On Contra I *b*-Continuity in Intuitionistic Topological Spaces

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Abstract. In this paper, some new class of functions, called intuitionistic contra b-continuous, intuitionistic contra b-irresolute, intuitionistic contra b-homeomorphism are introduced and studied their properties in intuitionistic topological space.

Key Words and Phrases: Intuitionistic contra b-continuous, intuitionistic contra b- irresolute, intuitionistic contra b-homeomorphism.

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I. Introduction

In 1986, Atanassov [4] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Later in 1996, Coker [6] introduced the concept of intuitionistic set and intuitionistic points. This is a discrete form of intuitionistic fuzzy sets where all the sets are crisp sets. Furthers in 2000, Coker [8] introduced the concept of intuitionistic topological space and investigated basic properties of continuous functions and compactness. Since, homeomorphism plays an important role in topology, Gnanambal Illango and Selvanayaki [10], introduced generalized pre regular closed sets in intuitionistic topological spaces, strong and weak form of IGPR-continuity, connectedness in intuitionistic topological space and study few properties of Igpr homeomorphism and Igpr*-homeomorphism in intuitionistic topological space. D. Andrijevic[2] introduced and discussed some more properties of semi preopen set in topological space.

In this article, we introduce the concepts of intuitionistic contra *b*-continuous and intuitionistic contra *b* and contra b^*-open(closed) function and investigate some properties of them. Also intuitionistic contra *b*-homeomorphism and contra irresolute functions and some of their basic properties are investigated.

II. Preliminaries

The following definitions and results are essential to proceed further.

Definition 2.1: [6] Let X be a non empty fixed set. An intuitionistic set (briefly. IS) A is an object of the form $A=(X,A_1,A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1\cap A_2=\varnothing$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

The family of all IS 's in X will be denoted by $IS(X)$. Every crisp set A on a non-empty set X is obviously an intuitionistic set.

Definition 2.2 [6] Let X be a non-empty set, $A=(X,A_1,A_2)$ and $B=(X,B_1,B_2)$ be intuitionistic sets on X, then

1. $A\subseteq B$ if and only if $A_1\subseteq B_1$ and $B_2\subseteq A_2$.
2. $A=B$ if and only if $A\subseteq B$ and $B\subseteq A$.
3. $A\subset B$ if and only if $A_1\cup A_2\supseteq B_1\cup B_2$.
4. $\overline{A}=(X,A_2,A_1)$.
5. $A\cup B=(X,A_1\cup B_1,A_2\cap B_2)$.
6. $A\cap B=(X,A_1\cap B_1,A_2\cup B_2)$.
7. $A-B=A\cap\overline{B}$
8. $\tilde{\phi}=(X,\phi,X)$ and $\tilde{X}=(X,X,\phi)$

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Corollary 2.1 [3] Let A, B, C and A_i be IS's in X. Then

1. $A_i \subseteq B$ for each i implies that $\bigcup A_i \subseteq B$.
2. $B \subseteq A_i$ for each i implies that $B \subseteq \bigcap A_i$.
3. $\overline{\bigcup A_i} = \bigcap \overline{A_i}$ and $\overline{\bigcap A_i} = \bigcup \overline{A_i}$.
4. $A \subseteq B \Leftarrow \overline{B} \subseteq \overline{A}$.
5. $\overline{(\overline{A})} = A, \overline{\tilde{\phi}} = \tilde{X}$ and $\overline{\tilde{X}} = \tilde{\phi}$

Definition 2.3 [8] An intuitionistic topology (briefly IT) on a non-empty set X is a family τ of IS's in X satisfying the following axioms

1. $\tilde{\phi}, \tilde{X} \in \tau$.
2. $A \cap B \in \tau$ for any $A, B \in \tau$.
3. $\bigcup A_i \in \tau$ for an arbitrary family in τ.

In this case the pair (X, τ) is called intuitionistic topological space (briefly ITS) and the IS's in τ are called the intuitionistic open set in X denoted by $I^{(\tau)}O$ and the complement of an $I^{(\tau)}O$ is called Intuitionistic closed set in X denoted by $I^{(\tau)}C$. The family of all $I^{(\tau)}O$ (resp. $I^{(\tau)}C$) sets in X will be denoted by $I^{(\tau)}O(X)$ (resp. $I^{(\tau)}C(X)$.)

