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Coupled Fixed Points For Jungck Type Maps With Rational Contraction On Dislocated Quasi B-Metric Spaces

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Abstract. In this paper, we prove a unique common coupled fixed point theorem for Jungck type maps satisfying certain rational contraction condition in dislocated quasi b-metric space and also we provide an example to illustrate our theorem. We also obtain some recent results as corollaries.

Keywords: Dislocated quasi b-metric spaces, compatible maps, coupled fixed points, point of coincidence.

I. INTRODUCTION

In 1992, S.Banach [4] established remarkable fixed point theorem known as " Banach contraction principle". This contraction principle assures the existence and uniqueness of fixed points of self maps on metric spaces. Hitzler [8] and Hitzler and Seda [7] introduced the notion of dislocated metric spaces and extended Banach contraction principle in such spaces. Zeyada et.al [13] initiated the concept of dislocated quasi metric spaces and generalized the results of Hitzler and Seda [7] in dislocated quasi metric spaces.
Bakhtin [3] introduced the concept of b-metric spaces. The concept of Bakhtin is extensively used by Czerwic [5] in connection with some problems concerning with the convergence of non measurable functions with respect to measure. Recently Klin-eam and Suanoom [9] introduced the concept of dislocated quasi b-metric spaces and proved some fixed point theorems in it by using cyclic contractions. The objective of this paper is improve the some previous results without using the continuity of maps.

First we recall some known definitions and lemmas. Throughout this paper, we assume that R^+ is the set of all non-negative real numbers.

Definition1.1 Let X be a non-empty set , $s \geq 1$ (a fixed real number) and $d: X \times X \to R^+$ be a function. Consider the following conditions on d.

(1.1.1) $d(x,x) = 0, \forall x \epsilon X$

(1.1.2) $d(x,y) = d(y,x) = 0 \Rightarrow x = y, \forall x, y \, \epsilon X$

(1.1.3) $d(x,y) = d(y,x), \forall x, y \epsilon X$

(1.1.4) $d(x,y) \leq d(x,z) + d(z,y), \forall x, y, z \, \epsilon X$

(1.1.5) $d(x,y) \leq s[d(x,z) + d(z,y)], \forall x, y, z \, \epsilon X$.

(i) If d satisfies (1.1.2), (1.1.3) and (1.1.4) then d is called a dislocated metric and (X,d) is called a dislocated metric space.

(ii) If d satisfies (1.1.1),(1.1.2) and (1.1.4) then d is called a quasi metric and (X,d) is called a quasi metric space.

(iii) If d satisfies (1.1.2) and (1.1.4) then d is called a dislocated quasi metric or dq-metric and (X,d) is called a dislocated quasi metric space.

(iv) If d satisfies (1.1.1),(1.1.2),(1.1.3) and (1.1.4) then d is called a metric and (X,d) is called a metric space.

(v) If d satisfies (1.1.1),(1.1.2),(1.1.3) and (1.1.5) then d is called a b-metric and (X,d) is called a b-metric space.

(vi) If d satisfies (1.1.2) and (1.1.5) then d is called a dislocated quasi b-metric and (X,d) is called a dislocated quasi b-metric space or a dq b-metric space.

Definition 1.2: Let (X,d) be a dq b-metric space. A sequence $\{x_n\}$ in (X,d) is said to be

(i) dq b-convergent if there exists some point $x \epsilon X$ such that $\lim_{n\to\infty} d(x_n, x) = 0 = \lim_{n\to\infty} d(x, x_n)$.

In this case x is called a dq b-limit of $\{x_n\}$ and we write $x_n \to x \text{ as } n \to \infty$.

(ii) Cauchy sequence if $\lim_{m,n\to\infty} d(x_n, x_m) = 0 = \lim_{m,n\to\infty} d(x_m, x_n)$.

The space (X,d) is called complete if every Cauchy sequence in X is dq b-convergent.

One can prove easily the following

Lemma 1.3: Let (X,d) be a dq b-metric space and $\{x_n\}$ be dq b-convergent to $x \epsilon X$ and $y \epsilon X$ be arbitrary.Then

$\frac{1}{s} d(x,y) \leq \lim_{n\to\infty} \inf d(x_n, y) \leq \lim_{n\to\infty} \sup d(x_n, y) \leq s \, d(x,y)$ and

$\frac{1}{s} d(y,x) \leq \lim_{n\to\infty} \inf d(y, x_n) \leq \lim_{n\to\infty} \sup d(y, x_n) \leq s \, d(y,x)$.

