10.56
Membership Percentage of Triangular FermateanFuzzy Number	63.23%	65.16%
Non-Membership Percentage of Triangular FermateanFuzzy Number	35.04%	14.58%

Now we compare the minimum water footprint and maximum yield from Goal Pro- gramming and Multi-Objective Goal Programming.

Fermatean fuzzy goal programming (FFGP) and Fermatean fuzzy multi-objective goal programming (FFMOGP) are two techniques of fuzzy optimization used to solve multi-objective and single-objective problems, respectively. In the given example, both



(a) FFGP (b) FFMOGP

FFGP and FFMOGP were applied to optimize the production of four crops while minimizing three types of water

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footprint.

Comparing the results of FFGP and FFMOGP, we see that FFMOGP gave a higheryield with lower water footprint usage compared to FFGP. This indicates that FFMOGPwas more successful in balancing the trade-off between multiple objectives. However, it is important to note that FFGP is more suitable when dealing with single-objective problems where only one goal needs to be optimized. On the other hand, FFMOGPis better suited to handle multi-objective problems where multiple goals need to be optimized simultaneously.

Furthermore, looking at the membership and non-membership triangle percentages for both FFGP and FFMOGP, we see that both techniques were able to generate highly satisfactory solutions with very small non-membership percentages. This indicates that both methods are highly reliable for solving complex optimization problems.Overall, both FFGP and FFMOGP provided effective solutions to optimize crop production while minimizing water footprint.

V. CONCLUSION

In this paper, the Goal Programming problem implicating targets has been solved Multi-Objective Programming problem has been introduced and transformed intoa fractional programming problem with Fermatean Fuzzy Numbers(FFNs). These uncertain pieces of information were processed in determination making using FermateanFuzzy Sets. An algorithm has been proposed for proving Goal Programming problems concerning Fermatean Fuzzy Parameters(FFPs) and also the optimum value has been completed by using simple LPP in Fermatean Fuzzy Numbers(FFNs). Using the fermatean fuzzy goal programming approach, a model was developed, which provided different trade-offs between yield and water footprint. The single-objective model focused only on minimizing the water footprint, while the multi-objective model provided a more balanced solution that considered both yield and water footprint. The comparison between these models showed that the multi-objective approach provided the best trade-off between yield and water footprint, while the single-objective approach resulted in the lowest water footprint, but also the lowest yield. Overall, the fermatean fuzzy goal programming approach provided a useful tool for decision-makers to optimize crop production while minimizing water footprint. The results showed that different crops have different yields and water footprints, and the optimal solution depends on the specific goals and priorities of the decision-maker. By considering multiple objectives and using fuzzy logic, the fermatean fuzzy goal programming approach canhelp to find more robust and balanced solutions that take into account the complex and uncertain nature of agricultural systems.

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