Definition 2.4 [7]Let (X, τ) be an ITS and $A \in IS(X)$. Then the intuitionistic interior (resp. intuitionistic closure) of A are defined by $int(A) = \bigcup\{K : K \in I^{(\tau)}O(X) \text{ and } K \subseteq A\}$ (resp. $cl(A) = \bigcap\{K : K \in I^{(\tau)}C(X) \text{ and } A \subseteq K\}$.)

In this study we use $I^{(\tau)}i(A)$ (resp. $I^{(\tau)}c(A)$) instead of $int(A)$ (resp. $cl(A)$).

Definition 2.5 [8] Let (X, τ) be an ITS and an IS A in X is said to be

1. intuitionistic regular-open [5] (briefly $I^{(\tau)}RO$) if $A = I^{(\tau)}i(I^{(\tau)}c(A))$ and intuitionistic $regular$-closed (briefly $I^{(\tau)}RC$) if $I^{(\tau)}c(I^{(\tau)}i(A)) = A$.

2. intuitionistic pre-open [5] (briefly $I^{(\tau)}PO$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A))$ and intuitionistic pre-closed (briefly $I^{(\tau)}PC$) if $I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$.

3. intuitionistic semi-open [5] (briefly $I^{(\tau)}SO$) if $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A))$ and intuitionistic semi-closed (briefly $I^{(\tau)}SC$) if $I^{(\tau)}i(I^{(\tau)}c(A)) \subseteq A$.

4. intuitionistic α-open [7] (briefly $I^{(\tau)}\alpha O$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A)))$ and intuitionistic α-closed (briefly $I^{(\tau)}\alpha C$) if $I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}c(A))) \subseteq A$.

5. intuitionistic β-open [7] (briefly $I^{(\tau)}\beta O$) if $A \subseteq I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}c(A)))$ and intuitionistic β-closed (briefly $I^{(\tau)}\beta C$) if $I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A))) \subseteq A$.

6. intuitionistic b-open [8] (briefly $I^{(\tau)}bO$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A))$ and intuitionistic b-closed (briefly $I^{(\tau)}bC$) if $I^{(\tau)}i(I^{(\tau)}c(A)) \cap I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$.

The family of all $I^{(\tau)}RO$ (resp. $I^{(\tau)}RC$, $I^{(\tau)}PO$, $I^{(\tau)}PC$, $I^{(\tau)}SO$, $I^{(\tau)}SC$, $I^{(\tau)}\alpha O$, $I^{(\tau)}\alpha C$, $I^{(\tau)}\beta O$, $I^{(\tau)}\beta C$, $I^{(\tau)}bO$ and $I^{(\tau)}bC$) sets in X will be denoted by $I^{(\tau)}RO(X)$ (resp. $I^{(\tau)}RC(X)$, $I^{(\tau)}PO(X)$, $I^{(\tau)}PC(X)$, $I^{(\tau)}SO(X)$, $I^{(\tau)}SC(X)$, $I^{(\tau)}\alpha O(X)$, $I^{(\tau)}\alpha C(X)$, $I^{(\tau)}\beta O(X)$, $I^{(\tau)}\beta C(X)$, $I^{(\tau)}bO(X)$ and $I^{(\tau)}bC(X)$.)

Definition 2.6 [8, 11, 13] Let (X, τ) be an ITS and A be an $IS(X)$, then

1. intuitionistic regular-interior (resp. intuitionistic pre- interior, intuitionistic semi-interior, intuitionistic α-interior and intuitionistic β-interior) of A is the union of all $I^{(\tau)}RO(X)$ (resp. $I^{(\tau)}PO(X)$, $I^{(\tau)}SO(X)$, $I^{(\tau)}\alpha O(X)$ and $I^{(\tau)}\beta O(X)$) contained in A, and is denoted by $I^{(\tau)}Ri(A)$ (resp. $I^{(\tau)}Pi(A)$, $I^{(\tau)}Si(A)$, $I^{(\tau)}\alpha i(A)$ and $I^{(\tau)}\beta i(A)$.)

i.e.

