P_4 - Decomposition in Boolean Function Graph B(G, L(G), NINC) of a graph

S. Muthammai

Principal(Retd), Alagappa Government Arts college, Karaikudi, Tamil Nadu, India.

muthammai.sivakami@gmail.com,

S. Dhanalakshmi

Government Arts College for Women (Autonomous), Pudukkottai, Tamil Nadu, India.

dhanalakshmi8108@gmail.com

Abstract - **For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B(G, L(G), NINC) of G is a graph with vertex set V(G)∪E(G) and two vertices in B(G, L(G), NINC) are adjacent if and only if they correspond to two adjacent vertices of G, two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by B_1(G). In this paper, P_4-decomposition of Boolean Function Graph B(G, L(G), NINC) of some standard graphs and corona graphs are obtained.**

Keywords: Boolean Function graph, Edge Domination Number, Decomposition

I. INTRODUCTION

Graphs discussed in this paper are undirected and simple graphs. For a graph G, let V(G) and E(G) denote its vertex set and edge set respectively. A graph with p vertices and q edges is denoted by G(p, q). A subset F ⊆ E(G) is called an edge dominating set of G, if every edge not in F is adjacent to some edge in F. The edge domination number γ'(G) of G is the minimum cardinality taken over all edge dominating sets of G. The corona G_1o G_2 of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2, and then joining the i^{th} vertex of G_1 to every vertex of in the i^{th} copy of G_2. For any graph G, GoK_1 is denoted by G^+.

A decomposition of a graph G is a family of edge-disjoint subgraphs{G_1, G_2, …, G_k} such that E(G) = E(G_1) ∪E(G_2) ∪ … ∪E(G_k). If each G_i is isomorphic to H, for some subgraph H of G, then the decomposition is called a H-decomposition of G. In particular, a P_4-decomposition of a graph G is a partition of the edge set of G into paths of length 3. In this case, G is said to be P_4-decomposable. Several authors studied various types of decomposition by imposing conditions on G_i in the decomposition. Heinrich, Liu and Yu[3] proved that a connected 4-regular graph admits a P_4-decomposition if and only if $|E(G)| \equiv 0 (\text{mod } 3)$. Sunil Kumar[10] proved that a complete r- partite graph is P_4-decomposable if and only if its size is a multiple of 3. P.Chithra devi and J. Paulraj Joseph [1] gave a necessary and sufficient condition for the decomposition of the total graph of standard graphs and corona of graphs into paths on three edges. Janakiraman et al., introduced the concept of Boolean function graphs [4 - 6]. For a real x, $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. For any graph G, let V(G) and E(G) denote the vertex set and edge set of G respectively. The Boolean function graph B(G, L(G), NINC) of G is a graph with vertex set V(G)∪E(G) and two vertices in B(G, L(G), NINC) are adjacent if and only if they correspond to two adjacent vertices of G, two adjacent edges of G or to a vertex and an edge not incident to it in G. For brevity, this graph is denoted by B_1(G).

In this paper, P_4 - decomposition of Boolean Function Graph B(G, L(G), NINC) of some standard graphs are obtained.

II. PRIOR RESULTS

Observation 2.1. [4]

Let G be a graph with p vertices and q edges.

1. G and L(G) are induced subgraphs of B_1(G).

2. Number of vertices in B_1(G) is p + q and if $d_i = \deg_G(v_i)$, $v_i \in V(G)$, then the number of edges in B_1(G) is $q(p - 2) + \frac{1}{2}\sum_{1\leq i\leq p} d_i^2$.

3. The degree of a vertex of G in B_1(G) is q and the degree of a vertex e′ of L(G) in B_1(G) is $\deg_{L(G)}(e') + p - 2$. Also if $d^*(e')$ is the degree of a vertex e′ of L(G) in B_1(G), then $0 \leq d^*(e') \leq p + q - 3$. The lower bound is attained, if $G \cong K_2$ and the upper bound is attained, if $G \cong K_{1,n}$, for $n \geq 2$.

Theorem 2.2 [4]. B_1(G) is disconnected if and only if G is one of the following graphs: nK_1, K_2, $2K_2$ and $K_2 \cup nK_1$, for $n \geq 1$.

Theorem 2.3. [7]. $\gamma'(B_1(P_n)) = n-1$, $\gamma'(B_1(C_n)) = n-1$, $n \geq 3$.

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Theorem 2.4. [7]. $\gamma'(B_1(K_{1,n})) = (n+4)/3$, $n \geq 2$.

Theorem 2.5. [8]. $\gamma'(B_1(P_n^{\ +})) = \left\lceil \frac{3n}{2} \right\rceil$, $n \geq 2$.

Theorem 2.6. [8]. $\gamma'(B_1(C_n^{\ +})) = \left\lceil \frac{3n}{2} \right\rceil$, $n \geq 3$.

Theorem 2.7. [8]. $\gamma'(B_1(K_{1,n}^{\ +})) = n+2$, $n \geq 2$.

III. MAIN RESULTS

In the following, P_4- decomposition of $B_1(P_n)$, $B_1(C_n)$, $B_1(K_{1,n})$ and corona graphs are found.

Theorem 3.1

- **(i)** For $n \geq 6$ and $n \equiv 0 \pmod 3$, the graph $B_1(P_n) - 2K_2$ is P_4 – decomposable .
- **(ii)** For $n \geq 7$ and $n \equiv 1 \pmod 3$, the graph $B_1(P_n) - (n+1)K_2$ is P_4 – decomposable.
- **(iii)** For $n \geq 8$ and $n \equiv 2 \pmod 3$, the graph $B_1(P_n) - 2nK_2$ is P_4 – decomposable.

