A Study on Mean time to Recruitment for a Two Graded Manpower System with Two Thresholds having Different Epochs for Decisions and Exits and Thresholds form an Extended Exponential Random Variables using Bivariate CUM policy of Recruitment

[1]D.Gowri, [2]Dr. S.Sendhamizh selvi, [3]L. Saral

[1]M.phil Scholar, [2]Assistant Professor, [3]Research Scholar, PG & Research Department of Mathematics, Government Arts College, Trichy, Tamil Nadu, India,

[1]gowridharmalingam95@gmail.com, [2]sendhamizhs@yahoo.co.in, [3]sarallucas1998@gmail.com

Abstract: In this paper, for a two graded manpower system, a mathematical model is constructed using a Bivariate CUM policy of recruitment based on shock model approach. The mean time to recruitment is obtained based on the assumptions,(i) the loss of manpower due to the attrition form a sequence of independent and identically distributed exponential random variables (ii) the inter exit times and decision times form an ordinary renewal process and (iii) each grade has two types of threshold namely optional and mandatory thresholds which follow independent and identically distributed exponential random variables. A different probabilistic analysis is used to derive the analytical results.

Keywords: ***Two graded manpower system , decision and exit epochs, bivariate policy of recruitment with two thresholds, renewal process and mean time to recruitment.***

I. INTRODUCTION

Manpower planning refers to complex task of forecasting and planning for the right number and the right kind of people at the right place and the right time to perform activities that will benefit both the organization and the individuals in it. Human resource management is an important aspect of study and the manpower planning is to be done taking into consideration the dynamics of manpower availability and requirements. Manpower planning depends not only on the highly unpredictable behavior of human beings but also on the uncertain environment of the organization. This uncertainly leads to propose stochastic models for manpower planning. These models incorporate several factors such as recruitment, training, promotion, demotion, and wastage. These factors are interlinked and their analysis become highly essential in order to analyze the cost involved in recruitment. In short manpower planning provides right size and structure of human resources which provides the basic infrastructure for smooth functioning of an organization. It minimizes the cost of employment and nullifies the effects of distributions in developing and utilizing the human resources. In [1], [2], [5] & [6] the authors have discussed the manpower planning models by markovian and renewal theoretic approach. In [7] the author has studied the problem of time to recruitment for a single grade manpower system and obtained mean time to recruitment when the loss of manpower forms a sequence of independent and identically distributed random variables, the inter-decision times form an ordinary renewal process and mandatory breakdown threshold for the cumulative loss of manpower is an exponential random variable by using the univariate CUM policy of recruitment. In [4] the author has initiated the study of the problem of time to recruitment for a single grade manpower system by incorporating alertness in the event of cumulative loss of manpower due to attrition crossing the threshold, by considering optional and mandatory threshold for the cumulative loss of manpower in this manpower system. In [12] the author has studied the problem of time to recruitment for a two graded manpower system, by considering optional and mandatory thresholds. In [8] the author has studied the problem of time to recruitment for a two graded manpower system, by considering optional and mandatory thresholds using different types for inter-decision times. In all the above cited work, it is assumed that attrition will take place instantaneously at decision epochs. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decision points. This aspect is taken into account for the first time in [3] the author has studied the problem of variance of time to recruitment is obtained when inter decision times and exit times are independent and identically

 DOI : 10.18231/2454-9150.2018.0849

distributed exponential random variables using univariate policy for recruitment and laplace transform in the analysis, In [9] the author has studied the work in [3] by considering optional and mandatory thresholds which considering non-instantaneous exits at decision epochs. In [11] and [13] the author have studied the problem of time to recruitment for a two grade manpower system using bivariate policy for recruitment. Recently, in [10] the author has studied the work in [9] by considering optional and mandatory thresholds which considering non-instantaneous exits at decision epochs.

In the present paper, for a two grade manpower system, a mathematical model is constructed in which attrition due to policy decision take place at exit points and there are optional and mandatory thresholds as control limits for the cumulative loss of manpower. A bivariate CUM policy of recruitment based on shock model approach is used to determine the expected time to recruitment when the system has different epochs for policy decisions and exits and the inter-decision times forms ordinary renewal processes and loss of manpower follows independent and identically exponential random variable each grade has two types of threshold namely optional and mandatory thresholds which follow independent and identically distributed exponential random variables, each grade has two types of threshold namely optional and mandatory thresholds which follow independent and identically distributed exponential random variables.

II. NOTATIONS

$\mathbf{X_i}$ **:** continuos random variable denoting the loss of manpower due to i^{th} exit point.

$\boldsymbol{S_k}$ **:** total loss of manpower upto the first k exit point.

$\mathbf{X_i}$ is are independent and identically distributed extended exponential random variable with density function m(.), distribution function M(.) and mean $\frac{1}{\alpha}$;$\alpha > 0$.

$\mathbf{U_j}$ **:** continuos random variable representing the time between the $(j-1)^{th}$ and j^{th} policy decisions.

j= 1,2,3,…

$\mathbf{U_j}$'s are independent and identically distributed random variable with density function f(.), distribution function F(.) with parameter η respectively.

$\mathbf{W_i}$ **:** continuos random variable representing the time between the $(i-1)^{th}$ and i^{th} exists. It is assumed that $W_i's$ are independent and identically distributed random variable with density function g(.) ,distribution function G(.) and mean $\frac{1}{\delta}$;$\delta > 0$ respectively.

$\boldsymbol{Y_1, Y_2}$ **:** extended exponential random variable denoting the optional threshold for grade 1 and grade 2 with density function $h_1(.), h_2(.)$ and distribution function $H_1(.), H_2(.)$ with parameter λ_1, λ_2 respectively.

$\boldsymbol{Z_1, Z_2}$ **:** extended exponential random variable denoting the mandatory threshold for grade1 and grade 2 with density function $h_1(.), h_2(.)$ and distribution function $H_1(.), H_2(.)$ with parameter μ_1, μ_2 respectively.