$$I^{(\tau)}Ri(A) = \bigcup\{G : G \in I^{(\tau)}RO(X) \text{ and } G \subseteq A\},$$

$$I^{(\tau)}Pi(A) = \bigcup\{G : G \in I^{(\tau)}PO(X) \text{ and } G \subseteq A\},$$

$$I^{(\tau)}Si(A) = \bigcup\{G : G \in I^{(\tau)}SO(X) \text{ and } G \subseteq A\},$$

$$I^{(\tau)}\alpha i(A) = \bigcup\{G : G \in I^{(\tau)}\alpha O(X) \text{ and } G \subseteq A\},$$

$$I^{(\tau)}\beta i(A) = \bigcup\{G : G \in I^{(\tau)}\beta O(X) \text{ and } G \subseteq A\}.$$

2. intuitionistic regular-closure (resp. intuitionistic pre-closure, intuitionistic semi-closure, intuitionistic α-closure, intuitionistic β-closure) of A is the intersection of all $I^{(\tau)}RC(X)$ (resp. $I^{(\tau)}PC(X)$, $I^{(\tau)}SC(X)$, $I^{(\tau)}\alpha C(X)$, $I^{(\tau)}\beta C(X)$) containing A, and is denoted by $I^{(\tau)}Rc(A)$ (resp. $I^{(\tau)}Pc(A)$, $I^{(\tau)}Sc(A)$,

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$I^{(\tau)}\alpha c(A)$, $I^{(\tau)}\beta c(A)$.)

i.e.

$$I^{(\tau)}Rc(A) = \bigcap\{G : G \in I^{(\tau)}RC(X) \text{ and } G \supseteq A\},$$

$$I^{(\tau)}Pc(A) = \bigcap\{G : G \in I^{(\tau)}PC(X) \text{ and } G \supseteq A\}$$

$$I^{(\tau)}Sc(A) = \bigcap\{G : G \in I^{(\tau)}SC(X) \text{ and } G \supseteq A\},$$

$$I^{(\tau)}\alpha c(A) = \bigcap\{G : G \in I^{(\tau)}\alpha C(X) \text{ and } G \supseteq A\},$$

$$I^{(\tau)}\beta c(A) = \bigcap\{G : G \in I^{(\tau)}\beta C(X) \text{ and } G \supseteq A\}.$$

Definition 2.7[8] Let X be a non empty set and pϵ X. Then the IS $\tilde{p}$ defined by $\tilde{p}$ = (X, {p}, $\{p\}^c$) is called an intuitionistic point (IP for short) in X. The intuitionistic point $\tilde{p}$ is s aid to be contained in A = (X, A_1, A_2) (i.e $\tilde{p}$ ϵ A) if and only if $\tilde{p}$ ϵ A_1.

Definition 2.8 [8] Let $f: (X, \tau) \to (Y, \sigma)$ be a function. If A = (X, A_1, A_2) is an intuitionistic set in X, then the image of A under f, denoted by f(A), is an intuitionistic set in Y defined by f(A) = (Y, f(A_1), f_(A_2)), where f_(A_2) = $(f(A_2)^c)^c$.

Definition 2.9 [8] Let $f: (X, \tau) \to (Y, \sigma)$ be a function. If A = (Y, A_1, A_2) is an intuitionistic set in Y, then the preimage of A under f, denoted by f^{-1} (A), is an intuitionistic set in X defined by f^{-1} (A) = (X, f^{-1} (A_1), f^{-1} (A_2)).

Definition 2.10[6, 8] Let $A, A_i (i \in J)$ be IS's in X, $B, B_j (j \in K)$ IS's in Y and $f: (X, \tau) \to (Y, \sigma)$ be a function. Then

1. $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
2. $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
3. $A \subseteq f^{-1}(f(A))$ and if f is one to one, then $A = f^{-1}(f(A))$
4. $f(f^{-1}(B)) \subseteq B$ and if f is onto, then $f(f^{-1}(B)) = B$
5. $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
6. $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
7. $f(\cup A_i) = \cup f(A_i)$
8. $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is one to one, then $f(\cap A_i) = \cap f(A_i)$
9. $f^{-1}(\tilde{Y}) = \tilde{X}$
10. $f^{-1}(\tilde{\phi}) = \tilde{\phi}$
11. $f(\tilde{X}) = \tilde{Y}$ if f is onto
12. $f(\tilde{\phi}) = \tilde{\phi}$
13. If f is onto, then $\overline{f(A)} \subseteq f(\bar{A})$: and if furthermore, f is $1-1$, we have $\overline{f(A)} \subseteq f(\bar{A})$
14. $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$
15. $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$

Definition 2.11[8] Let (X, τ) and (Y, δ) be two intuitionistic topological spaces and $f: (X, \tau) \to (Y, \delta)$ be a function. Then f is said to be intuitionistic continuous if and only if the preimage of every intuitionistic openset in Y is intuitionistic open in X.