Proof: Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = (v_i, v_{i+1})$, $i = 1, 2, \ldots, n-1$, be the edges of P_n. Then $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_{n-1} \in V(B_1(P_n))$. $B_1(P_n)$ has $2n-1$ vertices and $n^2 - n - 1$ edges. It is to be noted that, in all the sets defined below the suffix i in v_i is integer modulo n, and j in e_j is integer modulo n-1, $v_0 = v_n$ and $e_0 = e_{n-1}$.

Let $F = \bigcup_{i=1}^{n-1} F_i$, where $F_1 = \{(v_1, e_i), i = 2, 3, \ldots, n-1\}$ and

$F_i = \{(v_i, v_j), 1 \leq j \leq n-1 \text{ and } j \neq i-1, i\}$, $i = 2, 3, \ldots, n-2$. $F_{n-1} = \{(v_n, e_j), j = 1, 2, \ldots, n-2\}$.

$E(B_1(P_n)) = E(P_n) \cup E(L(P_n)) \cup F = E(P_n) \cup E(P_{n-1}) \cup F$.

Case 1. $n \equiv 0 \pmod 3$.

Then the edge set of $B_1(P_n)$ can be decomposed into $((n^2 - n - 3)/3)P_4$ and $2K_2$. The edge set of $2K_2$ is given by the set $\{(v_{n-1}, v_n), (v_1, e_{n-1})\}$. The edge sets of $((n^2 - n - 3)/3)P_4$ are given by the edge sets $A^{(1)}$, $A^{(2)}$, $A^{(3)}$ and $A^{(4)}$, where $A^{(1)} = \bigcup_{i=1}^{n-2} A_i^{(1)}$ and $A_i^{(1)} = \{(v_i, v_{i+1}), (v_{i+1}, e_{i+2}), (e_{i+2}, v_{i+4})\}$, $A^{(2)} = \bigcup_{i=1}^{n-2} A_i^{(2)}$, where $A_i^{(2)} = \{(v_n, e_i), (e_i, e_{i+1}), (e_{i+1}, v_{i+4})\}$,
$A^{(3)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-6)/3} A_{j,i}^{(3)})$, where $A_{j,i}^{(3)} = \{(e_{i+2j}, v_i), (v_i, e_{i+2j+1}), (e_{i+2j+1}, v_{i+3j+4})\}$ and
$A^{(4)} = \{(v_1, e_2), (e_2, v_4), (v_4, e_1)\}$. Here, $\langle A^{(1)} \rangle \cong (n-2)P_4$, $\langle A^{(2)} \rangle \cong (n-2)P_4$,
$\langle A^{(3)} \rangle \cong ((n-1)((n-6))/3)P_4$ and $\langle A^{(4)} \rangle \cong P_4$. Therefore, $B_1(P_n) - 2K_2$ is P_4 – decomposable.

Case 2. $n \equiv 1 \pmod 3$

Then the edge set of $B_1(P_n)$ can be decomposed into $((n^2 - 2n - 2)/3)$ P_4 and $(n+1)K_2$. The edge set of $(n+1)K_2$ is given by the set $\{(v_{n-1}, v_n), (v_1, e_{n-1})\} \cup (\bigcup_{i=1}^{n-1} v_i, e_{i+2})\})$. The edge sets of $((n^2 - 2n - 2)/3)P_4$ are given by the edge sets $A^{(1)}$, $A^{(2)}$, and $A^{(4)}$, as in Case1, and the set $A^{(5)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-7)/3} A_{j,i}^{(5)})$, where $A_{j,i}^{(5)} = \{(e_{i+2j+2}, v_i), (v_i, e_{i+2j+3}), (e_{i+2j+3}, v_{i+3j+6})\}$,
$\langle A^{(5)} \rangle \cong ((n-1)((n-7))/3)P_4$. Therefore, $B_1(P_n) - (n+1)K_2$ is P_4 – decomposable.

Case 3. $n \equiv 2 \pmod 3$

Then the edge set of $B_1(P_n)$ can be decomposed into $((n^2 - 3n - 1)/3)$ P_4 and $2nK_2$. The edge set of $2nK_2$ is given by the set $\{(v_{n-1}, v_n), (v_1, e_{n-1})\} \cup (\bigcup_{i=1}^{n-1} \{(v_i, e_{i+2}), (v_i, e_{i+3})\})$. The edge sets of $((n^2 - 3n - 1)/3)$ P_4 are given by the edge sets $A^{(1)}$, $A^{(2)}$, and $A^{(4)}$, as in Case1 , and the set $A^{(6)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-8)/3} A_{j,i}^{(6)})$ where $A_{j,i}^{(6)} = \{(e_{i+2j+2}, v_i), (v_i, e_{i+2j+3}), (e_{i+2j+3}, v_{i+3j+6})\}$,
$\langle A^{(6)} \rangle \cong ((n-1)((n-8))/3)P_4$. Therefore, $B_1(P_n) - 2nK_2$ is P_4 – decomposable.

Theorem 3.2

- **(i)** For $n \geq 3$ and $n \equiv 0 \pmod 3$, the graph $B_1(C_n)$ is P_4 – decomposable.
- **(ii)** For $n \geq 4$ and $n \equiv 1 \pmod 3$, the graph $B_1(C_n) - nK_2$ is P_4 – decomposable.
- **(iii)** For $n \geq 5$ and $n \equiv 2 \pmod 3$, the graph $B_1(C_n) - 2nK_2$ is P_4 – decomposable.

Proof: Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $e_i = (v_i, v_{i+1})$, $i = 1, 2, \ldots, n-1$, $e_n = (v_n, v_1)$ be the edges of C_n. Then $v_1, v_2, \ldots, v_n, e_1, e_2, \ldots, e_n \in V(B_1(C_n))$. $B_1(C_n)$ has $2n$ vertices and n^2 edges. In all the sets, suffices are integers modulo n, $v_0 = v_n$ and $e_0 = e_n$.