Note: $\frac{1}{2s} d(x,y) \leq max\{d(x,z), d(z,y)\}$ for all $x, y, z \in X$.

The notion of coupled fixed point is introduced by Bhaskar and Lakshmikantham [6] and studied some fixed point theorems in partially ordered metric spaces. Later Lakshmikantham and Ciric [10] defined coupled coincidence point and common coupled fixed points for a pair of maps and Abbas et.al [1] introduced the notion of w-compatible mappings.

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Definition 1.4 ([6]): Let X be a non-empty set .An element $(x,y)\epsilon X\times X$ is called a coupled fixed point of a mapping $F: X\times X\to X$ if $x=F(x,y)$ and $y=F(y,x)$.

Definition 1.5 ([10]): Let X be a non-empty set. An element $(x,y)\epsilon X\times X$ is called

(1) a coupled coincidence point of mappings $F: X\times X\to X$ and $f: X\to$ X if

fx = F(x,y) and fy = F(y,x).

(2) a common coupled fixed point of mappings $F: X\times X\to X$ and $f: X\to$ X if

x = fx = F(x,y) and y = fy = F(y,x).

(3) **([1]).** The pair (F, f) is called w-compatible if f(F(x,y)) = $F(fx,fy)$ *and* $f(F(y,x)) = F(fy,fx)$ whenever there exist $x,y\ \epsilon X$ *with* $fx = F(x,y)$ *and* $fy = F(y,x)$.

Definition 1.6 ([2]): Let X be a non-empty set and $f,g: X\to X$ be mappings.If there exists $x\epsilon X$ such that fx = gx then x is called a coincidence point of f and g and fx is called a point of coincidence of f and g.

we generalize Definition 1.6 for a pair of maps in which one is a coupled map as follows.

Definition 1.7: Let X be a non-empty set. Let $F: X\times X\to X$ and $S: X\to$ X. If there exists $(u,v)\in X\times X$ such that $F(u,v)=Su$ and $F(v,u)=Sv$ then (Su,Sv) is called a coupled point of coincidence of F and S and (u,v) is called a coupled coincidence point of F and S.

Definition 1.8: For a fixed real number $s\geq 1$, let $\mathbf{\Phi}_s$ denote the class of all functions $\varphi: R^+\to R^+$ satisfying the following

(φ_1) : φ is monotonically non-decreasing ,

(φ_2): $\sum_{n=1}^{\infty} s^n\varphi^n(t)<\infty$ for all t > 0,

(φ_3) : φ(t) < t for all t > 0.

(φ_4) : φ is continuous.

From (φ_1) and (φ_3) or (φ_3) and (φ_4) we have $\varphi(0) = 0$.

Recently Sumati Kumari et.al [12] proved the following

Theorem 1.9 [12]: Let (X,bd_q) be a complete dislocated quasi b-metric space with the coefficient $s\geq 1$, and let $T: X\to X$ be a mapping such that $d(Tx,Ty)\leq\varphi\big(d(x,y)\big)$ for all $x,y\in X$,where $\varphi: R^+\to R^+$ is a continuous mapping such that $\varphi(t)=0$ iff $t=0$ and $\varphi(t)<1$ for all $t>0$. If $\sum_{n=1}^{\infty}s^n\varphi^n(t)$ is bd_q converges for all $t>0$,where φ^n is the n^{th} iterate of φ then T has unique fixed point.

We note that in this theorem the authors inherently used the condition that φ is monotonically non-decreasing in proving $d(x_n,x_{n+1})\leq\varphi^n\big(d(x_0,x_1)\big)$ and $d(x_{n+1},x_n)\leq\varphi^n\big(d(x_1,x_0)\big)$.

Rahman [11] proved the following

Theorem 1.10 [11] : Let (X,d) be a complete d_q-b-metric space with fixed real number $s\geq 1$, and T be a continuous self mapping on X satisfying

(1.10.1) $d(Tx,Ty)\leq\alpha\, d(x,y)+\beta[d(x,Tx)+d(y,Ty)]+\mu\,[d(x,Ty)+d(y,Tx)]$

$\forall\, x,y\in X$, where $\alpha,\beta,\mu\geq 0$ with $s\alpha+(1+s)\beta+2(s^2+s)\mu<1$

Then T has a unique fixed point in X.

In this paper we mainly prove a unique common coupled fixed point theorem for Jungck type maps satisfying certain rational contraction condition in dislocated quasi b-metric spaces and also we provide an example to illustrate our theorem. We also obtain some recent results as corollaries.