Definition 2.12[6] A map $f: (X, \tau) \to (X, \sigma)$ is called intuitionistic open(closed) if the image $f(A)$ is intuitionistic open(closed) in Y for every intuitionistic open(closed) set in X.

III. CONTRA Ib-CONTINUOUS FUNCTIONS

Definition 3.1 Let (X, τ) and (Y, δ) be two ITS and $f: (X, \tau) \to (Y, \delta)$ be a function. Then f is said to be intuitionistic b-continuous (In short. Ib-continuous) if and only if the preimage of every intuitionistic open set in Y is intuitionistic b-open in X.

Definition 3.2 Let (X, τ) and (Y, δ) be two ITS and $f: (X, \tau) \to (Y, \delta)$ be a function. Then f is said to be intuitionistic contra b-continuous (In short.contra Ib-continuous) if only if the preimage of every intuitionistic open set in Y is intuitionistic b-closed in X.

Example3.1 Let$X = \{a, b, c\} = Y$,

$$\sigma = \left\{ \tilde{\phi}, \tilde{X}, \phi, \left(X, \phi, \{b, c\}\right)\right\}, \text{ and}$$

$$\tau = \{ \tilde{\phi}, \tilde{Y}, \phi, (Y, \phi, \{a\}), (Y, \{b\}, \{c\}), (Y, \{b\}, \phi), (Y, \phi, \{c\}), (Y, \phi, \{a, c\})\}$$

and $f: (X, \tau) \to (Y, \sigma)$ be a function such that $f(a) = c$, $f(b) = a$, $f(c) = b$. Then the function f is contra Ib-continuous

Theorem 3.1 Let $f: (X, \tau) \to (Y, \sigma)$ be intuitionistic contra continuous, then f is contra Ib-continuous.

Proof. Let $A = (X, A_1, A_2)$ be an $I^{(\sigma)}OS$ of Y. Since f is intuitionistic contra continuous, then $f^{-1}(A)$ is intuitionistic closed in X, we know that every intuitionistic closed set is Ib-closed set, then $f^{-1}(A)$ is $I^{(\tau)}b$-closed in X. Thus f is contra Ib-continuous.

Theorem 3.2 Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping, where X and Y are ITS, then the following are equivalent.

i. The function f is contra Ib-continuous.
ii. The inverse image of intuitionistic closed set of Y is $I^{(\tau)}b$-open set in X.
iii. $f(I^{(\tau)}bint(A)) \subseteq I^{(\sigma)}cl(f(A))$ for intuitionistic set A of X.
iv. $I^{(\tau)}bint(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}cl(B))$ for each intuitionistic set of Y.

Proof. (i) $\Rightarrow$ (ii): Let A be intuitionistic closed subset of Y, then $Y - A$ is intuitionistic open in Y. Since f is Ib-continuous, $f^{-1}(Y - A) = X - f^{-1}(A)$, is $I^{(\tau)}b$-open in X, which implies that $f^{-1}(A)$ is $I^{(\tau)}b$-closed in X.

(ii) $\Rightarrow$ (iii): Let A be an intuitionistic open set of X. The $I^{(\sigma)}cl(f(A))$ is intuitionistic closed in Y. By (ii) $f^{-1}(I^{(\sigma)}cl(f(A)))$ is $I^{(\tau)}b$-open in X and

$$f^{-1}(I^{(\sigma)}cl(f(A))) = I^{(\tau)}bint(f^{-1}(I^{(\sigma)}cl(f(A))))$$

since $A \subseteq f^{-1}(f(A))$

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we have $I^{(\tau)}bint(A) \subseteq I^{(\tau)}bint(f^{-1}(f(A)))$

$\subseteq I^{(\tau)}bint(f^{-1}(I^{(\sigma)}cl(f(A))))$

$= f^{-1}(I^{(\sigma)}cl(f(A)))$

$f(I^{(\tau)}bint(A)) \subseteq I^{(\sigma)}cl(f(A))$.

(iii) ⇒ (iv): Let B be an intuitionistic open set of Y. Then by (iii) we have

$f(I^{(\tau)}bint(f^{-1}(B))) \subseteq I^{(\sigma)}cl(f(f^{-1}(B)))$.

