$\tau_a\tau_b$- $^{\#}$g Closed sets in Bitopological Ordered Space

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Abstract **This paper is to launch an innovative category of sets called increasing $\tau_a\tau_b$ - $^{\#}$g-closed sets in topological ordered spaces and to analyze the properties of this set.**

- ***Keywords —*** *$\tau_a\tau_b$ - $^{\#}$g-i-closed sets, $\tau_a\tau_b$ - $^{\#}$g-i-open sets*

I. INTRODUCTION

1. In 1963 semi open sets are defined by Levin [11, 12] and also generalized closed sets in 1970. β-open sets are introduced by AbdEl-Monsef et al. [1]. And $^{\#}$g-closed sets are introduced by M.K.R.S Veerakumar [15]. In 1963 Kelley [7] pioneered the study of bitopological spaces. A bitopological space (X, τ_a, τ_b) means a non empty set X combined two topological spaces τ_a and τ_b. From that time many topologists generalized many of the outcomes in topological spaces to bitopological spaces. Generalized closed sets and semi open sets in bitopological spaces are first established by Fukutake [5,6]. Regular generalized closed sets are stated and analyzed by Rao and Mariasingam [3] in bitopological spaces. New class of sets called increasing $\tau_a\tau_b$- $^{\#}$g-closed sets in bitopological spaces and the study of its properties. Throughout this paper (X, τ_a, τ_b, ≤) means is a bitopological ordered space.

Definition 2.1. Subset A of a bitopological space (X, τ_a, τ_b) is named as a

1. If $A\subseteq\tau_b\, cl(\tau_a int(A))$ it is defined as $\tau_a\tau_b$ – semi open where it is stated as $\tau_a\tau_b$ – semi closed if $\tau_b int(\tau_a cl(A))\subseteq A$.
2. $\tau_a\tau_b$ -pre open if $A\subseteq\tau_b int(\tau_a cl(A))$ and $\tau_a\tau_b$– pre closed if $\tau_2 cl(\tau_1 int(A))\subseteq A$.
3. $\tau_a\tau_b$ -α-open if $A\subseteq\tau_a int(\tau_b cl(\tau_a int(A)))$.
4. $\tau_a\tau_b$ –semi preopen if $A\subseteq\tau_a cl(\tau_b int(\tau_a cl(A)))$.
5. $\tau_a\tau_b$– regular open if $A=\tau_b int(\tau_a cl(A))$.
6. $\tau_a\tau_b$ – regular closed if $A=\tau_b cl(\tau_a int(A))$.

Definition 2.2. Subset A of a bitopological space (X, τ_a, τ_b) is called a

1. If τ_b-cl(A)⊆U, whenever A⊆U, U is τ_1-open then the resultant is called $\tau_a\tau_b$ -g-closed set $\tau_a\tau_b$ -generalized closed set) .
2. If τ_b-scl(A)⊆U, whenever A⊆U, U is τ_a-semi open then the resultant is called $\tau_a\tau_b$ -sg-closed set $\tau_a\tau_b$ -semi generalized closed set) .
3. If τ_b-scl(A)⊆U, whenever A⊆U, U is τ_a-open then the resultant is called$\tau_a\tau_b$ -gs-closed set($\tau_a\tau_b$ generalized semi closed set)
4. If τ_b-αcl(A)⊆U, whenever A⊆U, U is τ_a-open then the resultant is called $\tau_a\tau_b$ -αg-closed set($\tau_a\tau_b$ -α-generalized closed set)
5. If τ_b-αcl(A)⊆U, whenever A⊆U, U is τ_a-α-open then the resultant is called $\tau_a\tau_b$ -gα-closed set ($\tau_a\tau_b$ -generalized α-closed set)
6. If τ_b-pcl(A)⊆U, whenever A⊆U, U is τ_a-open then the resultant is called $\tau_a\tau_b$ -gp-closed set ($\tau_a\tau_b$ -generalized pre-closed set) .
7. If τ_b-spcl(A)⊆U, whenever A⊆U, U is τ_a-open then the resultant is called $\tau_a\tau_b$ -gsp-closed set ($\tau_a\tau_b$ -generalized semi preclosed set)
8. If τ_b-pcl(A)⊆U, whenever A⊆U, U is τ_a-regularopen then the resultant is called $\tau_a\tau_b$ -gpr-closed set ($\tau_a\tau_b$ -generalized preregular closed set) .
9. If τ_b-cl(A))⊆U, whenever A⊆U, U is τ_a-gα*-open then the resultant is called $\tau_a\tau_b$ -μ-closed set.
10. If τ_b-scl(A))⊆U, whenever A⊆U, U is τ_a-sg-open then the resultant is called $\tau_a\tau_b$ -ψ-closed set.
11. If τ_b-spcl(A)⊆U, whenever A⊆U, U is τ_a-g-open then the resultant is called $\tau_a\tau_b$ -pre semi closed set
12. If τ_b-cl(A)⊆U, whenever A⊆U, U is τ_a-g-open then the resultant is called $\tau_a\tau_b$ -g*-closed set
13. If τ_b-pcl(A)⊆U, whenever A⊆U, U is τ_a-g-open then the resultant is called $\tau_a\tau_b$ -g*-pre closed set.
14. If τ_b-cl(A)⊆U, whenever A⊆U, U is τ_a-semiopen then the resultant is named as $\tau_a\tau_b$ -g^-closed set.
15. If τ_b-cl(A)⊆U, whenever A⊆U, U is τ_a-g^-open then the resultant is named as $\tau_a\tau_b$ -*g-closed set.
16. If τ_2-scl(A)⊆U, whenever A⊆U, U is τ_1-g^-open then the resultant is called $\tau_a\tau_b$ -*g-semi closed set.