Let $F_1 = \{(v_1, e_j), j = 2, 3, \ldots, n-1\}$ and $F_i = \{(v_i, e_j), 1 \leq j \leq n \text{ and } j \neq i, i-1\}$, $2 \leq i \leq n$.

$E(B_1(C_n)) = E(C_n) \cup E(L(C_n)) \cup F = E(C_n) \cup E(C_n) \cup F = E(2C_n) \cup F$.

Case 1. $n \equiv 0 \pmod 3$.

Then the edge set of $B_1(C_n)$ can be decomposed into $(n^2/3)$ P_4 . The edge sets of $(n^2/3)$ P_4 are given by the edge sets $B^{(1)}$ and $B^{(2)}$, where $B^{(1)} = \bigcup_{i=1}^{n} B_i^{(1)}$ and $B_i^{(1)} = \{(v_i, v_{i+1}), (v_{i+1}, e_{i+2}), (e_{i+2}, e_{i+3})\}$, $B^{(2)} = \bigcup_{i=1}^{n} (\bigcup_{j=1}^{(n-3)/3} B_{j,i}^{(2)})$, where $B_{j,i}^{(2)} = \{(v_i, e_{i+j+1}), (e_{i+j+1}, v_{i+3j+1}), (v_{i+3j+1}, e_{i+j})\}$.

Here $\langle B^{(1)} \rangle \cong nP_4$, $\langle B^{(2)} \rangle \cong n((n-3)/3)P_4$. Hence, $B_1(C_n)$ is P_4 – decomposable.

Conversely, suppose that $B_1(C_n)$ is P_4 – decomposable. Then $|E(B_1(C_n))| \equiv 0 \pmod 3$, which implies $n^2 \equiv 0 \pmod 3$ and hence $n \equiv 0 \pmod 3$.

Case 2. $n \equiv 1 \pmod 3$

Then the edge set of $B_1(C_n)$ can be decomposed into $((n^2 - n)/3)$ P_4 and nK_2 . The edge set of nK_2 is given by $\{(v_i,$

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e_{i+2}), i =1, 2, …, n }. The edge sets of $((n^2 - n)/3)$ P_4 are given by the sets $B^{(1)}$ as in Case1 and $B^{(3)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-4)/3} B_{j,i}^{(3)})$, where $B_{j,i}^{(3)}$ ={(v_i, e_{i+2j+1}), (e_{i+2j+1}, v_{i+4j+1}),(v_{i+4j+1}, e_{i+j+1})}. Here, < $B^{(3)}$ > ≅ n ((n-4)/3)P_4. Hence, $B_1(C_n) - nK_2$ is P_4 – decomposable.

Case 3. n ≡ 2(mod 3)

Then the edge set of $B_1(C_n)$ can be decomposed into $((n^2 - 2n)/3)P_4$ and $2nK_2$. The edge set of $2nK_2$ is given by the set of edges {(v_i, e_{i+2}), (v_i, e_{i+3}), i =1, 2, …, n }. The edge set $((n^2 - n)/3)P_4$ is given by the set $B^{(1)}$ as in Case1 and $B^{(4)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-5)/3} B_{j,i}^{(3)})$ and $B_{j,i}^{(3)}$ is as in Case 2.
Here , < $B^{(4)}$ > ≅ n ((n-5)/3)P_4. Hence, $B_1(C_n) - 2nK_2$ is P_4 – decomposable.

Theorem 3.3

For n ≥ 4, the graph $B_1(K_{1,n}) - nK_2$ is P_4 – decomposable.

Proof: Let $V(K_{1,n})$ = {v, v_1, v_2, …, v_n }. Let e_i = (v, v_i), i = 1, 2, …, n be the edges of $K_{1,n}$. Then v, v_1, v_2, …, v_n, e_1, e_2, …, e_n ∈ $V(B_1(K_{1,n}))$. $B_1(K_{1,n})$ has 2n+1 vertices and
(n(3n–1)) ⁄2) edges. In all the sets defined below, the suffices are integers modulo n, $v_o = v_n$ and $e_o = e_n$. Let F = $\bigcup_{i=1}^{n} F_i$, where F_i= { (v_i, e_j), 1 ≤ j ≤ n and j≠ i } .
$E(B_1(K_{1,n})) = E(K_{1,n}) \cup E(L(K_{1,n})) \cup F = E(K_{1,n}) \cup E(K_n) \cup F$.

Case 1. n ≡ 0 (mod 3)

Then the edge set of $B_1(K_{1,n})$ can be decomposed into $((n^2 - n)/2)P_4$ and nK_2 . The edge set of nK_2 is given by the set {(v_i, e_{i+n-1}), i=1, 2, …, n}. Let $D^{(1)} = \bigcup_{i=1}^{n} D_i^{(1)}$, where

$D_i^{(1)}$ ={(v, v_i), (v_i, e_{i+1}), (e_{i+1}, e_{i+2})} and $D^{(2)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-3)/3} D_{j,i}^{(2)})$ and $D_{j,i}^{(2)}$ = {(v_i, e_{i+j+1}), (e_{i+j+1}, v_{i+3j+1}), (v_{i+3j+1}, e_{i+j})}. Here, < $D^{(1)}$ > ≅ nP_4, < $D^{(2)}$ > ≅ n((n-3)/3) P_4.

Subcase 1.1. n ≡ 0 (mod 3), n ≥ 9 and n is odd.