Hence $I^{(\tau)}bint(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}cl(f(f^{-1}(B)))) \subseteq f^{-1}(I^{(\sigma)}cl(B))$

$I^{(\tau)}bint(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}cl(B))$.

(iv) ⇒ (i): Let B be an intuitionistic open set of Y. Then $B^c = C$ is intuitionistic closed subset in Y so that $I^{(\sigma)}cl(C) = C$. Now by(iv)

$I^{(\tau)}bint(f^{-1}(C)) \subseteq f^{-1}(I^{(\sigma)}cl(C))$

$= f^{-1}(C)$ ie., C is intuitionistic closed

we have $f^{-1}(C) \supseteq I^{(\tau)}bint(f^{-1}(C))$

$= (I^{(\tau)}bint(f^{-1}(C^c)))^c$.

Hence $(f^{-1}(C))^c$ is $I^{(\tau)}b$-open in X. That is $f^{-1}(C)$ is $I^{(\tau)}b$-closed. Therefore f is contra Ib-continuous.

Theorem 3.3 Let $f:(X,\tau) \to (Y,\sigma)$ be a mapping, where X and Y are intuitionistic topological spaces, then the followings are equivalent.

i. The function f is contra Ib-continuous.
ii. For each subset A of Y, $f^{-1}(I^{(\sigma)}int(A)) \subseteq I^{(\tau)}bcl(f^{-1}(A))$.

Proof. (i) ⇒ (ii): Let $A = (X, A_1, A_2)$ be any intuitionistic set of Y, $I^{(\sigma)}int(A)$ is open set in Y and $f^{-1}(I^{(\sigma)}int(A))$ is a $I^{(\tau)}b$-closed set in X. Since f is contra Ib-continuous. As $f^{-1}(I^{(\sigma)}int(A)) \subseteq f^{-1}(A)$

and $f^{-1}(I^{(\sigma)}int(A)) \subseteq I^{(\tau)}bcl(f^{-1}(A))$.

(ii) ⇒ (i): Let A be any intuitionistic open set of Y, so that $I^{(\sigma)}int(A) = A$. By condition $f^{-1}(I^{(\sigma)}int(A)) \subseteq I^{(\tau)}bcl(f^{-1}(A))$

⇒ $f^{-1}(A) = f^{-1}(I^{(\sigma)}int(A))$

$\subseteq I^{(\tau)}bcl(f^{-1}(A))$

⇒ $f^{-1}(A) \subseteq I^{(\tau)}bcl(f^{-1}(A))$.

Hence $f^{-1}(A)$ is $I^{(\tau)}b$-closed, where A is intuitionistic open in Y. Therefore f is contra Ib-continuous.

Theorem 3.4 Let $f:(X,\tau) \to (Y,\sigma)$ be a single valued function, where X and Y are intuitionistic topological spaces, and then the followings are equivalent.

i. The function f is contra Ib-continuous.
ii. For each element $p \in X$ and each intuitionistic open set V in Y with $f(\tilde{p}) \in V$, there is a Ib-closed set U in X, such that $\tilde{p} \in U$, $f(U) \subseteq V$.

Proof. (i)⇒(ii): Assume $f:(X,\tau) \to (Y,\sigma)$ is a single valued contra Ib-continuous function. Let $f(\tilde{p}) \in V$ and $V \subseteq Y$ an intuitionistic open set, then $\tilde{p} \in f^{-1}(V) \in I^{(\tau)}b$-closed set X. Since f is contra Ib-continuous, let $U = f^{-1}(V)$, then $\tilde{p} \in U$ and $f(U) \subseteq V$.

(ii)(i): Let V be an intuitionistic open set in Y and $\tilde{p} \in f^{-1}(V)$, then $f(\tilde{p}) \in V$, there exists a $U_{\tilde{p}} \in I^{(\tau)}b$-closed set of X, such that $\tilde{p} \in U_{\tilde{p}}$ and $f(U_{\tilde{p}}) \subseteq V$. Then $\tilde{p} \in U_{\tilde{p}} \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup U_{\tilde{p}}$. Since every intuitionistic contra continuous function is contra Ib-continuous function. Therefore, $f^{-1}(V)$ is $I^{(\tau)}b$-closed set in X. Therefore f is contra Ib-continuous function.