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17. If τ_b-αcl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-g^-open then the resultant is called $\tau_a\tau_b$ -α*g-closed set.
18. If τ_b-scl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-gα*-open then the resultant is called $\tau_a\tau_b$ -μ-semi closed set$\tau_a\tau_b$ -μ-closed)
19. If τ_b-pcl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-gα*-open then the resultant is called $\tau_a\tau_b$ -μ-pre closed set ($\tau_a\tau_b$ -μp-closed).
20. If τ_b-scl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-gα**-open then the resultant is called $\tau_a\tau_b$ -semi μ-closed set
21. If τ_b-cl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-*g-open then the resultant is called $\tau_a\tau_b$ -$^{\#}$g-closed set.
22. Subset A of a bitopological space (X, τ_a ,τ_b) is called a $\tau_a\tau_b$ -$^{\#}$g-closed set if τ_b-cl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-*g-open in (X, τ_a).
23. The complement of $\tau_a\tau_b$ -$^{\#}$g-closed set is called $\tau_a\tau_b$ -$^{\#}$g-open set.

Now we studied the subsequent definitions of increasing, decreasing and balanced types. Let (X,τ, $\leq$) be a topological ordered space. For any x$\in$X, x increase is indicated by [x,$\rightarrow$] = {y$\in$X/x$\leq$y} and x decrease is defined by [$\leftarrow$, x] = {y$\in$X/y$\leq$x} [16]. A subset A of a topological ordered space (X,τ, $\leq$) is supposed to be increasing [16] if A = i(A) and decreasing [16] if A = d(A), where i(A) = $\cup_{a\in A}$[a, $\rightarrow$] and d(A) = $\cup_{a\in A}$[$\leftarrow$, a]. A subset of a topological ordered space (X, τ, $\leq$) is said to be balanced [16] if it is together increasing and decreasing.

3. Vital properties of $\tau_a\tau_b$ -$^{\#}$g-i- closed sets

Definition 3.1 Subset A of (X, τ_a, τ_b, $\leq$) is called a $\tau_a\tau_b$ -$^{\#}$g-i--closed set if it is together $\tau_a\tau_b$ -$^{\#}$g-closed and increasing.

Example 3.2. Let X = {1, 2, 3}, Consider one topology τ_a= {X, Φ, (1)} and another topology τ_b = {X, Φ, (1), (1, 2)}, and followed with partial order $\leq$ = {(1, 1), (2, 2), (3, 3), (1, 3)}.

$\tau_a\tau_b$ -$^{\#}$g-i--closed sets are {X,Φ, (3), (2, 3)}.

Theorem3.3. Each increasing τ_b-closed set is an increasing $\tau_a\tau_b$ -$^{\#}$g-closed set.