II. Main Result

Now we give our main Theorem.

Theorem 2.1: Let (X,d) be a dislocated quasi b-metric space with fixed real number $s\geq 1$ and $F: X\times X\to X$ and $S: X\to X$ be mappings satisfying

(2.1.1) $d\big(F(x,y),F(u,v)\big)$

$$\leq\varphi\left(max\left\{\begin{array}{c} d(Sx,Su),d(Sy,Sv),\frac{1}{s}d\big(Sx,F(x,y)\big),\\ \frac{1}{s}d\big(Sy,F(y,x)\big),\frac{1}{s}d\big(Su,F(u,v)\big),\frac{1}{s}d\big(Sv,F(v,u)\big),\\ \frac{1}{2s^2}d\big(Sx,F(u,v)\big),\frac{1}{2s^2}d\big(Sy,F(v,u)\big),\\ \frac{1}{2s^2}d\big(Su,F(x,y)\big),\frac{1}{2s^2}d\big(Sv,F(y,x)\big),\\ \left[\frac{d(Sx,F(x,y))\,d(Su,F(u,v))}{1+d(Sx,Su)+d(Sy,Sv)}\right],\left[\frac{d(Sy,F(y,x))\,d(Sv,F(v,u))}{1+d(Sx,Su)+d(Sy,Sv)}\right],\\ \frac{1}{2s}\left[\frac{d(Sx,F(x,y))\,d(Sx,F(u,v))}{1+d(Sx,Su)+d(Sy,Sv)}\right],\frac{1}{2s}\left[\frac{d(Sy,F(y,x))\,d(Sy,F(v,u))}{1+d(Sx,Su)+d(Sy,Sv)}\right],\\ \frac{1}{2s}\left[\frac{d(Su,F(x,y))\,d(Su,F(u,v))}{1+d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}\right],\frac{1}{2s}\left[\frac{d(Sv,F(y,x))\,d(Sv,F(v,u))}{1+d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}\right]\end{array}\right\}\right)$$

for all $x,y,u,v\in X$, *where* $\varphi\in\mathbf{\Phi}_s$,

(2.1.2) $F(X\times X)\subseteq S(X)$ and S(X) is a complete subspace of X,

(2.1.3) the pair (F,S) is w-compatible

Then F and S have a unique common coupled fixed point in $X\times X$.

Proof: Let $(x_0,y_0)\in X\times X$.

From (2.1.2), there exist sequences $\{x_n\}$ *and* $\{y_n\}$, in X such that

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$F(x_n, y_n) = Sx_{n+1}, n = 0,1,2, \dots$

$F(y_n, x_n) = Sy_{n+1}, n = 0,1,2, \dots$

Case (i): Suppose

$\max\{d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n)\} \neq 0$ for all $n = 1,2,3, \dots$

Now from (2.1.1), (φ_1) and Note, we have

$$d(Sx_n, Sx_{n+1}) = d\big(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\big)$$

$$\leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sy_{n-1}, Sy_n), \frac{1}{s} d(Sx_{n-1}, Sx_n), \frac{1}{s} d(Sy_{n-1}, Sy_n), \\ \frac{1}{s} d(Sx_n, Sx_{n+1}), \frac{1}{s} d(Sy_n, Sy_{n+1}), \frac{1}{2s^2} d(Sx_{n-1}, Sx_{n+1}),, \\ \frac{1}{2s^2} d(Sy_{n-1}, Sy_{n+1}), \frac{1}{2s^2} d(Sx_n, Sx_n), \frac{1}{2s^2} d(Sy_n, Sy_n), \\ \frac{d(Sx_{n-1}, Sx_n)\, d(Sx_n, Sx_{n-1})}{1 + d(Sx_{n-1}, Sx_n) + d(Sy_{n-1}, Sy_n)}, \frac{d(Sy_{n-1}, Sy_n)\, d(Sy_n, Sy_{n+1})}{1 + d(Sx_{n-1}, Sx_n) + d(Sy_{n-1}, Sy_n)}, \\ \frac{1}{2s}\left[\frac{d(Sx_{n-1}, Sx_n)\, d(Sx_{n-1}, Sx_{n+1})}{1 + d(Sx_{n-1}, Sx_n) + d(Sy_{n-1}, Sy_n)}\right], \frac{1}{2s}\left[\frac{d(Sy_{n-1}, Sy_n)\, d(Sy_{n-1}, Sy_{n+1})}{1 + d(Sx_{n-1}, Sx_n) + d(Sy_{n-1}, Sy_n)}\right], \\ \frac{1}{2s}\left[\frac{d(Sx_n, Sx_n)\, d(Sx_n, Sx_{n+1})}{1 + d(Sx_n, Sx_{n+1}) + d(Sy_n, Sy_{n+1})}\right], \frac{1}{2s}\left[\frac{d(Sy_n, Sy_n)\, d(Sy_n, Sy_{n+1})}{1 + d(Sx_n, Sx_{n+1}) + d(Sy_n, Sy_{n+1})}\right] \end{array} \right\} \right)$$