The edge set of $((n^2 - n)/2)$ P_4 is given by the set $D^{(1)} \cup D^{(2)} \cup D^{(3)}$, where
$D^{(3)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-3)/6} D_{j,i}^{(3)})$ and $D_{j,i}^{(3)}$ = {(e_i, e_{i+2j}), (e_{i+2j}, e_{i+5j+1}), (e_{i+5j+1}, e_{i+j+2})}, < $D^{(3)}$ > ≅ n((n-3)/6) P_4.

Subcase 1.2. n ≡ 0 (mod 3), n ≥ 6 and n is even.

The edge set of $((n^2 - n)/2)$ P_4 is given by the set $D^{(1)} \cup D^{(2)} \cup D^{(4)} \cup D^{(5)}$, where
$D^{(4)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-6)/6} D_{j,i}^{(4)})$, where $D_{j,i}^{(4)}$ = {(e_i, e_{i+3j+1}), (e_{i+3j+1}, e_{i+5j+1}), (e_{i+5j+1}, e_{i+j+2})} and
$D^{(5)} = \bigcup_{i=1}^{n/2} D_i^{(5)}$, where $D_i^{(5)}$ = {(e_i, $e_{n/2+i-1}$), ($e_{n/2+i-1}$, e_{n+i-1}), (e_{n+i-1}, $e_{n/2+2-i}$)},
< $D^{(4)}$ > ≅ n((n-6)/6)P_4 , < $D^{(5)}$ > ≅ (n/2) P_4. Therefore, $B_1(K_{1,n}) - nK_2$ is P_4 – decomposable .

Case 2. n ≡ 1, 2 (mod 3) , n ≥ 4

Then the edge set of $B_1(K_{1,n})$ can be decomposed into $((n^2 - n)/2)P_4$ and nK_2's.

Subcase 2.1 n ≡ 1, 2 (mod 3), n ≥ 5 and n is odd.

The edge sets of $((n^2 - n)/2)$ P_4 are given by the set $D^{(1)} \cup D^{(6)}$, where
$D^{(6)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-3)/2} D_{j,i}^{(6)})$, where $D_{j,i}^{(6)}$ ={(e_{i+2j}, v_i), (v_i, e_{i+2j+1}), (e_{i+2j+1}, e_{i+j})},
< $D^{(6)}$ > ≅ n((n-3)/2) P_4.

Subcase 2.2 n ≡ 1, 2 (mod 3), n ≥ 8 and n is even.

The edge sets of $((n^2 - n)/2)P_4$ are given by the set $D^{(1)} \cup D^{(7)} \cup D^{(8)}$, where $D^{(7)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-4)/2} D_{j,i}^{(7)})$, where $D_{j,i}^{(7)}$ = {(e_{i+2j}, v_i), (v_i, e_{i+2j+1}), (e_{i+2j+1}, e_{i+j})} and
$D^{(8)} = \bigcup_{i=1}^{n/2} D_i^{(8)}$, where $D_i^{(8)}$ = {(v_i, e_{i+n-2}), (e_{i+n-2}, $e_{n/2-1}$), ($e_{n/2-1}$, $v_{n/2+1}$)}, < $D^{(7)}$ > ≅ n((n-4)/2)P_4,
< $D^{(8)}$ > ≅ (n/2) P_4. Therefore, $B_1(K_{1,n}) - nK_2$ is P_4 – decomposable .

In the following, P_4 – decomposition of Boolean function graph of corona of P_n, C_n and $K_{1,n}$ are found.

Theorem 3.4

(i) For n ≥ 6 and n ≡ 0 (mod 3), the graph $B_1(P_n^+) - (3n-3)K_2$ is P_4 – decomposable .

(ii) For n ≥ 4 and n ≡ 1 (mod 3), the graph $B_1(P_n^+) - (n+2)K_2$ is P_4 – decomposable .

(iii) For n ≥ 4 and n ≡ 2 (mod 3), the graph $B_1(P_n^+) - (2n+1)K_2$ is P_4 – decomposable .

Proof: Let $V(P_n^+)$ = { v_1, v_2, …, v_n , u_1, u_2, …,u_n} where v_1, v_2, …, v_n are the vertices of P_n and u_1, u_2, …,u_n are the pendant vertices of P_n^+ and e_i = (v_i, v_{i+1}), i = 1, 2, …, n-1and f_i = (v_i, u_i), i=1,2, …, n be the edges of P_n^+. Then v_1, v_2, …, v_n, u_1, u_2, …,u_n , e_1, e_2, …, e_{n-1}, f_1, f_2, …, f_n ∈$V(B_1(P_n^+))$. P_n^+ and $L(P_n^+)$ are induced subgraphs of $B_1(P_n^+)$. $B_1(P_n^+)$ has 4n-1 vertices and $4n^2$ - n -3 edges. In all the sets, the suffices i in v_i and j in e_j are integers modulo n and n-1 respectively, and suffices k in f_k, u_k, v_k are integers modulo n, $f_0 = f_n$, $u_0 = u_n$, $v_0 = v_n$, $e_0 = e_{n-1}$.

Let F_k= {(v_i, e_j) / for all i, 1≤ i≤ n, j≡ (i+k) (mod (n-1)) and $e_0 = e_{n-1}$} and F = $\bigcup_{k=1}^{n-3} F_k$.

Let H_i= {(v_i, f_j) (u_i, f_j), j=1, 2, …, n, j ≠ i } and H = $\bigcup_{i=1}^{n} H_i$ and │H│= 2n(n-1).

Let J_k= {(u_k, e_j) / for all j, 1≤ j≤ n-1}, J = $\bigcup_{k=1}^{n} J_k$ and │J│= n(n-1).

$E(B_1(P_n^+)) = E(P_n^+) \cup E(L(P_n^+)) \cup (F \cup H \cup J)$.

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Case 1. $n \equiv 0 \pmod 3$.