Proof: Let A be τ_b-i-closed set

$\Rightarrow$ A be τ_b closed and increasing

Then τ_b– cl(A)$\subseteq$U, when A$\subseteq$U, where U is τ_a-*g-open.

$\Rightarrow$A is $\tau_a\tau_b$ -$^{\#}$g-closed

Hence A is $\tau_a\tau_b$ -$^{\#}$g-i-closed set.

Theorem3.6. Each $\tau_a\tau_b$ –g*-i-closed set is $\tau_a\tau_b$ -$^{\#}$g-i-closed set.

Proof: Let A$\subseteq$U, where U is τ_1-*g-i-open

$\Rightarrow$ A$\subseteq$U, U is τ_a-*g-open and increasing.

Then U is τ_a-g-open.

$\Rightarrow\tau_b$ – cl(A)$\subseteq$U (By the assumption)

$\Rightarrow$ A is $\tau_a\tau_b$ -$^{\#}$g-closed set.

Hence A is $\tau_a\tau_b$ -$^{\#}$g-i-closed set.

Theorem3.7. Each $\tau_a\tau_b$ –g$^{\#}$-i-closed set is $\tau_a\tau_b$ -$^{\#}$g-i-closed set.

Proof: Let A$\subseteq$U, where U is τ_a-*g-i-open

This means A$\subseteq$U, where U is τ_a-*g—open and increasing then U is τ_1-*g-open

This means U is τ_a-αg-open.

$\Rightarrow\tau_b$– cl(A)$\subseteq$U (By our assumption)

$\Rightarrow$ A is $\tau_a\tau_b$ -$^{\#}$g-closed set.

Hence A is $\tau_a\tau_b$ -$^{\#}$g-i-closed set.

Theorem3.8. Each $\tau_a\tau_b$ –$^{\#}$g-i-closed set is $\tau_a\tau_b$ -gs-i-closed set.

Proof: Let A$\subseteq$U, where U is τ_a-open and increasing

Then U is τ_a-open $\Rightarrow$U is τ_a-*g-open.

$\Rightarrow\tau_b$ – cl(A)$\subseteq$U (By our assumption)

But τ_b– scl(A)$\subseteq$ τ_b – cl(A)$\subseteq$U

$\Rightarrow$A is $\tau_a\tau_b$ -gs-closed set.

Hence A is $\tau_a\tau_b$ -gs-i-closed set.

Theorem3.9. Each $\tau_a\tau_b$–$^{\#}$g-i-closed set is $\tau_a\tau_b$ -αg-i-closed set.

Proof: Let A$\subseteq$U, where U is τ_a-open and increasing.

Then U is τ_a-open $\Rightarrow$U is τ_a-*g-open.

$\Rightarrow\tau_b$– cl(A)$\subseteq$U (By our assumption)

But τ_b– αcl(A)$\subseteq$ τ_b – cl(A)$\subseteq$U

$\Rightarrow$A is $\tau_a\tau_b$ -αg-closed set.

Hence A is $\tau_a\tau_b$ -αg-i-closed set.

Theorem3.10. Each $\tau_a\tau_b$ –$^{\#}$g-i-closed set is $\tau_a\tau_b$ -gp-i-closed set.

Proof: Assume that A is $\tau_a\tau_b$ –$^{\#}$g-i-closed set.

This means A is $\tau_a\tau_b$ –$^{\#}$g--closed set and increasing

To confirm A is $\tau_a\tau_b$ gp-closed set.

Let A$\subseteq$U, where U is τ_a-open

Then U is τ_a-open

$\Rightarrow$U is τ_a-*g-open.

$\Rightarrow\tau_b$ – cl(A)$\subseteq$U (By our assumption)

But τ_b– pcl(A)$\subseteq$ τ_b – cl(A)$\subseteq$U

$\Rightarrow$A is $\tau_a\tau_b$ -gp-closed set.

Hence A is $\tau_a\tau_b$ -gp-i-closed set.

Theorem3.11. Each $\tau_a\tau_b$ –$^{\#}$g-i-closed set is $\tau_a\tau_b$ -gpr-i-closed set.

Proof: Suppose that A is $\tau_a\tau_b$–$^{\#}$g-i-closed set.

To prove A is $\tau_a\tau_b$ -gpr-closed set.