Assumed that $Y_1 < Z_1$ & $Y_2 < Z_2$

p : probability that the organization is not going for recruitment when optional threshold is exceeded by the cumulative loss of manpower.

q : the probability that every decision has exit of personnel when q=0 corresponds to the case where exists are impossible. It is assumed that q ≠ 0.

d : a non negative constant representing threshold for number of decisions.

T : random variable denoting the time to recruitment with density function l(.), distribution function L(.), mean E(T).

$\boldsymbol{N_e(t)}$ **:** number of exit points lying in (0,t].

$\boldsymbol{f_k(t), F_k(t)}$ **:** k fold convolution of f(.) and F(.) respectively.

$\boldsymbol{G^*}(.)$ & $\boldsymbol{\bar{g}}(.)$ **:** Laplace – stieltjes transform and laplace transform of G(.) & g(.) respectively.

The **Bivariate CUM policy of Recruitment** employed in this section is stated as follows:

Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold or the number of decisions crosses the corresponding threshold whichever is earlier. However ,the organization may or may not go for recruitment if the cumulative loss of manpower crosses the optional threshold.

III. MAIN RESULT

P(T > t) = $\sum_{k=0}^{d-1} P$ {there are exactly 'k' exits are taken in (0,t] k=0,1,2,3,… × probability that the total number of exits in these k-decision does not cross the optional level Y (or) the total number of exists in these k-decision crosses the optional level Y but lies below the mandatory level Z and the organization is not making recruitment }

$$P(T > t) = \sum_{k=0}^{d-1} P(S_{N_e(t)} \le Y) + p\sum_{k=0}^{d-1} P(Y < S_{N_e(t)} \le Z) \quad (1)$$

$$= \sum_{k=0}^{d-1} P(N_e(t) = k) P\left(\sum_{i=1}^{k} X_i < Y\right) + p\sum_{k=0}^{d-1} P(N_e(t) = k) P\left(\sum_{i=1}^{k} X_i > Y\right) P\left(\sum_{i=1}^{k} X_i \le Z\right)$$

From the renewal theory,

$$P\{N_e(t) = k\} = G_k(t) - G_{k+1}(t) \quad \text{and } G_0(t) = 1 \qquad (2)$$

 DOI : 10.18231/2454-9150.2018.0849

∴(1)becomes,

$$P(T>t)=\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}(t)\}P(S_k\leq Y)+p\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}(t)\}P(S_k>Y)P(S_k\leq Z)\quad(3)$$

Case (i): $Y=\max(Y_1,Y_2)\;\&\;Z=\max(Z_1,Z_2)$

If Y_1, Y_2 follows extended exponential distribution with scale parameter λ_1, λ_2 and shape parameter 2 then,

$$P(Y_1\leq x)=(1-e^{-\lambda_1 x})^2 \quad \& \quad P(Y_2\leq x)=(1-e^{-\lambda_2 x})^2$$

$$P(S_k\leq Y)=\int_0^\infty P\{Y>x\}m_k(x)dx \qquad (4)$$

$$P(Y>x)=2e^{-\lambda_1 x}+2e^{-\lambda_2 x}-4e^{-(\lambda_1+\lambda_2)x}+2e^{-(2\lambda_1+\lambda_2)x}+2e^{-(\lambda_1+2\lambda_2)x}-e^{-2\lambda_1 x}-e^{-2\lambda_2 x}-e^{-2(\lambda_1+\lambda_2)x} \qquad (5)$$

$$P(S_k\leq Y)=2b_1^k+2b_2^k-4b_3^k+2b_4^k+2b_5^k-b_6^k-b_7^k-b_8^k$$

Where,

$$b_1=E[e^{-\lambda_1 x}], b_2=E[e^{-\lambda_2 x}], b_3=E[e^{-(\lambda_1+\lambda_2)x}], b_4=E[e^{-(2\lambda_1+\lambda_2)x}],$$

$$b_5=E[e^{-(\lambda_1+2\lambda_2)x}], b_6=E[e^{-2\lambda_1 x}], b_7=E[e^{-2\lambda_2 x}], b_8=E[e^{-(2\lambda_1+2\lambda_2)x}] \qquad (6)$$

Similarly,

If Z_1, Z_2 follows extended exponential distribution with scale parameter μ_1, μ_2 and shape parameter 2 then,

$$P(Z_1\leq x)=(1-e^{-\mu_1 x})^2 \quad \& \quad P(Z_2\leq x)=(1-e^{-\mu_2 x})^2$$

$$P(S_k\leq Z)=\int_0^\infty P\{Z>x\}m_k(x)dx \qquad (7)$$

$$P(Z>x)=2e^{-\mu_1 x}+2e^{-\mu_2 x}-4e^{-(\mu_1+\mu_2)x}+2e^{-(2\mu_1+\mu_2)x}+2e^{-(\mu_1+2\mu_2)x}-e^{-2\mu_1 x}-e^{-2\mu_2 x}-e^{-2(\mu_1+\mu_2)x} \qquad (8)$$

$$p(S_k\leq Z)=2c_1^k+2c_2^k-4c_3^k+2c_4^k+2c_5^k-c_6^k-c_7^k-c_8^k$$

Where,

$$c_1=E[e^{-\mu_1 x}], c_2=E[e^{-\mu_2 x}], c_3=E[e^{-(\mu_1+\mu_2)x}], c_4=E[e^{-(2\mu_1+\mu_2)x}],$$

$$c_5=E[e^{-(\mu_1+2\mu_2)x}], c_6=E[e^{-2\mu_1 x}], c_7=E[e^{-2\mu_2 x}], c_8=E[e^{-(2\mu_1+2\mu_2)x}] \qquad (9)$$

Using (6) & (9) in (3),we get

$$p(T>t)=\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}(t)\}(2b_1^k+2b_2^k-4b_3^k+2b_4^k+2b_5^k-b_6^k-b_7^k-b_8^k)+$$
$$p\sum_{k=0}^{d-1}\{G_k(t)-G_{k+1}(t)\}(1-2b_1^k-2b_2^k+4b_3^k-2b_4^k-2b_5^k+b_6^k+b_7^k+b_8^k)$$
$$(2c_1^k+2c_2^k-4c_3^k+2c_4^k+2c_5^k-c_6^k-c_7^k-c_8^k) \qquad (10)$$

and L(t)=1-P(T>t)