Then the edge set of $B_1(P_n^+)$ can be decomposed into$((4n^2-4n)/3)P_4$ and $(3n-3)K_2$. The edge set of $(3n-3)K_2$ is given by the set$\{(u_1, f_n), (v_1, f_n), (v_1,e_{n-1})\} \cup \{(v_n,e_i), i=1, 2, \ldots, n-2\} \cup \{(e_i,e_{i+1}), i = 1,2, \ldots, n-2\} \cup \{(e_i, f_i), i=2,3, \ldots, n-1\}$. The edge sets of $((4n^2-4n)/3)P_4$ are given by the edge sets $M^{(1)}$, $M^{(2)}$, $M^{(3)}$ and $M^{(4)}$, where $M^{(1)} = \bigcup_{i=1}^{n-1} M_i^{(1)}$, where $M_i^{(1)} =\{(v_i, v_{i+1}),(v_{i+1}, f_i),(f_i, u_{i+1})\}$, $M^{(2)} =\bigcup_{i=1}^{n} M_i^{(2)}$, where $M_i^{(2)}= \{(v_i, u_i), (u_i, e_i), (e_i, f_{i+1})\}$and

$M^{(3)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-2)} M_{j,i}^{(3)})$, where $M_{j,i}^{(3)} = \{(v_i, f_{i+j}), (f_{i+j}, u_i), (u_i, e_{i+j})\}$ and

$M^{(4)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-3)/3} M_{j,i}^{(4)})$, where $M_{j,i}^{(4)} = \{(e_{i+3j-2}, v_i), (v_i, e_{i+j}), (e_{i+j}, v_{i+1})\}$.

Here, $< M^{(1)} > \cong (n-1)P_4$, $< M^{(2)} > \cong nP_4$, $< M^{(3)} > \cong n(n-2)P_4$ and $< M^{(4)} > \cong (n-1)((n-3)/3)P_4$. Therefore, $B_1(P_n^+) - (3n-3)K_2$ is P_4 - decomposable.

Case 2. $n \equiv 1 \pmod 3$.

Then the edge set of $B_1(P_n^+)$ can be decomposed into $((4n^2-2n-5)/3)P_4$ and $(n+2)K_2$. The edge set of $(n+2)K_2$ is given by the set $\{(u_1, f_n), (v_1, f_n), (v_1,e_{n-1}), (v_{(n+5)/3}, e_1)\} \cup (\bigcup_{i=2}^{n-1}\{(e_i, f_i)\})$. The edge sets of $((4n^2-2n-5)/3)P_4$ are given by the edge set $M^{(1)}$, $M^{(2)}$ and $M^{(3)}$ as in case 1, and the set $M^{(5)} = \bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-4)/3} M_{j,i}^{(5)})$ and $M_{j,i}^{(5)} = \{(e_{i+2j+1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, v_{i+3j+1})\}$.

Here, $< M^{(5)} > \cong (n-1)((n-4)/3)P_4$. Hence, $B_1(P_n^+) - (n+2)K_2$ is P_4 - decomposable.

Case 3. $n \equiv 2 \pmod 3$.

Then the edge set of $B_1(P_n^+)$ can be decomposed into $((4n^2-3n-4)/3)P_4$ and $(2n+1)K_2$. The edge set of $(2n+1)K_2$ is given by $\{(u_1, f_n), (v_1, f_n), (v_1,e_{n-1}),(v_{(n+4)/3}, e_1)\} \cup (\bigcup_{i=2}^{n-1}\{(e_i, f_i)\}) \cup (\bigcup_{i=1}^{n-1}\{(v_i, e_{i+1})\})$. The edge sets $((4n^2-3n-4)/3)P_4$ are given by the edge set $M^{(1)}$, $M^{(2)}$ and $M^{(3)}$ as in case1, and the set $M^{(6)} =\bigcup_{i=1}^{n-1} (\bigcup_{j=1}^{(n-5)/3} M_{j,i}^{(6)})$, where $M_{j,i}^{(6)} =\{(e_{i+2j}, v_i), (v_i, e_{i+2j+1}),(e_{i+2j+1}, v_{i+3j+2})\}$ and $M^{(7)} =\{(v_n, e_i), (e_i, e_{i+1}), (e_{i+1}, v_{i+(n+4)/3})\}$. Here, $< M^{(6)} > \cong (n-1)((n-5)/3)P_4$,
$< M^{(7)} > \cong (n-2)P_4$. Hence, $B_1(P_n^+) - (2n+1)K_2$ is P_4 - decomposable.

Theorem 3.5

(i) For $n \geq 6$ and $n \equiv 0 \pmod 3$, the graph $B_1(C_n^+) - nK_2$ is P_4 - decomposable .

(ii) For $n \geq 4$ and $n \equiv 1 \pmod 3$, the graph $B_1(C_n^+) - 2nK_2$ is P_4 - decomposable .

(iii) For $n \geq 5$ and $n \equiv 2 \pmod 3$, the graph $B_1(C_n^+)$ is P_4 - decomposable .

Proof: Let $V(C_n^+) = \{ v_1, v_2, \ldots, v_n, u_1, u_2, \ldots,u_n\}$where $v_1, v_2, \ldots, v_n$ are the vertices of C_n and $u_1, u_2, \ldots,u_n$ are the pendant vertices of C_n^+ and $e_i = (v_i, v_{i+1})$, i= 1, 2, …,n-1 , $e_n = (v_n, v_1)$ and $f_i = (v_i, u_i)$, i=1,2, …, n be the edges of C_n^+. Then $v_1, v_2, \ldots, v_n, u_1, u_2, \ldots,u_n, e_1, e_2, \ldots, e_n, f_1, f_2, \ldots, f_n, \in V(B_1(C_n^+))$. C_n^+ and $L(C_n^+)$ are induced subgraphs of $B_1(C_n^+)$. $B_1(C_n^+)$ has 2n vertices and $4n^2 + n$ edges. In all the sets, the suffices i in v_i , u_i, e_i and f_i are integers modulo n, $v_0=v_n$, $u_0= u_n$, $e_0=e_n$ and $f_0= f_n$.