Let A$\subseteq$U, where U is τ_1-regular open

Then U is τ_1-regular open $\Rightarrow$U is τ_a-open $\Rightarrow$ U is τ_a-*g-open. $\Rightarrow\tau_b$ – cl(A)$\subseteq$U (By our assumption),But τ_b – pcl(A)$\subseteq$ τ_b– cl(A)$\subseteq$U

$\Rightarrow\tau_b$ – pcl(A)$\subseteq$U, whenever A$\subseteq$U, U is τ_a-regular open.

$\Rightarrow$A is $\tau_a\tau_b$ -gpr-closed set.

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Hence A is $\tau_a\tau_b$ -gpr-i-closed set.

Theorem3.12. Each $\tau_a\tau_b$ –$^{\#}$g-i-closed set is $\tau_a\tau_b$ -gsp-i-closed set.

Proof: Assume that A is $\tau_a\tau_b$ –$^{\#}$g-i-closed set.

To prove A is $\tau_a\tau_b$ -gsp-closed set.

Let $A\subseteq U$, where U is τ_1-open

Then U is τ_a-open $\Rightarrow$U is τ_a-*g-open.

$\Rightarrow\tau_b$– cl(A)$\subseteq$U (By our assumption)

But τ_b– spcl(A)$\subseteq$ τ_b– cl(A)$\subseteq$U

$\Rightarrow$A is $\tau_a\tau_b$-gsp-closed set.

Hence A is $\tau_a\tau_b$ -gsp-i-closed set.

Theorem3.13. The converses of the above theorems are not true as can be seen by the following examples.

Example 3.14. Let X= (1, 2, 3), τ_a = {X, Φ, (1),(2, 3)} & τ_b= {X, Φ, (1)},
$\leq$ = {(1, 1), (2, 2), (3, 3), {1, 3)}.
$\tau_a\tau_b$ -$^{\#}$g-i--closed sets = {X,Φ, (2), (3), (1, 3), (2, 3)}.
Here (2), (3), (1, 3) are $\tau_a\tau_b$ -$^{\#}$g-i—closed but they are not τ_b-i-closed.
$\tau_a\tau_b$ –g*-i-closed sets are {X,Φ, (2, 3)} Here (2), (3), (1, 3) are $\tau_a\tau_b$ -$^{\#}$g—closed sets but they are not $\tau_a\tau_b$ –g*-i--closed set. $\tau_a\tau_b$ –g$^{\#}$-i-closed sets are {X, Φ, (2, 3)} now (2), (3), (2, 3),(1, 3) are $\tau_a\tau_b$ -$^{\#}$g-i—closed sets but they are not $\tau_a\tau_b$ –g$^{\#}$-i-closed.

Example 3.15. Let X= (1, 2, 3), τ_a = {X, Φ, (1)} & τ_b = {X, Φ, (1),(1, 2)},
$\leq$ = {(1, 1), (2, 2), (3, 3), (1, 3), (2, 3)}.
$\tau_a\tau_b$ -$^{\#}$g-i-closed sets are {X, Φ, (3), (2, 3)}.
$\tau_a\tau_b$ -g-i--closed sets are {X, Φ, (3), (2, 3), (1, 3)}.
Here (1, 3) are $\tau_a\tau_b$ -g-i—closed sets but they are not $\tau_a\tau_b$ -$^{\#}$g-i—closed.
$\tau_a\tau_b$ –gs-i-closed set = {X, Φ, (3), (2, 3), (1, 3)} Here (1, 3) are $\tau_a\tau_b$ –gs-i—closed sets but they are not $\tau_a\tau_b$ –$^{\#}$g-i-closed set. $\tau_a\tau_b$ –αg-i-closed sets = {X, Φ, (3), (2, 3), (1, 3)}= $\tau_a\tau_b$ –gp-i-closed set = $\tau_a\tau_b$–gsp-i-closed set. Here (1, 3) are $\tau_a\tau_b$ -gs-i—closed, $\tau_a\tau_b$ –gp-i-closed, $\tau_a\tau_b$–gsp-i-closed sets but they are not $\tau_a\tau_b$ –$^{\#}$g-i-closed.
$\tau_a\tau_b$ –gpr-i-closed sets = {X,Φ, (3), (2, 3), (1, 3)}.
Here (1, 3) are $\tau_a\tau_b$-gpr-i—closed but they are not $\tau_a\tau_b$–$^{\#}$g-i-closed.