$$\leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sy_{n-1}, Sy_n), d(Sx_n, Sx_{n+1}), d(Sy_n, Sy_{n+1}), \\ d(Sx_{n+1}, Sx_n), d(Sy_{n+1}, Sy_n), d(Sx_n, Sx_{n-1}), d(Sy_n, Sy_{n+1}), \\ \max\{d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n+1})\}, \\ max\{d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n+1})\}, \\ max\{d(Sx_n, Sx_{n-1}), d(Sx_{n-1}, Sx_n)\}, \\ max\{d(Sy_n, Sy_{n-1}), d(Sy_{n-1}, Sy_n)\} \end{array} \right\} \right)$$

$$\leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n-1}), \\ d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \right)$$

Similarly we can show that $d(Sx_{n+1}, Sx_n) \leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n-1}), \\ d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \right)$

$$d(Sy_n, Sy_{n+1}) \leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n-1}), \\ d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \right)$$

$$d(Sy_{n+1}, Sy_n) \leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n-1}), \\ d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \right)$$

Thus $max \left\{ \begin{array}{c} d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ , d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\}$

$$\leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), d(Sy_{n-1}, Sy_n), \\ d(Sy_n, Sy_{n-1}), d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \right) \text{ -------- (1)}$$

If $max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), \\ d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n-1}) \end{array} \right\} \leq max \left\{ \begin{array}{c} d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ , d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\}$

Then from (1), we have

$$max \left\{ \begin{array}{c} d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ , d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ , d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \right)$$

which is a contradiction from (φ_3) and Case (i).

Hence from (1), we have

$$max \left\{ \begin{array}{c} d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ , d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \leq \varphi\left(max \left\{ \begin{array}{c} d(Sx_{n-1}, Sx_n), d(Sx_n, Sx_{n-1}), \\ d(Sy_{n-1}, Sy_n), d(Sy_n, Sy_{n-1}) \end{array} \right\} \right)$$

which is true for n = 1,2,3, …

Continuing in this way, we obtain

$$max \left\{ \begin{array}{c} d(Sx_n, Sx_{n+1}), d(Sx_{n+1}, Sx_n), \\ , d(Sy_n, Sy_{n+1}), d(Sy_{n+1}, Sy_n) \end{array} \right\} \leq \varphi^n(t) \text{ ------------- (2)}$$

where $t = max \left\{ \begin{array}{c} d(Sx_0, Sx_1), d(Sx_1, Sx_0) \\ d(Sy_0, Sy_1), d(Sy_1, Sy_0) \end{array} \right\}$.

Now for all positive integers n and p, consider, using (5),

$$d(Sx_n, Sx_{n+p}) \leq s\, d(Sx_n, Sx_{n+1}) + s^2 d(Sx_{n+1}, Sx_{n+2}) + \cdots + s^p d(Sx_{n+p-1}, Sx_{n+p})$$

$$\leq s\, \varphi^n(t) + s^2 \varphi^{n+1}(t) + \cdots + s^p \varphi^{n+p-1}(t), \quad \text{where } t \text{ is as in above}$$

$$\leq s^n\, \varphi^n(t) + s^{n+1} \varphi^{n+1}(t) + \cdots + s^{n+p-1} \varphi^{n+p-1}(t) \text{ since } s \geq 1$$

$$= \sum_{i=n}^{n+p-1} s^i\, \varphi^i(t) \leq \sum_{i=n}^{\infty} s^i\, \varphi^i(t).$$

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Since $\sum_{i=n}^{\infty} s^i\, \varphi^i(t)$ converges for all $t > 0$, we have $\lim_{n\to\infty} d(Sx_n, Sx_{n+p}) = 0$.