IV. Contra Ib-Homeomorphisms and Contra Ib-Irresolute Functions

Definition 4.1 A bijection $f:(X,\tau) \to (Y,\sigma)$ is called contra Ib-homeomorphism if both f and , f^{-1} are contra Ib-continuous.

Example4.1 Let $X = \{a, b, c\} = Y$,

$\sigma = \{\tilde{\phi}, \tilde{X}, \phi, (X, \phi, \{b, c\})\}$,

$\tau = \{\tilde{\phi}, \tilde{Y}, \phi, (Y, \phi, \{a\}), (Y, \{b\}, \{c\}), (Y, \{b\}, \phi), (Y, \phi, \{c\}), (Y, \phi, \{a, c\})\}$

and $f:(X,\tau) \to (Y,\sigma)$ be a function such that $f(a) = c$, $f(b) = a$, $f(c) = b$. Then the function f is contra Ib-homeomorphism.

Theorem 4.2 For a bijective contra Ib-continuous map $f:(X,\tau) \to (Y,\sigma)$, the following are equivalent,

a. f is contra Ib-open
b. f is contra Ib-homeomorphism
c. f is contra Ib-closed.

Proof. (a) ⇒ (b): Since f is intuitionistic bijective, contra Ib-continuous and contra Ib-open map, by definition, f is contra Ib-homeomorphism.

(b) ⇒ (c): Let f be contra Ib-homeomorphism. Then f is contra Ib-open. By ? f is contra Ib-closed.

(c) ⇒ (a): Let (X,τ) and (Y,σ) be any two intuitionistic topological spaces and $f:(X,\tau) \to (Y,\sigma)$ be a function. Then f is said to be Ib-irresolute if the pre-image of every $I^{(\sigma)}b$-closed set of Y is $I^{(\tau)}b$-closed in X.

Definition 4.2 Let (X,τ) and (Y,σ) be any two intuitionistic topological spaces and $f:(X,\tau) \to (Y,\sigma)$ be a function. Then f is said to be contra Ib-irresolute if the pre-image of every $I^{(\sigma)}b$-closed set of Y is $I^{(\tau)}b$-open in X.

Theorem 4.2 Let $f:(X,\tau) \to (Y,\sigma)$ be an intuitionistic contra b-continuous and intuitionistic contra b-open . Then f is contra Ib-irresolute functions.

Proof. Let $A = (X, A_1, A_2)$ be any $I^{(\sigma)}b$-open set. Then $A \subseteq I^{(\sigma)}int(I^{(\sigma)}cl(A)) \cup I^{(\sigma)}cl(I^{(\sigma)}int(A))$, since f is intuitionistic contra b-continuous and intuitionistic contra b-open it follows that

$f^{-1}(A) \subseteq f^{-1}(I^{(\tau)}int(I^{(\tau)}cl(A)) \cap I^{(\tau)}cl(I^{(\tau)}int(A)))$

$\subseteq I^{(\tau)}int(I^{(\tau)}cl(f^{-1}(A))) \cap I^{(\tau)}cl(I^{(\tau)}int(f^{-1}(A)))$

⇒ $f^{-1}(A) \subseteq I^{(\tau)}int(I^{(\tau)}cl(f^{-1}(A))) \cap I^{(\tau)}cl(I^{(\tau)}int(f^{-1}(A)))$.

Therefore $f^{-1}(A)$ is

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$I^{(\tau)}int(I^{(\tau)}clf^{-1}(A)) \cap I^{(\tau)}cl(I^{(\tau)}int(f^{-1}(A)))b$-closed. This shows that f is contra Ib-irresolute functions.

Theorem 4.2 Let $f:(X,\tau)\to(Y,\sigma)$ be a contra Ib-irresolute $\leftrightarrow$ for all intuitionistic set A of Y, $I^{(\tau)}bint(f^{-1}(A)) \subseteq f^{-1}(I^{(\sigma)}bcl(A))$.

Proof. Let f is contra Ib-irresolute function, now $I^{(\sigma)}bcl(A)$ is an $I^{(\sigma)}b$-closed set. Since $f^{-1}(A) \subset f^{-1}(I^{(\sigma)}bcl(A))$, it follows from the definition $I^{(\tau)}bint(f^{-1}(A)) \subseteq f^{-1}(I^{(\sigma)}bcl(A))$. Conversely suppose that $A=(X,A_1,A_2)$ is $I^{(\sigma)}b$-closed set in Y, then $I^{(\sigma)}bcl(A)=A$. Now by hypothesis

$$I^{(\tau)}bint(f^{-1}(A)) \subseteq f^{-1}(I^{(\sigma)}bcl(A)) = f^{-1}(A)$$
$$\Rightarrow I^{(\hat{a}u)}bint(f^{-1}(A)) = f^{-1}(A).$$

Thus $f^{-1}(A)$ is $I^{(\tau)}b$-open set and so f is an contra Ib-irresolute functions.