Expanding and taking laplace transform $\bar{l}(s)$ is given by,

$$\bar{l}(s)=2\bar{b}_1\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_1^{d-1}}{1-b_1\bar{g}(s)}\right]+2\bar{b}_2\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_2^{d-1}}{1-b_2\bar{g}(s)}\right]$$
$$-4\bar{b}_3\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_3^{d-1}}{1-b_3\bar{g}(s)}\right]+2\bar{b}_4\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_4^{d-1}}{1-b_4\bar{g}(s)}\right]$$
$$+2\bar{b}_5\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_5^{d-1}}{1-b_5\bar{g}(s)}\right]-\bar{b}_6\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_6^{d-1}}{1-b_6\bar{g}(s)}\right]$$
$$-\bar{b}_7\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_7^{d-1}}{1-b_7\bar{g}(s)}\right]-\bar{b}_8\left[\frac{\bar{g}(s)-\bar{g}(s)^d b_8^{d-1}}{1-b_8\bar{g}(s)}\right]$$
$$+p\{2\bar{c}_1\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_1^{d-1}}{1-c_1\bar{g}(s)}\right]+2\bar{c}_2\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_2^{d-1}}{1-c_2\bar{g}(s)}\right]$$
$$-4\bar{c}_3\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_3^{d-1}}{1-c_3\bar{g}(s)}\right]+2\bar{c}_4\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_4^{d-1}}{1-c_4\bar{g}(s)}\right]$$
$$+2\bar{c}_5\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_5^{d-1}}{1-c_5\bar{g}(s)}\right]-\bar{c}_6\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_6^{d-1}}{1-c_6\bar{g}(s)}\right]$$
$$-\bar{c}_7\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_7^{d-1}}{1-c_7\bar{g}(s)}\right]-\bar{c}_8\left[\frac{\bar{g}(s)-\bar{g}(s)^d c_8^{d-1}}{1-c_8\bar{g}(s)}\right]$$

 DOI : 10.18231/2454-9150.2018.0849

$$4\overline{b_1c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_1)^{d-1}}{1-b_1c_1\,\bar g(s)}\right]-4\overline{b_1c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_2)^{d-1}}{1-b_1c_2\,\bar g(s)}\right]$$

$$+8\overline{b_1c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_3)^{d-1}}{1-b_1c_3\,\bar g(s)}\right]-4\overline{b_1c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_4)^{d-1}}{1-b_1c_4\,\bar g(s)}\right]$$

$$-4\overline{b_1c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_5)^{d-1}}{1-b_1c_5\,\bar g(s)}\right]+2\overline{b_1c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_6)^{d-1}}{1-b_1c_6\,\bar g(s)}\right]$$

$$+2\overline{b_1c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_7)^{d-1}}{1-b_1c_7\,\bar g(s)}\right]+2\overline{b_1c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_1c_8)^{d-1}}{1-b_1c_8\,\bar g(s)}\right]$$

$$-4\overline{b_2c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_1)^{d-1}}{1-b_2c_1\,\bar g(s)}\right]-4\overline{b_2c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_2)^{d-1}}{1-b_2c_2\,\bar g(s)}\right]$$

$$+8\overline{b_2c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_3)^{d-1}}{1-b_2c_3\,\bar g(s)}\right]-4\overline{b_2c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_4)^{d-1}}{1-b_2c_4\,\bar g(s)}\right]$$

$$-4\overline{b_2c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_5)^{d-1}}{1-b_2c_5\,\bar g(s)}\right]+2\overline{b_2c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_6)^{d-1}}{1-b_2c_6\,\bar g(s)}\right]$$

$$+2\overline{b_2c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_7)^{d-1}}{1-b_2c_7\,\bar g(s)}\right]+2\overline{b_2c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_2c_8)^{d-1}}{1-b_2c_8\,\bar g(s)}\right]$$

$$+8\overline{b_3c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_1)^{d-1}}{1-b_3c_1\,\bar g(s)}\right]+8\overline{b_3c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_2)^{d-1}}{1-b_3c_2\,\bar g(s)}\right]$$

$$-16\overline{b_3c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_3)^{d-1}}{1-b_3c_3\,\bar g(s)}\right]+8\overline{b_3c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_4)^{d-1}}{1-b_3c_4\,\bar g(s)}\right]$$

$$+8\overline{b_3c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_5)^{d-1}}{1-b_3c_5\,\bar g(s)}\right]-4\overline{b_3c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_6)^{d-1}}{1-b_3c_6\,\bar g(s)}\right]$$

$$-4\overline{b_3c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_7)^{d-1}}{1-b_3c_7\,\bar g(s)}\right]-4\overline{b_3c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_3c_8)^{d-1}}{1-b_3c_8\,\bar g(s)}\right]$$

$$-4\overline{b_4c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_1)^{d-1}}{1-b_4c_1\,\bar g(s)}\right]-4\overline{b_4c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_2)^{d-1}}{1-b_4c_2\,\bar g(s)}\right]$$

$$+8\overline{b_4c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_3)^{d-1}}{1-b_4c_3\,\bar g(s)}\right]-4\overline{b_4c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_4)^{d-1}}{1-b_4c_4\,\bar g(s)}\right]$$

$$-4\overline{b_4c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_5)^{d-1}}{1-b_4c_5\,\bar g(s)}\right]+2\overline{b_4c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_6)^{d-1}}{1-b_4c_6\,\bar g(s)}\right]$$

$$+2\overline{b_4c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_7)^{d-1}}{1-b_4c_7\,\bar g(s)}\right]+2\overline{b_4c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_4c_8)^{d-1}}{1-b_4c_8\,\bar g(s)}\right]$$

$$-4\overline{b_5c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_1)^{d-1}}{1-b_5c_1\,\bar g(s)}\right]-4\overline{b_5c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_2)^{d-1}}{1-b_5c_2\,\bar g(s)}\right]$$