$$\begin{aligned}
d(Sx_{n+p}, Sx_n) &\le s\, d(Sx_{n+p}, Sx_{n+1}) + s\, d(Sx_{n+1}, Sx_n)\\
&\le s^2\, d(Sx_{n+p}, Sx_{n+2}) + s^2\, d(Sx_{n+2}, Sx_{n+1}) + s\, d(Sx_{n+1}, Sx_n)\\
&\le s^3\, d(Sx_{n+p}, Sx_{n+3}) + s^3\, d(Sx_{n+3}, Sx_{n+2}) + s^2\, d(Sx_{n+2}, Sx_{n+1}) + s\, d(Sx_{n+1}, Sx_n)\\
&\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\le s^{p-1} d(Sx_{n+p}, Sx_{n+p-1}) + s^{p-1} d(Sx_{n+p-1}, Sx_{n+p-2}) + \cdots + s^2\, d(Sx_{n+2}, Sx_{n+1})\\
&\qquad + s\, d(Sx_{n+1}, Sx_n)\\
&\le s^{p-1}\, \varphi^{n+p-1}(t) + s^{p-1}\varphi^{n+p-2}(t) + \cdots + s^2\, \varphi^{n+1}(t) + s\, \varphi^n(t)\\
&\le s^{n+p-1}\, \varphi^{n+p-1}(t) + s^{n+p-2}\varphi^{n+p-2}(t) + \cdots . + s^{n+1}\, \varphi^{n+1}(t) + s^n\, \varphi^n(t)\\
&= \sum_{i=n}^{n+p-1} s^i\, \varphi^i(t)
\end{aligned}$$

Since $\sum_{i=n}^{n+p-1} s^i\, \varphi^i(t)$ converges for all$t > 0$, we have $\lim_{n\to\infty} d(Sx_{n+p}, Sx_n) = 0$.

Similarly we can show that $\lim_{n\to\infty} d(Sy_n, Sy_{n+p}) = 0$ and $\lim_{n\to\infty} d(Sy_{n+p}, Sy_n) = 0$.

Thus $\{Sx_n\}$ and $\{Sy_n\}$ are Cauchy sequences in X.

Since S(X) is a complete subspace of X, there exist u and v in X such that

$Sx_n \to Su$ and $Sy_n \to Sv$.

By Lemma (1.3),(φ_1) and (φ_4),we have

$$\frac{1}{s}\, d(F(u,v), Su) \le \lim_{n\to\infty} \inf d(F(u,v), F(x_n, y_n))$$

$$\le \lim_{n\to\infty} \inf \varphi\left(\max\left\{\begin{array}{c}
d(Su, Sx_n), d(Sv, Sy_n), \frac{1}{s} d(Su, F(u,v)),\\
\frac{1}{s} d(Sv, F(v,u)), \frac{1}{s} d(Sx_n, Sx_{n+1}), \frac{1}{s} d(Sy_n, Sy_{n+1}),\\
\frac{1}{2s^2} d(Su, Sx_{n+1}), \frac{1}{2s^2} d(Sv, Sy_{n+1}), \frac{1}{2s^2} d(Sx_n, F(u,v)),\\
\frac{1}{2s^2} d(Sy_n, F(v,u)), \frac{d(Su,F(u,v))\, d(Sx_n,Sx_{n+1})}{1+d(Su,Sx_n)+d(Sv,Sy_n)},\\
\frac{d(Sv,F(v,u))\, d(Sy_n,Sy_{n+1})}{1+d(Su,Sx_n)+d(Sv,Sy_n)}, \frac{1}{2s}\left[\frac{d(Su,F(u,v))\, d(Su,Sx_{n+1})}{1+d(Su,Sx_n)+d(Sv,Sy_n)}\right],\\
\frac{1}{2s}\left[\frac{d(Sv,F(v,u))\, d(Sv,Sy_{n+1})}{1+d(Su,Sx_n)+d(Sv,Sy_n)}\right], \frac{1}{2s}\left[\frac{d(Sx_n,F(u,v))\, d(Sx_n,Sx_{n+1})}{1+d(F(u,v),Sx_{n+1})+d(F(v,u),Sy_{n+1})}\right],\\
\frac{1}{2s}\left[\frac{d(Sy_n,F(v,u))\, d(Sy_n,Sy_{n+1})}{1+d(F(u,v),Sx_{n+1})+d(F(v,u),Sy_{n+1})}\right]
\end{array}\right\}\right)$$

$$\le \varphi\left(\max\left\{\begin{array}{l}
0,0,\frac{1}{s} d(Su, F(u,v)), \frac{1}{s} d(Sv, F(v,u)), 0,0,0,0,\\
\frac{1}{2s} d(Su, F(u,v)), \frac{1}{2s} d(Sv, F(v,u)), 0,0,0,0,0,0
\end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s} max\{d(Su, F(u,v)), d(Sv, F(v,u))\}\right).$$

