A Economic Order Quantity Model with Shortage Using Pentagonal Fuzzy Number

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Abstract - In this paper, an economic order quantity model with shortage is formulated. The cost parameters are represented as Pentagonal fuzzy number. The model is defuzzified by K-preference graded mean integration representation method. The optimum order quantity and shortage level have been obtained. A numerical illustration is given to support the problem.

Keywords: Inventory, economic order quantity, Pentagonal fuzzy number, k-Preference.

I. INTRODUCTION

For several years, classical economic order quantity (EOQ) problems with different variations were solved by many researchers and had been reported in the reference books and survey papers (e.g. Cheng[2], De and Goswami [3] etc,).

But all these inventory problems are solved with the assumption that the co-efficient or cost parameters are specified in a precise way. In real life, there are many situations due to uncertainty. Here inventory costs are imprecise, that is fuzzy in nature.

Early works in using fuzzy concept in decision making were done by Zadeh [8] and Bellman [1] through introducing fuzzy goals, costs and constraints. Later, the fuzzy linear programming models was formulated and an approach for solving linear programming model with fuzzy numbers has been presented by Zimmermann [9].

In 1987, Park [7] used fuzzy set concept to treat the inventory problem with fuzzy inventory cost under arithmetic operations of Extension Principle. In 2003, Hsieh [4] proposed a optimization of Fuzzy Inventory Models under K- preference. In [5] C.H. Hsieh discussed Optimization of fuzzy inventory models under fuzzy demand and fuzzy lead time.

Till now there is no fuzzy inventory model using k-preference of the pentagonal fuzzy number. So that in this paper, the economic order quantity inventory model with shortage using k-preference of the pentagonal fuzzy number has been considered in a fuzzy environment. The fuzzy holding cost, ordering cost and shortage cost have been represented by the pentagonal fuzzy number. The model is defuzzified by k- preference Graded mean Integration technique. This paper is organized as follows.

In section 2, assumptions and notations for both crisp and fuzzy model under consideration are given. The main results are obtained in section 3 and 5. The Numerical example is presented in section 6. To illustrate the model and the results have been compared with the crisp model.

II. ASSUMPTIONS AND NOTATIONS

ASSUMPTIONS

(i) A single item is considered.

(ii) Lead time is zero.

(iii) Shortages are allowed.

(iv) Demand rate is determined.

NOTATIONS

$\tilde{C}_1$ - fuzzy holding cost per unit per unit time

$\tilde{C}_3$ - fuzzy ordering cost per order

$\tilde{C}_2$ - fuzzy Shortage cost per unit per unit time

D - demand rate

T - planning horizon

t - cycle time

t_2 - the time in which the shortage.

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Q - economic order quantity

S - shortage quantity

III. MATHEMATICAL MODEL IN CRISP ENVIRONMENT

The inventory model is formulated to minimize the average total cost, which includes the ordering cost, holding cost and the shortage cost.

Average total cost

$$Min\quad TC = \frac{C_2 S^2}{2Q} + \frac{C_1 (Q-S)^2}{2Q} + \frac{C_3 D}{Q}$$

By using calculus technique, the analytical expression for optimum lot size and shortage level have been derived.

Q= $\sqrt{\frac{2C_3 D (C_1 + C_2)}{C_1 C_2}}$ **S=** $\sqrt{\frac{2C_1 C_3 D}{(C_1 + C_2) C_2}}$

IV. PENTAGONAL FUZZY NUMBER AND ITS K-PREFERENCE GRADED MEAN INTEGRATION METHOD

Pentagonal fuzzy number: A pentagonal fuzzy number $\tilde{A}$ described as a fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad a \le x \\ w_A \dfrac{x-a}{b-a} & , \quad a \le x \le b \\ w_A + (1-w_A)\dfrac{x-b}{c-b} & , \quad b \le x \le c \\ w_A + (1-w_A)\dfrac{d-x}{d-c} & , \quad c \le x \le d \\ w_A \dfrac{e-x}{e-d} & , \quad d \le x \le e \\ 0 & , \quad e \ge x \end{cases}$$

where w_A , $0.6 \le w_A < 1$.