Theorem3.16. $\tau_a\tau_b$ –$^{\#}$g-i-closedness is free of $\tau_a\tau_b$ –α-i-closedness, $\tau_a\tau_b$ -i-semi closedness,$\tau_a\tau_b$ -i-semi preclosedness, and $\tau_a\tau_b$ -i-pre closedness.

Proof: It can be seen from the subsequent example.

Example 3.17. Let , τ_a = {X, Φ, (1), (1, 3)} & τ_b = {X, Φ, (1),(1, 2)},
$\leq$ = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}.
$\tau_a\tau_b$ -$^{\#}$g-i-closed sets = {X, Φ, (2), (1, 2), (2, 3)}.
$\tau_a\tau_b$ –α-i-closed sets = {X, Φ, (2), (2, 3)} =$\tau_a\tau_b$ -i-semi closed sets = $\tau_a\tau_b$ -i-pre closed sets = $\tau_a\tau_b$ -i-semi pre closed sets. Here (1, 2) is $\tau_a\tau_b$ -$^{\#}$g-i--closed set but is not a $\tau_a\tau_b$ –α-i-closed set, $\tau_a\tau_b$ -i-semi closed set, $\tau_a\tau_b$ -i-pre closed set, $\tau_a\tau_b$ -i-semi pre closed set.

Example 3.18. Let X= (1, 2, 3), τ_a = {X, Φ, (1)} & τ_b= {X, Φ, (1),(1, 2)},
$\leq$ = {(1, 1), (2, 2), (3, 3), (1, 2), (1, 3)}.
$\tau_a\tau_b$ -$^{\#}$g-i-closed sets are {X,Φ, (3), (2, 3)}.
$\tau_a\tau_b$ –α-i-closed sets are{X, Φ, (2), (3), (2, 3)} $\tau_a\tau_b$ -i-semi closed sets = $\tau_a\tau_b$ -i-pre closed sets = $\tau_a\tau_b$ -i-semi pre closed sets. Here (2) is not a $\tau_a\tau_b$ -$^{\#}$g-i--closed set but it is a $\tau_a\tau_b$ –α-i-closed set, $\tau_a\tau_b$ -i-semi closed set, $\tau_a\tau_b$ -i-pre closed set, $\tau_a\tau_b$ -i-semi pre closed set.

Theorem3.19. $\tau_a\tau_b$–$^{\#}$g-i-closedness is independent of $\tau_a\tau_b$ –ψ-i-closedness, $\tau_a\tau_b$ -gα-i- closedness, $\tau_a\tau_b$ –sg-i-closedness, $\tau_a\tau_b$–*g-i-closedness, $\tau_a\tau_b$–*gs-i-closedness, $\tau_a\tau_b$ –α-*g-i-closedness, $\tau_a\tau_b$–μ-i-closedness, $\tau_a\tau_b$ –μs-i-closedness, $\tau_a\tau_b$ –μp-i-closedness.

Proof: It can be seen from the next example.

Example 3.20. Let X= (1, 2, 3), τ_1 = {X, Φ, (1)} & τ_2 = {X, Φ, (1),(1, 2)},
$\leq$ = {(1, 1), (2, 2), (3, 3), (1, 2), (1, 3)}.
$\tau_a\tau_b$ -$^{\#}$g-i-closed sets are {X, Φ, (3), (2, 3)}.
$\tau_a\tau_b$ –ψ-i-closed sets are{X, Φ, (2), (3), (2, 3)} =$\tau_a\tau_b$ -gα-i-closed sets = $\tau_a\tau_b$ –sg-i-closed sets. Here (2) is a $\tau_a\tau_b$ –ψ-i-closed set, $\tau_a\tau_b$ -gα-i-closed set, $\tau_a\tau_b$ –sg-i-closed set. But is not a $\tau_a\tau_b$ -$^{\#}$g-i-closed set. $\tau_a\tau_b$ –*g-i-closed sets = {X, Φ, (2), (3), (2, 3)} = $\tau_a\tau_b$ –*gs-i-closed sets = $\tau_a\tau_b$–α-*g-i-closed sets = $\tau_a\tau_b$ –μ-i-closed sets = $\tau_a\tau_b$ –μs-i-closed sets = $\tau_a\tau_b$–μp-i-closed sets. Here (2) are not $\tau_a\tau_b$ -$^{\#}$g-i-closed sets.