Similarly we can show that

$$\frac{1}{s} d(Su, F(u,v)) \le \varphi\left(\frac{1}{s} max\{d(Su, F(u,v)), d(Sv, F(v,u))\}\right)$$

$$\frac{1}{s}\, d(F(v,u), Sv) \le\ \varphi\left(\frac{1}{s} max\{d(Su, F(u,v)), d(Sv, F(v,u))\}\right)$$

$$\frac{1}{s} d(Sv, F(v,u)) \le \varphi\left(\frac{1}{s} max\{d(Su, F(u,v)), d(Sv, F(v,u))\}\right)$$

Thus using (φ_1) , we obtain

$$\frac{1}{s} max\left\{\begin{array}{l} d(F(u,v), Su), d(Su, F(u,v)),\\ d(F(v,u), Sv), d(Sv, F(v,u))\end{array}\right\} \le \varphi\left(\frac{1}{s} max\left\{\begin{array}{l} d(F(u,v), Su), d(Su, F(u,v)),\\ d(F(v,u), Sv), d(Sv, F(v,u))\end{array}\right\}\right)$$

which in turn yields from (φ_3) and (1.1.2) that $F(u,v) = Su$ and $F(v,u) = Sv$.

Thus (Su, Sv) is a coupled point of coincidence of F and S.

We will show that $d(Su, Su) = d(Sv, Sv) = 0$ if (Su, Sv) is a coupled point of coincidence of F and S.

$$\begin{aligned}
d(Su, Su) &= d(F(u,v), F(u,v))\\
&\le \varphi\left(\max\left\{\begin{array}{c} d(Su, Su), d(Sv, Sv), \frac{1}{s} d(Su, Su), \frac{1}{s} d(Sv, Sv), \frac{1}{s} d(Su, Su),\\ \frac{1}{s} d(Sv, Sv), \frac{1}{2s^2} d(Su, Su), \frac{1}{2s^2} d(Sv, Sv), \frac{1}{2s^2} d(Su, Su), \frac{1}{2s^2} d(Sv, Sv)\end{array}\right\}\right)\\
&\le \varphi(max\{d(Su, Su), d(Sv, Sv)\})
\end{aligned}$$

Similarly we can show that

$$d(Sv, Sv) \le \varphi(max\{d(Su, Su), d(Sv, Sv)\})$$

Thus $\quad max\{d(Su, Su), d(Sv, Sv)\} \le \varphi(max\{d(Su, Su), d(Sv, Sv)\})$

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which in turn yields from (φ_3) and (1.1.2) that $d(Su,Su) = d(Sv,Sv) = 0$.

Suppose (Sp,Sq) is another coupled point of coincidence of F and S.

Hence Sp = F(p,q) and Sq = F(q,p).Also $d(Sp,Sq) = d(Sq,Sp) = 0$.

Now consider

$$d(Su,Sp) = d\big(F(u,v),F(p,q)\big)$$

$$\leq \varphi\left(max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq), \frac{1}{s}d(Su,Su), \frac{1}{s}d(Sv,Sv), \frac{1}{s}d(Sp,Sp), \\ \frac{1}{s}d(Sq,Sq), \frac{1}{2s^2}d(Su,Sp), \frac{1}{2s^2}d(Sv,Sv), \frac{1}{2s^2}d(Sp,Su), \\ \frac{1}{2s^2}d(Sv,Sv), \frac{d(Su,F(u,v))\, d(Sp,F(p,q))}{1+d(Su,Sp)+d(Sv,Sq)}, \\ \frac{d(Sv,F(v,u))\, d(Sq,F(q,p))}{1+d(Su,Sp)+d(Sv,Sq)}, \frac{1}{2s}\left[\frac{d(Su,F(u,v))\, d(Su,F(p,q))}{1+d(Su,Sp)+d(Sv,Sq)}\right], \\ \frac{1}{2s}\left[\frac{d(Sv,F(v,u))\, d(Sv,F(q,p))}{1+d(Su,Sp)+d(Sv,Sq)}\right], \frac{1}{2s}\left[\frac{d(Sp,F(u,v))\, d(Sp,F(p,q))}{1+d\big(F(u,v),F(p,q)\big)+d\big(F(v,u),F(q,p)\big)}\right], \\ \frac{1}{2s}\left[\frac{d(Sq,F(v,u))\, d(Sq,F(q,p))}{1+d\big(F(u,v),F(p,q)\big)+d\big(F(v,u),F(q,p)\big)}\right] \end{array} \right\}\right)$$

$$\leq \varphi\left(max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq) \\ d(Sp,Su), d(Sq,Sv) \end{array} \right\}\right)$$