$$+8\overline{b_5c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_3)^{d-1}}{1-b_5c_3\,\bar g(s)}\right]-4\overline{b_5c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_4)^{d-1}}{1-b_5c_4\,\bar g(s)}\right]$$

$$-4\overline{b_5c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_5)^{d-1}}{1-b_5c_5\,\bar g(s)}\right]+2\overline{b_5c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_6)^{d-1}}{1-b_5c_6\,\bar g(s)}\right]$$

$$+2\overline{b_5c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_7)^{d-1}}{1-b_5c_7\,\bar g(s)}\right]+2\overline{b_5c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_5c_8)^{d-1}}{1-b_5c_8\,\bar g(s)}\right]$$

$$+2\overline{b_6c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_1)^{d-1}}{1-b_6c_1\,\bar g(s)}\right]+2\overline{b_6c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_2)^{d-1}}{1-b_6c_2\,\bar g(s)}\right]$$

$$-4\overline{b_6c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_3)^{d-1}}{1-b_6c_3\,\bar g(s)}\right]+2\overline{b_6c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_4)^{d-1}}{1-b_6c_4\,\bar g(s)}\right]$$

$$+2\overline{b_6c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_5)^{d-1}}{1-b_6c_5\,\bar g(s)}\right]-\overline{b_6c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_6)^{d-1}}{1-b_6c_6\,\bar g(s)}\right]$$

$$-\overline{b_6c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_7)^{d-1}}{1-b_6c_7\,\bar g(s)}\right]-\overline{b_6c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_6c_8)^{d-1}}{1-b_6c_8\,\bar g(s)}\right]$$

$$+2\overline{b_7c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_1)^{d-1}}{1-b_7c_1\,\bar g(s)}\right]+2\overline{b_7c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_2)^{d-1}}{1-b_7c_2\,\bar g(s)}\right]$$

$$-4\overline{b_7c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_3)^{d-1}}{1-b_7c_3\,\bar g(s)}\right]+2\overline{b_7c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_4)^{d-1}}{1-b_7c_4\,\bar g(s)}\right]$$

$$+2\overline{b_7c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_5)^{d-1}}{1-b_7c_5\,\bar g(s)}\right]-\overline{b_7c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_6)^{d-1}}{1-b_7c_6\,\bar g(s)}\right]$$

$$-\overline{b_7c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_7)^{d-1}}{1-b_7c_7\,\bar g(s)}\right]-\overline{b_7c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_7c_8)^{d-1}}{1-b_7c_8\,\bar g(s)}\right]$$

$$+2\overline{b_8c_1}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_1)^{d-1}}{1-b_8c_1\,\bar g(s)}\right]+2\overline{b_8c_2}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_2)^{d-1}}{1-b_8c_2\,\bar g(s)}\right]$$

$$-4\overline{b_8c_3}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_3)^{d-1}}{1-b_8c_3\,\bar g(s)}\right]+2\overline{b_8c_4}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_4)^{d-1}}{1-b_8c_4\,\bar g(s)}\right]$$

$$+2\overline{b_8c_5}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_5)^{d-1}}{1-b_8c_5\,\bar g(s)}\right]-\overline{b_8c_6}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_6)^{d-1}}{1-b_8c_6\,\bar g(s)}\right]$$

$$-\overline{b_8c_7}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_7)^{d-1}}{1-b_8c_7\,\bar g(s)}\right]-\overline{b_8c_8}\left[\frac{\bar g(s)-\bar g(s)^d\,(b_8c_8)^{d-1}}{1-b_8c_8\,\bar g(s)}\right]\Bigg\} \qquad (11)$$

It can be shown that distribution function G(.) of the inter exit time satisfy the relation,

$$2G(s)=\sum_{n=1}^{\infty}(1-q_1)^{n-1}q_1F_n(x)+\sum_{n=1}^{\infty}(1-q_2)^{n-1}q_2F_n(x) \qquad (12)$$

Taking Laplace transform on both sides, and simplifying

$$2G^*(s)=q_1\sum_{n=1}^{\infty}(1-q_1)^{n-1}F_n^{\;*}(x)+q_2\sum_{n=1}^{\infty}(1-q_2)^{n-1}F_n^{\;*}(x) \qquad (13)$$

$$\bar g(0)=1, \qquad \bar g'(0)=-\frac{1}{2\eta q_1q_2} \qquad (14)$$

$$E(T)=-\left\{\frac{d}{ds}\bar l(s)\right\}_{s=0} \qquad (15)$$

Using (11) and (14) in (15)