Example 3.21. Let X= (1, 2, 3), τ_a = {X, Φ, (1), (2, 3)} & τ_b = {X, Φ, (1)},
$\leq$ = {(1, 1), (2, 2), (3, 3), (1, 3)}.
$\tau_a\tau_b$ -$^{\#}$g-i-closed sets are {X, Φ, (2), (3), (1, 3), (2, 3)}.
$\tau_a\tau_b$ –ψ-i-closed sets are {X, Φ, (2), (3), (2, 3)} = $\tau_a\tau_b$ –μs-i-closed sets = $\tau_a\tau_b$ –μp-i-closed sets = $\tau_a\tau_b$ –*gs-i-closed sets = $\tau_a\tau_b$ –α-*g-i-closed sets. Here (1, 3) are $\tau_a\tau_b$ -$^{\#}$g-i-closed sets. But they are not $\tau_a\tau_b$ –ψ-i-closed sets. $\tau_a\tau_b$ –μs-i-closed sets, $\tau_a\tau_b$ –μp-i-closed sets, $\tau_a\tau_b$–*gs-i-closed sets, $\tau_a\tau_b$ –α-*g-i-closed sets.
$\tau_a\tau_b$ –μ-i-closed sets = {X, Φ, (2, 3)} = $\tau_a\tau_b$ –*g-i-closed sets. Here (2), (3), (1, 3) are $\tau_a\tau_b$ -$^{\#}$g-i-closed sets. But they are not $\tau_a\tau_b$ –μ-i-closed sets, $\tau_a\tau_b$ –*g-i-closed sets.

Example 3.22. Let X= (1, 2, 3), τ_a = {X, Φ, (1), (2), (1, 2)} & τ_b = {X, Φ, (1),(1, 2)},
$\leq$ = {(1, 1), (2, 2), (3, 3), (1, 3)}.

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$\tau_a\tau_b$ -#g-i-closed sets = {X, Φ, (3), (1, 3), (2, 3)}.
$\tau_a\tau_b$ –sg-i-closed sets = {X, Φ, (2), (3), (2, 3)} =$\tau_a\tau_b$ -gα-i-closed sets. Here (1, 3) is a $\tau_a\tau_b$ -#g-i-closed set. But it is not a $\tau_a\tau_b$ –sg-i-closed set and $\tau_a\tau_b$ –gα-i-closed set. Also (2) is $\tau_a\tau_b$ –sg-i-closed set and $\tau_a\tau_b$ –gα-i-closed set. But it is not a $\tau_a\tau_b$ -#g-i-closed set.

4. Properties of $\tau_a\tau_b$ -#g-i-closed sets and $\tau_a\tau_b$ -#g-i-open sets

Theorem 4.1. Union of two $\tau_a\tau_b$ -#g-i-closed set is $\tau_a\tau_b$ -#g-i-closed sets.

Proof: suppose that P and Q are $\tau_a\tau_b$ –#g-i-closed sets.
$\Rightarrow$ P and Q are $\tau_a\tau_b$ –#g-i-closed and increasing.
Let $P\cup Q \subset U$, where U is τ_a-*g open.
Then $P\subset U$ and $Q\subset U$.
$\Rightarrow \tau_b-cl(P)\subset U$ and $\tau_b-cl(Q)\subset U$
$\Rightarrow \tau_b-cl(P) \cup \tau_b-cl(Q)\subset U$
But $\tau_b-cl(P\cup Q) = \tau_b-cl(P) \cup \tau_b-cl(Q)\subset U$
$\Rightarrow P\cup Q$ is $\tau_a\tau_b$ -#g-closed set.
Hence $P\cup Q$ is $\tau_a\tau_b$ -#g-i-closed set.

Theorem 4.2. Intersection of two $\tau_a\tau_b$-#g-i-closed sets need not be $\tau_a\tau_b$-#g-i-closed sets.

Proof: This can be seen from the following example.