k-PREFERENCE GRADED MEAN INTEGRATION METHOD OF PENTAGANOL FUZZY NUMBER

Graded k-preference Integration Representation method, L^{-1} and $R^{-1}$are the inverse function L and R respectively, then the graded k-preference h-level value of generalized fuzzy number

$\tilde{A} = (a_1, a_2, a_3, a_4, a_5; w_A)_{LR}$ is $h[kL^{-1}(h)+(1-k)R^{-1}(h)]$

.Then the graded k-preference integration representation of $\tilde{A}$ is $P_k(\tilde{A})$ with grade w_A, where

$$P^k(\tilde{A}) = \frac{\int_0^1 h\left[k L^{-1}(h) + (1-k) R^{-1}(h)\right] dh}{\int_0^1 h\, dh}$$

$0 < h \le w_A$ and $0 \le k \le 1$.

$$\therefore P^k(\tilde{A}) = \frac{1}{3(1-w_A)}\left[k\begin{Bmatrix} b + 2c - 3c\,w_A + {w_A}^2 (a-b) \\ - {w_A}^3 (a-c) \end{Bmatrix} + (1-k)\begin{Bmatrix} d + 2c - 3c w_A \\ + {w_A}^2 (e-d) - \\ {w_A}^3 (e-c) \end{Bmatrix} \right]$$

V. MATHEMATICAL MODEL IN FUZZY ENVIRONMENT

The above model is fuzzified by pentaganal fuzzy number

$$Min\ \tilde{T}C = \frac{\tilde{C}_2 S^2}{2Q} + \frac{\tilde{C}_1 (Q-S)^2}{2Q} + \frac{\tilde{C}_3 D}{Q}$$

Defuzzfying the fuzzy average total cost by using k-Preference Graded Mean Integration Representation method is given by

$$P_k(\tilde{T}C) = \frac{P_k(\tilde{C}_2) S^2}{2Q} + \frac{P_k(\tilde{C}_1)(Q-S)^2}{2Q} + \frac{P_k(\tilde{C}_3) D}{Q}$$

By using calculus technique, the analytical expression for optimum lot size and shortage level have been derived.

$$Q^* = \sqrt{\frac{2P_k(C_3) D \left(P_k(C_1) + P_k(C_2)\right)}{P_k(C_1) P_k(C_2)}}\ ;$$

$$S^* = \sqrt{\frac{2P_k(C_1) P_k(C_3) D}{\left(P_k(C_1) + P_k(C_2)\right) P_k(C_2)}}$$

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VI. NUMERICAL EXAMPLE

To illustrate the above fuzzy model and for comparing with the corresponding crisp model, the following data are considered.

A particular item has a demand of 8000 units/year. The cost of one procurement is Rs.1.5, the holding cost per unit is Rs.0.3 per year and the set-up cost is 12000 per year .The cost of shortage is Rs.1.10 per unit per year. Find the optimal solution of Q, S and T(Q,S).

w_A=0.75.

$$\tilde{C}_1 = [0.1,\ 0.2,\ 0.3,\ 0.4\ 0.5]$$
$$\tilde{C}_2 = [0.7,\ 0.9,\ 1.1,\ 1.3, 1.5]$$
$$\tilde{C}_3 = [10000,\ 11000,\ 12000,\ 13000, 14000]$$
$$\tilde{D} = [7000,\ 7500,\ 8000,\ 8500, 9000]$$

TABLE 1: OPTIMAL SOLUTION FOR CRISP AND FUZZY MODEL

Case	k	Q*	S*	Avg. TC	TC/ Unit
Crisp	-	40,000	35,000	12,600	0.3150
Crisp	-	31,754	25,981	15,696	0.4943
Crisp	-	28,540	22,424	18,724	0.6562
Crisp	-	26,879	20,555	21,822	0.8118
Crisp	-	25,923	19,442	25,021	0.9652
Fuzzy	0.1	28,190	22,034	19,269	0.6835
Fuzzy	0.5	29,854	23,883	17,269	0.5785
Fuzzy	0.9	32,587	26,897	15,222	0.4671

VII. CONCLUSION

- When compare the optimum lot size, shortage level and total average minimum cost in crisp and fuzzy model, fuzzy order quantity was higher than crisp order quantity
- And also fuzzy shortage level and average minimum total cost were lower than crisp shortage level and average minimum total cost.
- Finally, we conclude that these models can be executable in the real world.

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