 DOI : 10.18231/2454-9150.2018.0849

$$
\begin{aligned}
E(T)=\frac{1}{2\eta q_1 q_2}\Big\{&-\left[\frac{2\bar{b}_1}{(1-b_1)^2}\right]\left[db_1^d\,\bar{b}_1-1+b_1^d\right]-\left[\frac{2\bar{b}_2}{(1-b_2)^2}\right]\left[db_2^d\,\bar{b}_2-1+b_2^d\right]\\
&+\left[\frac{4\bar{b}_3}{(1-b_3)^2}\right]\left[db_3^d\,\bar{b}_3-1+b_3^d\right]-\left[\frac{2\bar{b}_4}{(1-b_4)^2}\right]\left[db_4^d\,\bar{b}_4-1+b_4^d\right]\\
&-\left[\frac{2\bar{b}_5}{(1-b_5)^2}\right]\left[db_5^d\,\bar{b}_5-1+b_5^d\right]+\left[\frac{\bar{b}_6}{(1-b_6)^2}\right]\left[db_6^d\,\bar{b}_6-1+b_6^d\right]\\
&+\left[\frac{\bar{b}_7}{(1-b_7)^2}\right]\left[db_7^d\,\bar{b}_7-1+b_7^d\right]+\left[\frac{\bar{b}_8}{(1-b_8)^2}\right]\left[db_8^d\,\bar{b}_8-1+b_8^d\right]\\
&+p\Big\{-\left[\frac{2\bar{c}_1}{(1-c_1)^2}\right]\left[dc_1^d\,\bar{c}_1-1+c_1^d\right]-\left[\frac{2\bar{c}_2}{(1-c_2)^2}\right]\left[dc_2^d\,\bar{c}_2-1+c_2^d\right]\\
&+\left[\frac{4\bar{c}_3}{(1-c_3)^2}\right]\left[dc_3^d\,\bar{c}_3-1+c_3^d\right]-\left[\frac{2\bar{c}_4}{(1-c_4)^2}\right]\left[dc_4^d\,\bar{c}_4-1+c_4^d\right]\\
&-\left[\frac{2\bar{c}_5}{(1-c_5)^2}\right]\left[dc_5^d\,\bar{c}_5-1+c_5^d\right]+\left[\frac{\bar{c}_6}{(1-c_6)^2}\right]\left[dc_6^d\,\bar{c}_6-1+c_6^d\right]\\
&+\left[\frac{\bar{c}_7}{(1-c_7)^2}\right]\left[dc_7^d\,\bar{c}_7-1+c_7^d\right]+\left[\frac{\bar{c}_8}{(1-c_8)^2}\right]\left[dc_8^d\,\bar{c}_8-1+c_8^d\right]\\
&+\left[\frac{4\overline{b_1c_1}}{(1-b_1c_1)^2}\right]\left[d(b_1c_1)^d\,\overline{b_1c_1}-1+(b_1c_1)^d\right]+\left[\frac{4\overline{b_1c_2}}{(1-b_1c_2)^2}\right]\left[d(b_1c_2)^d\,\overline{b_1c_2}-1+(b_1c_2)^d\right]\\
&-\left[\frac{8\overline{b_1c_3}}{(1-b_1c_3)^2}\right]\left[d(b_1c_3)^d\,\overline{b_1c_3}-1+(b_1c_3)^d\right]+\left[\frac{4\overline{b_1c_4}}{(1-b_1c_4)^2}\right]\left[d(b_1c_4)^d\,\overline{b_1c_4}-1+(b_1c_4)^d\right]\\
&+\left[\frac{4\overline{b_1c_5}}{(1-b_1c_5)^2}\right]\left[d(b_1c_5)^d\,\overline{b_1c_5}-1+(b_1c_5)^d\right]-\left[\frac{2\overline{b_1c_6}}{(1-b_1c_6)^2}\right]\left[d(b_1c_6)^d\,\overline{b_1c_6}-1+(b_1c_6)^d\right]\\
&-\left[\frac{2\overline{b_1c_7}}{(1-b_1c_7)^2}\right]\left[d(b_1c_7)^d\,\overline{b_1c_7}-1+(b_1c_7)^d\right]-\left[\frac{2\overline{b_1c_8}}{(1-b_1c_8)^2}\right]\left[d(b_1c_8)^d\,\overline{b_1c_8}-1+(b_1c_8)^d\right]\\
&+\left[\frac{4\overline{b_2c_1}}{(1-b_2c_1)^2}\right]\left[d(b_2c_1)^d\,\overline{b_2c_1}-1+(b_2c_1)^d\right]+\left[\frac{4\overline{b_2c_2}}{(1-b_2c_2)^2}\right]\left[d(b_2c_2)^d\,\overline{b_2c_2}-1+(b_2c_2)^d\right]\\
&-\left[\frac{8\overline{b_2c_3}}{(1-b_2c_3)^2}\right]\left[d(b_2c_3)^d\,\overline{b_2c_3}-1+(b_2c_3)^d\right]+\left[\frac{4\overline{b_2c_4}}{(1-b_2c_4)^2}\right]\left[d(b_2c_4)^d\,\overline{b_2c_4}-1+(b_2c_4)^d\right]\\
&+\left[\frac{4\overline{b_2c_5}}{(1-b_2c_5)^2}\right]\left[d(b_2c_5)^d\,\overline{b_2c_5}-1+(b_2c_5)^d\right]-\left[\frac{2\overline{b_2c_6}}{(1-b_2c_6)^2}\right]\left[d(b_2c_6)^d\,\overline{b_2c_6}-1+(b_2c_6)^d\right]\\
&-\left[\frac{2\overline{b_2c_7}}{(1-b_2c_7)^2}\right]\left[d(b_2c_7)^d\,\overline{b_2c_7}-1+(b_2c_7)^d\right]-\left[\frac{2\overline{b_2c_8}}{(1-b_2c_8)^2}\right]\left[d(b_2c_8)^d\,\overline{b_2c_8}-1+(b_2c_8)^d\right]\\
&-\left[\frac{8\overline{b_3c_1}}{(1-b_3c_1)^2}\right]\left[d(b_3c_1)^d\,\overline{b_3c_1}-1+(b_3c_1)^d\right]-\left[\frac{8\overline{b_3c_2}}{(1-b_3c_2)^2}\right]\left[d(b_3c_2)^d\,\overline{b_3c_2}-1+(b_3c_2)^d\right]\\
&+\left[\frac{16\overline{b_3c_3}}{(1-b_3c_3)^2}\right]\left[d(b_3c_3)^d\,\overline{b_3c_3}-1+(b_3c_3)^d\right]-\left[\frac{8\overline{b_3c_4}}{(1-b_3c_4)^2}\right]\left[d(b_3c_4)^d\,\overline{b_3c_4}-1+(b_3c_4)^d\right]\\
&-\left[\frac{8\overline{b_3c_5}}{(1-b_3c_5)^2}\right]\left[d(b_3c_5)^d\,\overline{b_3c_5}-1+(b_3c_5)^d\right]+\left[\frac{4\overline{b_3c_6}}{(1-b_3c_6)^2}\right]\left[d(b_3c_6)^d\,\overline{b_3c_6}-1+(b_3c_6)^d\right]\\
&+\left[\frac{4\overline{b_3c_7}}{(1-b_3c_7)^2}\right]\left[d(b_3c_7)^d\,\overline{b_3c_7}-1+(b_3c_7)^d\right]+\left[\frac{4\overline{b_3c_8}}{(1-b_3c_8)^2}\right]\left[d(b_3c_8)^d\,\overline{b_3c_8}-1+(b_3c_8)^d\right]\\
&+\left[\frac{4\overline{b_4c_1}}{(1-b_4c_1)^2}\right]\left[d(b_4c_1)^d\,\overline{b_4c_1}-1+(b_4c_1)^d\right]+\left[\frac{4\overline{b_4c_2}}{(1-b_4c_2)^2}\right]\left[d(b_4c_2)^d\,\overline{b_4c_2}-1+(b_4c_2)^d\right]\\
&-\left[\frac{8\overline{b_4c_3}}{(1-b_4c_3)^2}\right]\left[d(b_4c_3)^d\,\overline{b_4c_3}-1+(b_4c_3)^d\right]+\left[\frac{4\overline{b_4c_4}}{(1-b_4c_4)^2}\right]\left[d(b_4c_4)^d\,\overline{b_4c_4}-1+(b_4c_4)^d\right]\\
&+\left[\frac{4\overline{b_4c_5}}{(1-b_4c_5)^2}\right]\left[d(b_4c_5)^d\,\overline{b_4c_5}-1+(b_4c_5)^d\right]-\left[\frac{2\overline{b_4c_6}}{(1-b_4c_6)^2}\right]\left[d(b_4c_6)^d\,\overline{b_4c_6}-1+(b_4c_6)^d\right]\\
&-\left[\frac{2\overline{b_4c_7}}{(1-b_4c_7)^2}\right]\left[d(b_4c_7)^d\,\overline{b_4c_7}-1+(b_4c_7)^d\right]-\left[\frac{2\overline{b_4c_8}}{(1-b_4c_8)^2}\right]\left[d(b_4c_8)^d\,\overline{b_4c_8}-1+(b_4c_8)^d\right]\\
&+\left[\frac{4\overline{b_5c_1}}{(1-b_5c_1)^2}\right]\left[d(b_5c_1)^d\,\overline{b_5c_1}-1+(b_5c_1)^d\right]+\left[\frac{4\overline{b_5c_2}}{(1-b_5c_2)^2}\right]\left[d(b_5c_2)^d\,\overline{b_5c_2}-1+(b_5c_2)^d\right]\\
&-\left[\frac{8\overline{b_5c_3}}{(1-b_5c_3)^2}\right]\left[d(b_5c_3)^d\,\overline{b_5c_3}-1+(b_5c_3)^d\right]+\left[\frac{4\overline{b_5c_4}}{(1-b_5c_4)^2}\right]\left[d(b_5c_4)^d\,\overline{b_5c_4}-1+(b_5c_4)^d\right]\\
&+\left[\frac{4\overline{b_5c_5}}{(1-b_5c_5)^2}\right]\left[d(b_5c_5)^d\,\overline{b_5c_5}-1+(b_5c_5)^d\right]-\left[\frac{2\overline{b_5c_6}}{(1-b_5c_6)^2}\right]\left[d(b_5c_6)^d\,\overline{b_5c_6}-1+(b_5c_6)^d\right]\\
&-\left[\frac{2\overline{b_5c_7}}{(1-b_5c_7)^2}\right]\left[d(b_5c_7)^d\,\overline{b_5c_7}-1+(b_5c_7)^d\right]-\left[\frac{2\overline{b_5c_8}}{(1-b_5c_8)^2}\right]\left[d(b_5c_8)^d\,\overline{b_5c_8}-1+(b_5c_8)^d\right]
\end{aligned}
$$