Theorem 4.3 Let $f:(X,\tau)\to(Y,\sigma)$ be a contra Ib-irresolute $\leftrightarrow$ for all intuitionistic set A of Y, $f(I^{(\tau)}bint(A)) \subseteq I^{(\sigma)}bcl(f(A))$.

Proof. Let f is contra Ib-irresolute function, now $f(I^{(\tau)}bint(A))$ is an $I^{(\sigma)}b$-closed set. By hypothesis $f^{-1}(I^{(\sigma)}bcl(A))$ is an contra $I^{(\tau)}b$-closed $A \subseteq f^{-1}(I^{(\sigma)}bcl(f(A)))$. That is $f(I^{(\sigma)}bcl(A)) \subseteq I^{(\sigma)}bcl(f(A))$.

Conversely suppose that $A=(X,A_1,A_2)$ is contra Ib-closed set in Y. Now by hypothesis $f(I^{(\tau)}bcl(f^{-1}(A))) \subseteq I^{(\sigma)}bcl(f(f^{-1}(A))) = A$.

This implies that $I^{(\tau)}bcl(f^{-1}(A)) \subseteq f^{-1}(A)$

and so, $f^{-1}(A) = I^{(\tau)}bcl(f^{-1}(A))$.

That is $f^{-1}(A)$ is an contra $I^{(\tau)}b$-closed set and so f is an contra Ib-irresolute functions.

Theorem 4.4 Let $f:(X,\tau)\to(Y,\sigma)$ be a contra Ib-irresolute $\leftrightarrow$ for all intuitionistic set A of Y, $f(I^{(\tau)}bint(A)) \subseteq I^{(\sigma)}bcl(f(A))$.

Proof. Let f is contra Ib-irresolute function, now $f(I^{(\tau)}bint(A))$ is an $I^{(\sigma)}b$-closed set. By hypothesis $f^{-1}(I^{(\sigma)}bcl(A))$ is an contra $I^{(\tau)}b$-closed $A \subseteq f^{-1}(I^{(\sigma)}bcl(f(A)))$. That is $f(I^{(\sigma)}bcl(A)) \subseteq I^{(\sigma)}bcl(f(A))$.

Conversely suppose that $A=(X,A_1,A_2)$ is contra Ib-closed set in Y. Now by hypothesis $f(I^{(\tau)}bcl(f^{-1}(A))) \subseteq I^{(\sigma)}bcl(f(f^{-1}(A))) = A$.

This implies that $I^{(\tau)}bcl(f^{-1}(A)) \subseteq f^{-1}(A)$

and so, $f^{-1}(A) = I^{(\tau)}bcl(f^{-1}(A))$.

That is $f^{-1}(A)$ is an contra $I^{(\tau)}b$-closed set and so f is an contra Ib-irresolute functions.

Proposition 4.1 Suppose $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\omega)$ are both contra Ib-irresolute, then $gof:(X,\tau)\to(Z,\omega)$ is contra Ib-irresolute function.

Proof. Let $E=(X,E_1,E_2)$ be an $I^{(\omega)}b$-open in (Z,ω), since g is contra Ib-irresolute, $g^{-1}(E)$ is an $I^{(\sigma)}b$-closed in (Y,σ). Since f is also contra Ib-irresolute $f^{-1}(g^{-1}(E)) = (gof)^{-1}(E)$ is an $I^{(\tau)}b$-open in (X,τ). Thus (gof) is contra Ib-irresolute functions.

Theorem 4.6 Suppose $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic contra Ib-continuous and intuitionistic contra Ib-open, then $f^{-1}(I^{(\sigma)}bint(A)) \subseteq I^{(\tau)}bcl(f^{-1}(A))$.

If $f:(X,\tau)\to(Y,\sigma)$ is an Ib-continuous function and $f^{-1}(I^{(\sigma)}int(B)) \subseteq (f^{-1}(B))^{-1}$ for each B belongs to $I^{(\sigma)}bO(Y)$, then f is contra Ib-irresolute function.