Let $F_i= \{(v_i, e_{i+j}) / 1\leq j\leq n-2\}$ and $F = \bigcup_{i=1}^{n} F_i$. $|F| = n(n-2)$.

Let $H_i= \{(v_i, f_j) (u_i, f_j) / 1\leq j\leq n, j\neq i \}$ and $H = \bigcup_{i=1}^{n} H_i$ and $|H| = 2n(n-1)$.

Let $J_k= \{(u_k, e_j) / 1\leq j \leq n\}$, $J = \bigcup_{k=1}^{n} J_k$ and $|J| = n^2$.

$E(B_1(C_n^+)) = E(C_n^+) \cup E(L(C_n^+)) \cup (F \cup H \cup J)$.

Let $N^{(1)} = \bigcup_{i=1}^{n} N_i^{(1)}$, where $N_i^{(1)} = \{(v_i, v_{i+1}), (v_{i+1}, e_{i+2}), (e_{i+2}, f_{i+2}\}$and

$N^{(2)} = \bigcup_{i=1}^{n} N_i^{(2)}$, where $N_i^{(2)} = \{(v_i , u_i), (u_i, e_i), (e_i, f_{i+1})\}$,

$N^{(3)} =\bigcup_{i=1}^{n} \bigcup_{j=1}^{(n-1)} N_{j,i}^{(3)})$, where $N_{j,i}^{(3)} =\{(v_i, f_{i+j}), (f_{i+j}, u_i), (u_i, e_{i+j-1})\}$.

Case 1. $n \equiv 0 \pmod 3$, $n \geq 6$.

The edge set of $B_1 (C_n^+)$ can be decomposed into $(4n^2)/3\ P_4$ and nK_2 . The edge sets of nK_2 is given by the set $\{(e_i, e_{i+1})\}$, i = 1, 2, 3,…, n, $e_{n+1} = e_1$. The edge set of $(4n^2)/3\ P_4$'s are given by the edge sets $N^{(1)}$, $N^{(2)}$, $N^{(3)}$ and $N^{(4)}$, where $N^{(4)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-3)/3} N_{j,i}^{(4)})$and $N_{j,i}^{(4)} = \bigcup_{j=1}^{n} (v_i, e_{i+j+1}), (e_{i+j+1}, v_{i+3j+1}), (v_{i+3j+1}, e_{i+j})\}$. Here, $< N^{(1)} > \cong nP_4$, $< N^{(2)} > \cong nP_4$, $< N^{(3)} > \cong n(n-1)P_4$ and $< N^{(4)} > \cong n((n-3)/3)P_4$. Therefore, $B_1(C_n^+) - nK_2$ is P_4 - decomposable.

Case 2. $n \equiv 1 \pmod 3$, $n \geq 7$.

The edge set of $B_1(C_n^+)$ can be decomposed into$((4n^2- n)/3)P_4$ and $2nK_2$. The edge set of $2nK_2$ is given by $\bigcup_{i=1}^{n} \{(v_i, e_{i+2}), (e_i, e_{i+1})\}$. The edge sets of $((4n^2 -n)/3)P_4$ are given by the edge sets $N^{(1)}$, $N^{(2)}$, $N^{(3)}$ and $N^{(5)}$ where $N^{(5)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-4)/3} N_{j,i}^{(5)})$ and $N_{j,i}^{(5)} =\{(v_i, e_{i+j+1}), (e_{i+j+1}, v_{i+3j+1}), (v_{i+3j+1}, e_{i+j})\}$. Here, $< N^{(1)} > \cong nP_4$, $< N^{(2)} > \cong nP_4$, $< N^{(3)} > \cong n(n-1)P_4$ and

$< N^{(5)} > \cong n((n-4)/3) P_4$. Therefore, $B_1(C_n^+) - 2nK_2$ is P_4 - decomposable.

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Case 3. $n \equiv 2 \pmod 3$, $n \geq 5$

The edge set of $B_1(C_n^+)$ can be decomposed into$((4n^2 + n)/3)P_4$ whose edge sets are given by the sets $N^{(1)}$, $N^{(2)}$, $N^{(3)}$, $N^{(6)}$ and $N^{(7)}$, where $N^{(6)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-5)/3} N_{j,i}^{(6)})$ and

$N_{j,i}^{(6)} = \{(v_i, e_{i+j+2}), (e_{i+j+2}, v_{i+3j+3}), (v_{i+3j+3}, e_{i+j+1})\}$, and $N^{(7)} = \bigcup_{i=1}^{n} N_i^{(7)}$, and

$N_i^{(7)} = \{(v_i, e_{i+2}), (e_{i+2}, e_{i+1}), (e_{i+1}, v_{i+3})\}$. Here, $< N^{(1)} > \cong nP_4$, $< N^{(2)} > \cong nP_4$, $< N^{(3)} > \cong n(n-1)P_4$, $< N^{(6)} > \cong (n((n-5))/3)P_4$ and $< N^{(7)} > \cong n\, P_4$. Therefore, $B_1(C_n^+)$ is P_4 – decomposable.

Theorem 3.6

For $n \geq 3$, the graph $B_1(K_{1,n}^+) - (n+1)K_2$ is P_4 – decomposable .