Example .4.3. Let X = {a, b, c}, τ_a = {X,Φ, (1), (2, 3)} & τ_b = {X, Φ, (1)},
$\leq$ = {(1, 1), (2, 2), (3, 3)}.
$\tau_a\tau_b$ -#g-i-closed sets = {X, Φ, (2), (3), (1, 2), (1, 3), (2, 3)}. Here (1, 2), (1, 3) are $\tau_a\tau_b$ -#g-i-closed set, But their intersection is not $\tau_a\tau_b$-#g-i-closed set.

Theorem 4.4. Let A be $\tau_a\tau_b$ -#g-i-closed and $A\subset B\subset \tau_2-cl(A)$, then B is $\tau_a\tau_b$ -#g-i-closed.

Proof: Let $B \subset U$, where U is τ_a-*g-i open.
$\Rightarrow B \subset U$, where U is τ_a-*g- open and increasing
Then $A\subset B\subset U \Rightarrow \tau_b-cl(A)\subset U$.
Given $B\subset\tau_b-cl(A)$, But $\tau_b-cl(B)$ is the smallest closed set containing B.
$\therefore$ $B\subset\tau_b-cl(B)\subset\tau_b-cl(A)\subset U \Rightarrow \tau_b-cl(B)\subset U \Rightarrow$ B is $\tau_a\tau_b$ -#g-closed.
Hence B is $\tau_a\tau_b$ -#g-i-closed.

Theorem 4.5. If A is $\tau_a\tau_b$ -#g-i-closed then $\tau_b-cl(A)$-A does not contain any non-empty $\tau_a\tau_b$ -*g-i-closed set.

Proof: Suppose $\tau_b-cl(A)$-A contains a non-empty τ_a-*g-i-closed set F. That is $F\subset\tau_b-cl(A)$-A.
$\Rightarrow F\subset\tau_b-cl(A)$ but $F\not\subset A \Rightarrow F\subset A^c$
$\Rightarrow A\subset F^c$, where F^c is τ_a -*g-open$\Rightarrow\tau_b-cl(A) \subset F^c \Rightarrow F\subset(\tau_b-cl(A))^c$
We have $F\subset\tau_b-cl(A)\cap(\tau_b-cl(A))^c = \Phi$.
$\tau_b-cl(A)$-A does not contain any non-empty τ_a -*g-i-closed set.

Theorem 4.6. Let A is $\tau_a\tau_b$ -#g-i-closed. Then A is τ_b-i-closed if and only if $\tau_b-cl(A)$-A is τ_a -*g-i-closed set.

Proof: Suppose that A is $\tau_a\tau_b$ -#g-i-closed and τ_b- i-closed then $\tau_b-cl(A) = A$.
$\Rightarrow\tau_b-cl(A)-A = \Phi$, which is τ_a -*g-i-closed.
Conversely assume that A is $\tau_a\tau_b$ -#g-i-closed and $\tau_b-cl(A)$-A is τ_a -*g-i-closed.
Since A is $\tau_a\tau_b$ -#g-i-closed, $\tau_b-cl(A)$-A does not contain any non-empty τ_a -*g-i-closed set. $\Rightarrow\tau_b-cl(A)-A = \Phi \Rightarrow\tau_b-cl(A) = A \Rightarrow$A is τ_b- closed. Hence A is A is τ_b-i- closed

Theorem 4.7. If A is $\tau_a\tau_b$ -#g-i-closed and $A\subset B\subset\tau_b-cl(A)$, then $\tau_b-cl(B)$-B contains no non-empty τ_a-*g-i-closed set.

Proof: By known theorem 4.4, the verification is as follows.

Theorem 4.8. For each $x\in X$, the singleton {x} is either τ_a -*g-i-closed or its complement $\{x\}^c$ is $\tau_a\tau_b$ -#g-i-closed.

Proof: Suppose {x} is not τ_a -*g-i-closed, then $\{x\}^c$ will not be τ_a -*g-i-open.
$\Rightarrow$X is the only τ_a -*g-i-open set containing $\{x\}^c$.
$\Rightarrow\tau_b-cl\{x\}^c\subset X \Rightarrow\{x\}^c$ is $\tau_a\tau_b$ -#g-i-closed.
$\Rightarrow${x} is $\tau_a\tau_b$ -#g-i-open set.