Proof. Let B belongs to $I^{(\sigma)}bO(Y)$,

$$f^{-1}(B) \subseteq f^{-1}(I^{(\sigma)}cl(I^{(\sigma)}int(B)) \cap I^{(\sigma)}int(I^{(\sigma)}cl(B)))$$
$$\subseteq (I^{(\sigma)}cl(f^{-1}(I^{(\sigma)}int(B)) \cap I^{(\sigma)}int(f^{-1}(I^{(\sigma)}cl(B)))$$
$$\subseteq I^{(\sigma)}cl(I^{(\sigma)}int(f^{-1}(B)) \cap I^{(\sigma)}int(I^{(\sigma)}cl(f^{-1}(B)))))$$
$$\Rightarrow f^{-1}(B) \subseteq I^{(\sigma)}cl(I^{(\sigma)}int(f^{-1}(B)) \cap I^{(\sigma)}int(I^{(\sigma)}cl(f^{-1}(B))))\ .$$

Hence the result.

Proposition 4.2 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be two functions such that gof is contra Ib^*-open. Then the following two statements hold.

i. If f is contra Ib-irresolute surjection, then g is contra Ib^*-open function.
ii. If g is contra Ib-irresolute injection, then f is contra Ib^*-open function.

Proof. (i) Let A be any $I^{(\sigma)}b$-open set in Y, since f is contra Ib-irresolute function, $f^{-1}(A)$ is an $I^{(\tau)}b$-closed set in X. As gof is contra Ib^*-open function and f is surjective, $(gof)^{-1}(f^{-1}(A)) = g(A)$, which is an $I^{(\mu)}b$-closed set in Z. This implies g is contra Ib^*-open function.

(ii). Obvious.

Proposition 4.3 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be two functions such that gof is contra Ib^*-closed function. Then the following two statements are hold.

i. If g is contra Ib-irresolute injection, then f is contra Ib^*-closed function.
ii. If f is contra Ib-irresolute surjection, then g is contra Ib^*-closed function.

Theorem 4.7 For a bijective map $f:(X,\tau)\to(Y,\sigma)$, the following statements are equivalent,

a. $f^{-1}:(X,\tau)\to(Y,\sigma)$ is contra Ib-irresolute
b. f is contra Ib^*-open
c. f is contra Ib^*-closed.

Proof. (a) $\Rightarrow$ (b): Let $A=(X,A_1,A_2)$ be $I^{(\tau)}b$-open in (X,τ). By (a), $(f^{-1})^{-1}(A) = f(A)$ is $I^{(\sigma)}b$-closed in (Y,σ). So, f is contra Ib^*-open map.

(b) $\Rightarrow$ (c): Let A be $I^{(\tau)}b$-open in (X,τ). Then $X-A=(X,A_2,A_1)$ is $I^{(\tau)}b$-closed in X and by (b),

$$f(X-A) = f((X,A_2,A_1)) = (Y,f(A_2),f(A_1)) = (Y,f(A_2),Y-f(X-A_1))$$

is $I^{(\sigma)}b$-closed in Y.

So $(Y,Y-f(X-A_1),f(A_2))$ is $I^{(\sigma)}b$-open in Y.

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Since $Y - f(X - A_1) = A_1$,

$$(Y, Y - f(X - A_1), f(A_2)) = (Y, f(A_1), f(A_2))$$'

$f(A)$ is $I^{(\sigma)}b$-closed in (Y, σ) and so, f is an contra Ib^*-closed map.

(c) ⇒ (a): Let A be contra $I^{(\tau)}b$-closed in (X, τ). By (c), $f(A)$ is contra $I^{(\sigma)}b$-closed in (Y, σ). But $f(A) = (f^{-1})^{-1}(A)$. Therefore, f^{-1} is contra Ib-irresolute.

Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \tilde{\phi}, \tilde{X}, (X, \{b\}, \{c\}), (X, \{b\}, \phi), (X, \phi, \{c\}), (X, \phi, \{a, c\})\}$. $\sigma = \{\phi, \tilde{\phi}, \tilde{X}, (Y, \phi, \{b, c\})\}$. Define $f: (X, \tau) \to (Y, a)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$ then f and f^{-1} are contra Ib-irresolute. So f is contra Ib-homeomorphism.

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