Proof: Let $V(K_{1,n}^+) = \{v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, u_{n+1}\}$, where v is the central vertex and $< \{v, v_1, v_2, \ldots, v_n\}> \cong K_{1,n}$ and $u_1, u_2, \ldots, u_n, u_{n+1}$ are the pendant vertices of $K_{1,n}^+$ and $e_i = (v, v_i)$, i= 1, 2, …,n and $f_i = (v_i, u_i)$, i=1,2, …, n, be the edges of $K_{1,n}^+$. Then $v, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n, u_{n+1}, f_1, f_2, \ldots, f_n, e_1, e_2, \ldots, e_{n-1}, e_n \in V(B_1(K_{1,n}^+))$. $K_{1,n}^+$ and $L(K_{1,n}^+)$ are induced subgraphs of $B_1(K_{1,n}^+)$ has 4n+3 vertices and $\frac{1}{2}(9n^2 + 11n+2)$ edges. In all the sets, the suffices i in v_i, u_i, e_i and f_i are integers modulo n, $v_0=v_n$, $u_0= u_n$, $e_0=e_n$ and $f_0= f_n$.

Let $F_i = \{(v_i, e_j), (v_i, f_j), (u_i, f_j) \;/\; 1 \leq j \leq n \text{ and } j \neq i\}$. $F = \bigcup_{i=1}^{n} F_i$, $|F| = 3n(n-1)$

Let $H_k= \{(u_k, e_j) \;/\; 1\leq i\leq n\}$, $H = \bigcup_{k=1}^{n} H_k$ and $|H| = n^2$. Let $J = \{(v, f_j) \;/\; 1 \leq j \leq n\}$.

Let $L = \{(v_i, e), (u_i, e), (u, e_i), (u, f_i) \;/\; 1 \leq i \leq n \}$

$E(B_1(K_{1,n}^+))= E(K_{1,n}^+) \cup E(L(K_{1,n}^+)) \cup (F \cup H \cup J \cup L)$.

Let $Q^{(1)} = \bigcup_{i=1}^{n} Q_i^{(1)}$, where $Q_i^{(1)} = \{(v, v_i), (v_i, f_i), (f_i, v_{i+1}\}$and

$Q^{(2)} = \bigcup_{i=1}^{n+1} Q_i^{(2)}$, where $Q_i^{(2)} = \{(v, u_i), (u_i, e_i), (e_i, f_{i+2})\}$and

$Q^{(3)} = \bigcup_{i=1}^{n} Q_i^{(3)}$,where $Q_i^{(3)} = \{(v, f_{i+1}), (f_{i+1}, u_i), (u_i, e_{i+1})\}$

$Q^{(4)}= \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-2)} Q_{j,i}^{(4)})$,where $Q_{j,i}^{(4)} = \{(v_i, f_{i+j+1}), (f_{i+j+1}, u_i), (u_i, e_{i+j+1})\}$.

Here, $< Q^{(1)} > \cong nP_4$, $< Q^{(2)} > \cong (n+1)P_4$, $< Q^{(3)}> \cong nP_4$ and $< Q^{(4)} > \cong n(n-2)P_4$.

Case 1. $n \equiv 0 \pmod 3$.

Then the edge set of $B_1(K_{1,n}^+)$ can be decomposed into $(3n(n+1)/2)P_4$ and $(n+1)K_2$. The edge sets of $(n+1)K_2$ is given by $(\bigcup_{i=1}^{n-1} (e_{i+1}, f_i)) \cup \{(u_1, f_n), (v_1, f_n)\}$.

Subcase 1.1. $n \equiv 0 \pmod 3$ and n is odd, $n \geq 3$.

The edges sets of $(3n(n+1)/2)P_4$ are given by the set $Q^{(1)} \cup Q^{(2)} \cup Q^{(3)} \cup Q^{(4)} \cup Q^{(5)} \cup Q^{(6)}$,

where $Q^{(5)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-1)/2} Q_{j,i}^{(5)})$, where $Q_{j,i}^{(5)} = \bigcup_{i=1}^{n} \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, v_i)\}$,

$< Q^{(5)} > \cong n\,((n-1)/2)\, P_4$ and $Q^{(6)} = \bigcup_{i=1}^{n-1} Q_i^{(6)}$, where $Q_i^{(6)} = \{(e_{i+1}, v_{n+1}), (v_{n+1}, f_i), (f_i, v_{i+1})\}$,

$< Q^{(6)} > \cong (n-1)P_4$

Subcase 1.2. $n \equiv 0 \pmod 3$, $n \geq 6$ and n is even.

The edge sets of $(3n(n+1)/2)\, P_4$'s are given by the set $Q^{(1)} \cup Q^{(2)} \cup Q^{(3)} \cup Q^{(4)} \cup Q^{(7)} \cup Q^{(8)} \cup Q^{(9)} \cup Q^{(10)}$ where $Q^{(7)} = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{(n-4)/4} Q_{j,i}^{(7)})$, where $Q_{j,i}^{(7)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_{i+j-1})\}$,

$Q^{(8)} = \bigcup_{i=1}^{n/2} Q_i^{(8)}$, where $Q_i^{(8)} = \{(v_i, e_{i+5}), (e_{i+5}, e_{i+2}), (e_{i+2}, v_{i+3})\}$,

$Q^{(9)} = \bigcup_{i=1}^{n-1} Q_i^{(9)}$, where $Q_i^{(9)} = \{(e_{i+1}, u_{n+1}), (u_{n+1}, f_i), (f_i, v_{i+1})\}$,

$Q^{(10)} = \bigcup_{i=1}^{n} Q_i^{(10)}$, where $Q_i^{(10)} = \{(e_{i+(n-3)}, v_i), (v_i, e_{i+(n-2)}), (e_{i+(n-2)}, e_{i+(n-1)})\}$,

Here, $< Q^{(7)} > \cong n((n-4)/4)\, P_4$, $< Q^{(8)}> \cong (n/2)P_4$, $< Q^{(9)} > \cong (n-1)P_4$, $< Q^{(10)} > \cong nP_4$.