DOI : 10.18231/2454-9150.2018.0849

$$-\left[\frac{2\overline{b_6c_1}}{(1-b_6c_1)^2}\right]\left[d(b_6c_1)^d\,\overline{b_6c_1}-1+(b_6c_1)^d\right]-\left[\frac{2\overline{b_6c_2}}{(1-b_6c_2)^2}\right]\left[d(b_6c_2)^d\,\overline{b_6c_2}-1+(b_6c_2)^d\right]$$

$$+\left[\frac{4\overline{b_6c_3}}{(1-b_6c_3)^2}\right]\left[d(b_6c_3)^d\,\overline{b_6c_3}-1+(b_6c_3)^d\right]-\left[\frac{2\overline{b_6c_4}}{(1-b_6c_4)^2}\right]\left[d(b_6c_4)^d\,\overline{b_6c_4}-1+(b_6c_4)^d\right]$$

$$-\left[\frac{2\overline{b_6c_5}}{(1-b_6c_5)^2}\right]\left[d(b_6c_5)^d\,\overline{b_6c_5}-1+(b_6c_5)^d\right]+\left[\frac{\overline{b_6c_6}}{(1-b_6c_6)^2}\right]\left[d(b_6c_6)^d\,\overline{b_6c_6}-1+(b_6c_6)^d\right]$$

$$+\left[\frac{\overline{b_6c_7}}{(1-b_6c_7)^2}\right]\left[d(b_6c_7)^d\,\overline{b_6c_7}-1+(b_6c_7)^d\right]+\left[\frac{\overline{b_6c_8}}{(1-b_6c_8)^2}\right]\left[d(b_6c_8)^d\,\overline{b_6c_8}-1+(b_6c_8)^d\right]$$

$$-\left[\frac{2\overline{b_7c_1}}{(1-b_7c_1)^2}\right]\left[d(b_7c_1)^d\,\overline{b_7c_1}-1+(b_7c_1)^d\right]-\left[\frac{2\overline{b_7c_2}}{(1-b_7c_2)^2}\right]\left[d(b_7c_2)^d\,\overline{b_7c_2}-1+(b_7c_2)^d\right]$$

$$+\left[\frac{4\overline{b_7c_3}}{(1-b_7c_3)^2}\right]\left[d(b_7c_3)^d\,\overline{b_7c_3}-1+(b_7c_3)^d\right]-\left[\frac{2\overline{b_7c_4}}{(1-b_7c_4)^2}\right]\left[d(b_7c_4)^d\,\overline{b_7c_4}-1+(b_7c_4)^d\right]$$