Similarly we can show that

$$d(Sp,Su) \leq \varphi\left(max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq) \\ d(Sp,Su), d(Sq,Sv) \end{array} \right\}\right)$$

$$d(Sv,Sq) \leq \varphi\left(max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq) \\ d(Sp,Su), d(Sq,Sv) \end{array} \right\}\right)$$

$$d(Sq,Sv) \leq \varphi\left(max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq) \\ d(Sp,Su), d(Sq,Sv) \end{array} \right\}\right).$$

Thus we have

$$max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq), \\ d(Sp,Su), d(Sq,Sv) \end{array} \right\} \leq \varphi\left(max \left\{ \begin{array}{c} d(Su,Sp), d(Sv,Sq), \\ d(Sp,Su), d(Sq,Sv) \end{array} \right\}\right)$$

which in turn yields from (φ_3) and (1.1.2) that Su = Sp and Sv = Sq.

Thus the coupled point of coincidence of F and S is unique.

Let $\alpha = Su$ and $\beta = Sv$.

Since (F,S) is w-compatible, we have

$S\alpha = S(Su) = S\big(F(u,v)\big) = F(Su,Sv) = F(\alpha,\beta)$ and

$S\beta = S(Sv) = S\big(F(v,u)\big) = F(Sv,Su) = F(\beta,\alpha)$.

Thus $(S\alpha, S\beta)$ is a coupled point of coincidence of F and S.

Since (α,β) is the unique coupled point of coincidence of F and S

It follows that $S\alpha = \alpha$ and $S\beta = \beta$.

Thus $\alpha = S\alpha = F(\alpha,\beta)$ and $\beta = S\beta = F(\beta,\alpha)$.

Hence (α,β) is a common coupled fixed point of F and S.

If (z, w) is another common coupled fixed point of F and S then

$z = F(z,w) = Sz$ and $w = F(w,z) = Sw$.

Hence (z,w) is a coupled point of coincidence of F and S.

Since (α,β) is the unique coupled point of coincidence of F and S,

We have $z = \alpha$ and $w = \beta$.

Thus (α,β) is the unique common coupled fixed point of F and S.

Case (ii): Suppose

max $\{d(Sx_n,Sx_{n+1}), d(Sx_{n+1},Sx_n), d(Sy_n,Sy_{n+1}), d(Sy_{n+1},Sy_n)\} = 0$ for some n.

Then $Sx_n = Sx_{n+1}$ and $Sy_n = Sy_{n+1}$.

Hence $Sx_n = F(x_n,y_n)$ and $Sy_n = F(y_n,x_n)$.

Thus $Su = F(u,v)$ and $Sv = F(v,u)$ where $u = x_n$ and $v = y_n$.

The rest of the proof follows as in Case (i).

Now we give an example to illustrate Theorem 2.1.

Example 2.2: Let X = [0,1] and $d(x,y) = (x+2y)^2$. Let F: $X \times X \to X$ and $S: X \to X$ be defined by $F(x,y) = \frac{x^2+y^2}{16}$ and $Sx = \frac{x^2}{4}$. Let $\varphi: R^+ \to R^+$ be defined by $\varphi(t) = \frac{t}{4}$, for all $t \in R^+$ Then d is dislocated quasi b-metric with s = 2.

Consider $d\big(F(x,y),F(u,v)\big) = \left(\frac{x^2+y^2}{16} + \frac{2(u^2+v^2)}{16}\right)^2 = \left(\frac{x^2+2u^2+y^2+2v^2}{16}\right)^2$