Theorem 4.9. Arbitrary union of $\tau_a\tau_b$ -#g-i-closed sets $\{A_i, i\in I\}$ in a bitopological ordered space. $(X, \tau_a, \tau_b, \leq)$ is $\tau_a\tau_b$ -#g-i-closed if the family $\{A_i, i\in I\}$ is locally finite on X.

Proof: Let $\{A_i, i\in I\}$ be locally finite in X and each A_i be $\tau_a\tau_b$ –#g-i-closed in X.
To show $\cup A_i$ is $\tau_a\tau_b$ –#g-i-closed.
Let $\cup A_i\subset U$, where U is τ_a -*g-i-open.
$\Rightarrow A_i\subset U$, for every $i\in I \Rightarrow\tau_b-cl(A_i)\subset U$ for every $i\in I \Rightarrow\tau_b-cl(A_i)\subset U$.
$\Rightarrow\tau_b-cl(\cup A_i)\subset U \Rightarrow\cup A_i$ is $\tau_a\tau_b$ -#g-i-closed.

Theorem 4.10. If A and B are $\tau_a\tau_b$ -#g-i-open sets in a bitopological space (X, τ_a, τ_b) then their intersection be a $\tau_a\tau_b$ -#g-i-open set.

Proof: If A and B are $\tau_a\tau_b$ -#g-i-open sets, then A^c and B^c are $\tau_a\tau_b$ -#g-i-closed sets.
$A^c \cup B^c$ is $\tau_a\tau_b$ -#g-i-closed by the theorem 4.1. That is $(A\cup B)^c$ is $\tau_a\tau_b$ -#g-i-closed.
$\Rightarrow A\cup B$ is $\tau_a\tau_b$ -#g-i-open set.

Theorem 4.11. The Union of two $\tau_a\tau_b$ -#g-i-open sets is need not be $\tau_a\tau_b$ -#g-i-open in X.

Proof: This can be seen from the subsequent example.

Example .4.12. Let X= (1, 2, 3), τ_a = {X, Φ, (1), (2, 3)} & τ_b= {X, Φ, (1)},
$\leq$ = {(1, 1), (2, 2), (3, 3)}.

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$\tau_a\tau_b$ -$^{\#}$g-i-closed sets are {X, Φ, (2), (3), (1, 2), (1, 3), (2, 3)}. Here (2), (3) are $\tau_a\tau_b$ -$^{\#}$g-i-open set, But their combination (union) is not $\tau_a\tau_b$ -$^{\#}$g-open set.

Theorem 4.13. If τ_b–int A⊂B⊂A and A is $\tau_a\tau_b$ -$^{\#}$g-i-open in X, then B is also $\tau_a\tau_b$ -$^{\#}$g-i-open in X.

Proof: Suppose τ_b –int A⊂B⊂A and A is $\tau_a\tau_b$ -$^{\#}$g-i-open in X.

Then $A^c \subset B^c \subset X$-$\tau_b$ –int A = τ_b –cl(X-A) = τ_b –cl (A^c).

Since A^c is $\tau_a\tau_b$ -$^{\#}$g-i-closed by known theorem 4.4.

B^c is $\tau_a\tau_b$ -$^{\#}$g-i-closed ⇒B is a$\tau_a\tau_b$ -$^{\#}$g-i-open set.

Theorem 4.14. A set A is $\tau_a\tau_b$ -$^{\#}$g-i-open if and only if F⊂τ_b –int A, where F is τ_a -*g-i-closed and F⊂A.

Proof: If F⊂τ_b –int A, where F is τ_a-*g-i-closed and F⊂A.

⇒ $A^c \subset F^c$ = G, where G is τ_a -*g-i-open and τ_b–cl (A^c)⊂G.

⇒A^c is $\tau_a\tau_b$ -$^{\#}$g-i-closed is $\tau_a\tau_b$ -$^{\#}$g-i-open.

Conversely presuppose that A is increasing$\tau_a\tau_b$ -$^{\#}$g-open and F⊂A, where F is τ_a -*g-i-closed.

Then $A^c \subset F^c$⇒τ_b–cl ($A^c \subset F^c$). (Since A^c is $\tau_a\tau_b$ -$^{\#}$g-i-closed).

⇒F⊂X-τ_b –cl (A^c) = τ_b –int(A).

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