$$-\left[\frac{2\overline{b_7c_5}}{(1-b_7c_5)^2}\right]\left[d(b_7c_5)^d\,\overline{b_7c_5}-1+(b_7c_5)^d\right]+\left[\frac{\overline{b_7c_6}}{(1-b_7c_6)^2}\right]\left[d(b_7c_6)^d\,\overline{b_7c_6}-1+(b_7c_6)^d\right]$$

$$+\left[\frac{\overline{b_7c_7}}{(1-b_7c_7)^2}\right]\left[d(b_7c_7)^d\,\overline{b_7c_7}-1+(b_7c_7)^d\right]+\left[\frac{\overline{b_7c_8}}{(1-b_7c_8)^2}\right]\left[d(b_7c_8)^d\,\overline{b_7c_8}-1+(b_7c_8)^d\right]$$

$$-\left[\frac{2\overline{b_8c_1}}{(1-b_8c_1)^2}\right]\left[d(b_8c_1)^d\,\overline{b_8c_1}-1+(b_8c_1)^d\right]-\left[\frac{2\overline{b_8c_2}}{(1-b_8c_2)^2}\right]\left[d(b_8c_2)^d\,\overline{b_8c_2}-1+(b_8c_2)^d\right]$$

$$+\left[\frac{4\overline{b_8c_3}}{(1-b_8c_3)^2}\right]\left[d(b_8c_3)^d\,\overline{b_8c_3}-1+(b_8c_3)^d\right]-\left[\frac{2\overline{b_8c_4}}{(1-b_8c_4)^2}\right]\left[d(b_8c_4)^d\,\overline{b_8c_4}-1+(b_8c_4)^d\right]$$

$$-\left[\frac{2\overline{b_8c_5}}{(1-b_8c_5)^2}\right]\left[d(b_8c_5)^d\,\overline{b_8c_5}-1+(b_8c_5)^d\right]+\left[\frac{\overline{b_8c_6}}{(1-b_8c_6)^2}\right]\left[d(b_8c_6)^d\,\overline{b_8c_6}-1+(b_8c_6)^d\right]$$

$$+\left[\frac{\overline{b_8c_7}}{(1-b_8c_7)^2}\right]\left[d(b_8c_7)^d\,\overline{b_8c_7}-1+(b_8c_7)^d\right]+\left[\frac{\overline{b_8c_8}}{(1-b_8c_8)^2}\right]\left[d(b_8c_8)^d\,\overline{b_8c_8}-1+(b_8c_8)^d\right]\}\} \quad (16)$$

This equation (16) gives the mean time to recruitment for case (i).

Case (ii): $Y = \min(Y_1, Y_2)$ & $Z = \min(Z_1, Z_2)$

$$P(Y > x) = 4e^{-(\lambda_1+\lambda_2)x} - 2e^{-(2\lambda_1+\lambda_2)x} - 2e^{-(\lambda_1+2\lambda_2)x} + e^{-2(\lambda_1+\lambda_2)x} \quad (17)$$

$$P(S_k \le Y) = 4b_3^k - 2b_4^k - 2b_5^k + b_8^k$$

Where,

$$b_3 = E[e^{-(\lambda_1+\lambda_2)x}], b_4 = E[e^{-(2\lambda_1+\lambda_2)x}], b_5 = E[e^{-(\lambda_1+2\lambda_2)x}], b_8 = E[e^{-2(\lambda_1+\lambda_2)x}] \quad (18)$$

Similarly,

$$P(Z > x) = 4e^{-(\mu_1+\mu_2)x} - 2e^{-(2\mu_1+\mu_2)x} - 2e^{-(\mu_1+2\mu_2)x} + e^{-2(\mu_1+\mu_2)x} \quad (19)$$

$$P(S_k \le Z) = 4c_3^k - 2c_4^k - 2c_5^k + c_8^k$$ Where,

$$c_3 = E[e^{-(\mu_1+\mu_2)x}], c_4 = E[e^{-(2\mu_1+\mu_2)x}], c_5 = E[e^{-(\mu_1+2\mu_2)x}], c_8 = E[e^{-2(\mu_1+\mu_2)x}] \quad (20)$$

Using (18) and (20) in (3)

$$P(T > t) = \sum_{k=0}^{d-1}\{G_k(t) - G_{k+1}(t)\}(4b_3^k - 2b_4^k - 2b_5^k + b_8^k) + p\sum_{k=0}^{d-1}\{G_k(t) - G_{k+1}(t)\}$$
$$(1 - 4b_3^k + 2b_4^k + 2b_5^k - b_8^k)(4c_3^k - 2c_4^k - 2c_5^k + c_8^k) \quad (21)$$

and L(t)=1-P(T>t)

As in case (i), the mean time to recruitment is obtained as,

$$E(T) = \frac{1}{2\eta q_1 q_2}\{-\left[\frac{4\overline{b_3}}{(1-b_3)^2}\right]\left[db_3^d\,\overline{b_3}-1+b_3^d\right]+\left[\frac{2\overline{b_4}}{(1-b_4)^2}\right]\left[db_4^d\,\overline{b_4}-1+b_4^d\right]+$$

$$\left[\frac{2\overline{b_5}}{(1-b_5)^2}\right]\left[db_5^d\,\overline{b_5}-1+b_5^d\right]-\left[\frac{\overline{b_8}}{(1-b_8)^2}\right]\left[db_8^d\,\overline{b_8}-1+b_8^d\right]$$

$$+p\{-\left[\frac{4\overline{c_3}}{(1-c_3)^2}\right]\left[dc_3^d\,\overline{c_3}-1+c_3^d\right]+\left[\frac{2\overline{c_4}}{(1-c_4)^2}\right]\left[dc_4^d\,\overline{c_4}-1+c_4^d\right]$$

$$+\left[\frac{2\overline{c_5}}{(1-c_5)^2}\right]\left[dc_5^d\,\overline{c_5}-1+c_5^d\right]-\left[\frac{\overline{c_8}}{(1-c_8)^2}\right]\left[dc_8^d\,\overline{c_8}-1+c_8^d\right]$$