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$$= \left[\frac{\left(\frac{x^2}{4}+\frac{u^2}{2}\right)+\left(\frac{y^2}{4}+\frac{v^2}{2}\right)}{16}\right]^2$$

$$\leq \frac{1}{8}\left[\left(\frac{x^2}{4}+\frac{u^2}{2}\right)^2+\left(\frac{y^2}{4}+\frac{v^2}{2}\right)^2\right]$$

$$= \frac{1}{4} max\{d(Sx,Su), d(Sy,Sv)\}$$

$$\leq \varphi\left(max\left\{\begin{array}{c} d(Sx,Su), d(Sy,Sv), \frac{1}{2s}d(Sx,F(x,y)), \\ \frac{1}{s}d(Sy,F(y,x)), \frac{1}{s}d(Su,F(u,v)), \frac{1}{s}d(Sv,F(v,u)), \\ \frac{1}{2s^2}d(Sx,F(u,v)), \frac{1}{2s^2}d(Sy,F(v,u)), \\ \frac{1}{2s^2}d(Su,F(x,y)), \frac{1}{2s^2}d(Sv,F(y,x)), \\ \left[\frac{d(Sx,F(x,y))\, d(Su,F(u,v))}{1+d(Sx,Su)+d(Sy,Sv)}\right], \left[\frac{d(Sy,F(y,x))\, d(Sv,F(v,u))}{1+d(Sx,Su)+d(Sy,Sv)}\right], \\ \frac{1}{2s}\left[\frac{d(Sx,F(x,y))\, d(Sx,F(u,v))}{1+d(Sx,Su)+d(Sy,Sv)}\right], \frac{1}{2s}\left[\frac{d(Sy,F(y,x))\, d(Sy,F(v,u))}{1+d(Sx,Su)+d(Sy,Sv)}\right], \\ \frac{1}{2s}\left[\frac{d(Su,F(x,y))\, d(Su,F(u,v))}{1+d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}\right], \frac{1}{2s}\left[\frac{d(Sv,F(y,x))\, d(Sv,F(v,u))}{1+d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}\right] \end{array}\right\}\right)$$

for all $x,y,u,v \in X$, *where* $\varphi \in \mathbf{\Phi}_s$,

Thus (2.1.1) is satisfied. One can easily verify all the remaining conditions of Theorem 2.1

Clearly (0,0) is the unique common coupled fixed point of F and S.

Corollary 2.3: Let (X,d) be a dislocated quasi b-metric space with fixed real number $s \geq 1$ and $f,g: X \to X$ be mappings satisfying

$$(2.3.1)\ d(fx,fy) \leq \varphi\left(max\left\{\begin{array}{c} d(gx,gy), \frac{1}{s}d(gx,fx), \frac{1}{s}d(gy,fy), \frac{1}{2s^2}d(gx,fy), \\ \frac{1}{2s^2}d(gy,fx), \frac{d(gx,fx)\, d(gy,fy)}{1+d(gx,gy)}, \frac{1}{2s}\left[\frac{d(gx,fx)\, d(gx,fy)}{1+d(gx,gy)}\right], \\ \frac{1}{2s}\left[\frac{d(gy,fy)\, d(gy,fx)}{1+d(fx,fy)}\right] \end{array}\right\}\right)$$

$\forall\, x,y \in X$, where $\varphi \in \mathbf{\Phi}_s$,

(2.3.2) $f(X) \subseteq g(X)$ and g(X) is a complete subspace of X,

(2.3.3) the pair (f,g) is weakly compatible.

Then f and g have a unique common fixed point in X.

Corollary 2.4: Let (X,d) be a complete dislocated quasi b-metric space with fixed real number $s \geq 1$ and $f: X \to X$ be mapping satisfying

(2.4.1)

$$d(fx,fy) \leq \varphi\left(max\left\{\begin{array}{c} d(x,y), \frac{1}{s}d(x,fx), \frac{1}{s}d(y,fy), \frac{1}{2s^2}d(x,fy), \\ \frac{1}{2s^2}d(y,fx), \frac{d(x,fx)\, d(y,fy)}{1+d(x,y)}, \frac{1}{2s}\left[\frac{d(x,fx)\, d(x,fy)}{1+d(x,y)}\right], \\ \frac{1}{2s}\left[\frac{d(y,fy)\, d(y,fx)}{1+d(fx,fy)}\right] \end{array}\right\}\right)$$

$\forall\, x,y \in X$, where $\varphi \in \mathbf{\Phi}_s$.

Then f has a unique fixed point in X.

Remark 2.5: Corollary 2.4 is a generalization of Theorem 4.1 of [12].

Remark 2.6 : In Theorem 1.10 (i.e Thorem 3.1 of [11]) the author used the continuity of T with the condition $\alpha s + (1+s)\beta + 2(s^2+s)\mu < 1$. Theorem 1.10 is true without continuity of T if we assume $\alpha + 2\beta s + 4\mu s^2 < 1$. It follows from Corollary 2.4 with $\varphi(t) = (\alpha + 2\beta s + 4\mu s^2)(t)$.

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