Therefore, $B_1(K_{1,n}^+) - (n+1)K_2$ is P_4 – decomposable .

Case 2. $n \equiv 1 \pmod 3$.

Then the edge set of $B_1(K_{1,n}^+)$ can be decomposed into $(3n(n+1)/2)\, P_4$ and $(n+1)K_2$.

The edge set of $(n+1)K_2$'s is given by $(\{\bigcup_{i=1}^{n-1} (e_{i+1}, f_i)) \cup \{(u_1, f_n), (v_1, f_n)\}\}$.

Let $Q^{(11)} = \bigcup_{i=1}^{n-1} Q_i^{(11)}$, where $Q_i^{(11)} = \{(e_{i+1}, u_{n+1}), (u_{n+1}, f_i), (f_i, v_{i+1})\}$, $< Q^{(11)} > \cong (n-1)P_4$.

Subcase 2.1. $n \equiv 1 \pmod 3$ and n is odd, $n \geq 7$.

The edge set of $(3n(n+1)/2)\, P_4$ are given by the set $Q^{(1)} \cup Q^{(2)} \cup Q^{(3)} \cup Q^{(4)} \cup Q^{(11)} \cup Q^{(12)}$ where $Q^{(12)}$

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$= \cup_{i=1}^{n}(\cup_{j=1}^{(n-1)/2} Q_{j,i}^{(12)})$, where $Q_{j,i}^{(12)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_i)\}$,

$< Q^{(12)} > \cong n((n-1)/2) P_4$

Subcase 2.2. $n \equiv 1 \pmod 3$, $n \geq 10$ and n is even.

The edge set of P_4's are given by the set $Q^{(1)} \cup Q^{(2)} \cup Q^{(4)} \cup Q^{(11)} \cup Q^{(13)} \cup Q^{(14)} \cup Q^{(15)}$

where $Q^{(13)} = \cup_{i=1}^{n}(\cup_{j=1}^{(n-4)/2} Q_{j,i}^{(13)})$, where $Q_{j,i}^{(13)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_{i+j-1})\}$,

$Q^{(14)} = \cup_{i=1}^{n/2} Q_i^{(14)}$, where $Q_i^{(14)} = \{(v_i, e_{i+5}), (e_{i+5}, e_{i+2}), (e_{i+2}, v_{i+3})\}$, $Q^{(15)} = \cup_{i=1}^{n} Q_i^{(15)}$, where $Q_i^{(15)} = \{(e_{i+(n-3)}, v_i), (v_i, e_{i+(n-2)}), (e_{i+(n-2)}, e_{i+(n-1)})\}$. Here, $< Q^{(13)} > \cong n((n-4)/2) P_4$,

$< Q^{(14)} > \cong (n/2)P_4$, $< Q^{(15)} > \cong nP_4$. Therefore, $B_1(K_{1,n}^{+}) - (n+1)K_2$ is P_4 – decomposable .

Case 3. $n \equiv 2 \pmod 3$.

Then the edge set of $B_1(K_{1,n}^{+})$ can be decomposed into $(3n(n+1)/2)P_4$ and $(n+1)K_2$'s. The edge set of $(n+1)K_2$'s is given by $(\bigcup_{i=1}^{n-1} (e_{i+1}, f_i)) \cup \{(u_1, f_n), (v_1, f_n)\}$.

Subcase 3.1. $n \equiv 2 \pmod 3$ and n is odd, $n \geq 5$.

The edge set of $(3n(n+1)/2)P_4$ are given by the set $Q^{(1)} \cup Q^{(2)} \cup Q^{(3)} \cup Q^{(4)} \cup Q^{(11)} \cup Q^{(12)}$, where $Q^{(12)} = \cup_{i=1}^{n}(\cup_{j=1}^{(n-1)/2} Q_{j,i}^{(12)})$, where $Q_{j,i}^{(12)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_i)\}$,
$< Q^{(12)} > \cong n((n-1)/2) P_4$

Subcase 3.2. $n \equiv 2 \pmod 3$, $n \geq 8$ and n is even.

The edge set of P_4 are given by the set $Q^{(1)} \cup Q^{(2)} \cup Q^{(4)} \cup Q^{(11)} \cup Q^{(13)} \cup Q^{(14)} \cup Q^{(15)}$

where $Q^{(13)} = \cup_{i=1}^{n}(\cup_{j=1}^{(n-4)/2} Q_{j,i}^{(13)})$, where $Q_{j,i}^{(13)} = \{(e_{i+2j-1}, v_i), (v_i, e_{i+2j}), (e_{i+2j}, e_{i+j-1})\}$,

$Q^{(14)} = \cup_{i=1}^{n/2} Q_i^{(14)}$, where $Q_i^{(14)} = \{(v_i, e_{i+5}), (e_{i+5}, e_{i+2}), (e_{i+2}, v_{i+3})\}$and $Q^{(15)} = \cup_{i=1}^{n} Q_i^{(15)}$, where $Q_i^{(15)} = \{(e_{i+(n-3)}, v_i), (v_i, e_{i+(n-2)}), (e_{i+(n-2)}, e_{i+(n-1)})\}$, $< Q^{(13)} > \cong n((n-4)/2) P_4$, $< Q^{(14)} > \cong (n/2)P_4$,

$< Q^{(15)} > \cong nP_4$. Therefore, $B_1(K_{1,n}^{+}) - (n+1)K_2$ is P_4 – decomposable .

IV. Conclusion

In this paper, P_4-Decomposition of Boolean Function Graph B(G, L(G), NINC) of path, cycle, stars and corona graphs are obtained.

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