$$+\left[\frac{16\overline{b_3c_3}}{(1-b_3c_3)^2}\right]\left[d(b_3c_3)^d\,\overline{b_3c_3}-1+(b_3c_3)^d\right]-\left[\frac{8\overline{b_3c_4}}{(1-b_3c_4)^2}\right]\left[d(b_3c_4)^d\,\overline{b_3c_4}-1+(b_3c_4)^d\right]$$

$$-\left[\frac{8\overline{b_3c_5}}{(1-b_3c_5)^2}\right]\left[d(b_3c_5)^d\,\overline{b_3c_5}-1+(b_3c_5)^d\right]+\left[\frac{4\overline{b_3c_8}}{(1-b_3c_8)^2}\right]\left[d(b_3c_8)^d\,\overline{b_3c_8}-1+(b_3c_8)^d\right]$$

$$-\left[\frac{8\overline{b_4c_3}}{(1-b_4c_3)^2}\right]\left[d(b_4c_3)^d\,\overline{b_4c_3}-1+(b_4c_3)^d\right]+\left[\frac{4\overline{b_4c_4}}{(1-b_4c_4)^2}\right]\left[d(b_4c_4)^d\,\overline{b_4c_4}-1+(b_4c_4)^d\right]$$

$$+\left[\frac{4\overline{b_4c_5}}{(1-b_4c_5)^2}\right]\left[d(b_4c_5)^d\,\overline{b_4c_5}-1+(b_4c_5)^d\right]-\left[\frac{2\overline{b_4c_8}}{(1-b_4c_8)^2}\right]\left[d(b_4c_8)^d\,\overline{b_4c_8}-1+(b_4c_8)^d\right]$$

$$-\left[\frac{8\overline{b_5c_3}}{(1-b_5c_3)^2}\right]\left[d(b_5c_3)^d\,\overline{b_5c_3}-1+(b_5c_3)^d\right]+\left[\frac{4\overline{b_5c_4}}{(1-b_5c_4)^2}\right]\left[d(b_5c_4)^d\,\overline{b_5c_4}-1+(b_5c_4)^d\right]$$

$$+\left[\frac{4\overline{b_5c_5}}{(1-b_5c_5)^2}\right]\left[d(b_5c_5)^d\,\overline{b_5c_5}-1+(b_5c_5)^d\right]-\left[\frac{2\overline{b_5c_8}}{(1-b_5c_8)^2}\right]\left[d(b_5c_8)^d\,\overline{b_5c_8}-1+(b_5c_8)^d\right]$$

$$+\left[\frac{4\overline{b_8c_3}}{(1-b_8c_3)^2}\right]\left[d(b_8c_3)^d\,\overline{b_8c_3}-1+(b_8c_3)^d\right]-\left[\frac{2\overline{b_8c_4}}{(1-b_8c_4)^2}\right]\left[d(b_8c_4)^d\,\overline{b_8c_4}-1+(b_8c_4)^d\right]$$

$$-\left[\frac{2\overline{b_8c_5}}{(1-b_8c_5)^2}\right]\left[d(b_8c_5)^d\,\overline{b_8c_5}-1+(b_8c_5)^d\right]+\left[\frac{\overline{b_8c_8}}{(1-b_8c_8)^2}\right]\left[d(b_8c_8)^d\,\overline{b_8c_8}-1+(b_8c_8)^d\right]\}\}$$

IV. CONCLUSION

The model discussed in this paper based on Bivariate, (i.e) loss of manpower and number of decisions taken by organization if found to the more realistic in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs and (ii) associating a probability for any decision to have

 DOI : 10.18231/2454-9150.2018.0849

exit points. From the organization's point of view, our model is more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

The analytical result obtained can be illustrated numerically and from the illustration it may be identified the realistic of the problem.

REFERENCES

[1] Bartholomew.D.J.(1973), Stochastic model for social processes, (John Wiley and Sons, New York).

[2] Bartholomew. D.J., and Andrew Forbes.F,(1979) Statistical techniques for manpower planning. (John Wiley and Sons, New York) .

[3] Devi. A., and Srinivasan. A,(2014) Variance of time to recruitment for single grade manpower system with different epochs for decisions and exits, *International Journal of Research in Mathematics and Computations, 2, 23-27.*

[4] Esther Clara. J.B.(2012), Contributions to the study on some stochastic models in manpower planning, Bharathidasan University, Tiruchirappalli.

[5] Girnold.R.C., and Marshall.K.T.(1977), Manpower planning models, (North-Holland, New York).

[6] Gurland J.(1955), "Distribution of Maximum of the Arithmetic Mean Correlated random variables". *Ann. Math. Statist.* (26) , 294-300.

[7] Ishwarya .G., and Srinivasan.A.(2015),Time to recruitment in a Two Graded Manpower System with different Epochs for Decisions and Exits , *International Journal of Science , Technology and Management*,4(3), 1- 10.

[8] Parameswari.K., Sridharan.J., and Srinivasan.A.(2013), Stochastic model on time to recruitment in a two graded manpower system , Proceedings of National conference on Recent Advances in Mathematical Analysis and Applications, 378-387.

[9] Ravichandran.G., and Srinivasan.A.(2015), Variance of time to recruitment for a single grade manpower system with two thresholds having different epochs for decisions and exits, *Indian Journal of Applied Research* 5(1),60-64.

[10] Saral.L.,Sendhamizhselvi.S., and Srinivasan.A.(2016),Mean Time to Recruitment for a two Graded manpower system with two thresholds, Different Epochs for Exits and Two types of inter-decisions , *International Journal of Innovative Research in Science , Engineering & Technology,5(11),19980-19986.*

[11] Selvambigai.D., and Srinivasan.A.,(2011), "Stochastic model for mean time to recruitment in a two graded organization-IV",*Acta Ciencia Indica,* 37(2),321-324.

[12] Srinivasan, A., and Vasudevan, V.(2011), Variance of the time to recruitment in an Organization with two grades, *Recent Research in Science and Technology,* 3(1),128-131.

[13] Uma.K.P.,(2010), "A study on manpower models with univariate and bivariate policies of recruitment", Ph.D. Thesis, Avinasilingam University for women, Coimbatore.

 DOI : 10.18231/2454-9150.2018.0849