Superior Eccentric Domination in Some Triangular Snake Graphs

M. Bhanumathi, Principal, Government Arts College for Women, Sivagangai-630562,Tamil Nadu, India. ***bhanu_ksp@yahoo.com***

R.Meenal Abirami, Research Scholar, Government Arts College for Women(Autonomous), Pudukkottai-622001, Tamil Nadu, India. meegeeabirami@gmail.com

ABSTRACT - In 2017 we define superior eccentric domination in graphs. A superior dominating set S of vertices of G is called a superior eccentric dominating set if every vertex of V (G) – S has some superior eccentric vertex in S. A superior eccentric dominating set of G of minimum cardinality is a minimum superior eccentric dominating set and its cardinality is called the superior eccentric domination number and is denoted by γ_{sed}(G). In this paper we initiate the study of superior eccentric dominating sets in triangular snake graphs, alternate triangular snake graphs, double triangular snake graphs, double alternate triangular snake graphs.

Keywords **- Domination, eccentricity, Superior distance, Superior eccentric vertex, superior dominating set, superior eccentric dominating set.**

Mathematics subject classification(2010) : **05C12, 05C69.**

I. INTRODUCTION

Let G be a finite, simple, undirected (a, b) graph with vertex set V(G) and edge set E(G), |V(G)| = a, |E(G)| = b . For graph theoretic terminology refer Harary [3], Buckley and Harary [1]. In 2010, Janakiraman, Bhanumathi and Muthammai defined eccentric domination in graphs [5]. K. M Kathiresan and G.Marimuthu introduced the superior domination in graphs and superior distance in graphs [5,6].

Definition 1.1: A set D ⊆ V is said to be a dominating set in G, if every vertex in V– D is adjacent to some vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by γ(G) . For two vertices u and v in a graph G, the distance from u to v is denoted by d(u, v) and defined as the length of a shortest u–v path in graph G. Let G be a connected graph and v be a vertex of G. The eccentricity e(v) of v is the distance to a vertex farthest from v. Thus, e(v) = max{d(u, v) : u∈V}. A set D ⊆ V(G) is an eccentric dominating set if D is a dominating set of G and for every v∈ V–D, there exist at least one eccentric vertex of v in D. The minimum cardinality of an eccentric dominating set is called the eccentric domination number and is denoted by γ_{ed}(G).

Definition 1.2[7]: For distinct vertices u and v of a non-trivial connected graph G, let $D_{u,v}$ = N(u) ∪ N(v). Define a $D_{u,v}$ – walk as a u–v walk in G that contains every vertex of $D_{u,v}$.

Definition 1.3[7]: The superior distance d_D(u, v) from u to v is the length of a shortest $D_{u,v}$ walk. For each vertex u ∈ V(G), define d_D(u) = min{d_D(u, v) : v∈ V(G) – {u}}. A vertex v (≠u) is called a superior neighbor of u if d_D(u, v) = d_D(u).

Definition 1.4[6]: A vertex u is said to superior dominate a vertex v if v is a superior neighbor of u. A set S of vertices of G is called a superior dominating set if every vertex of V(G) – S is superior dominated by some vertex in S.

Definition 1.5[6]: A superior dominating set of G of minimum cardinality is a minimum superior dominating set and its cardinality is called the superior domination number of G and is denoted by γ_{sd}(G).

Definition 1.6[7]: The superior eccentricity of v is e_D(v) = max{d_D(u,v) : u ∈ V(G)}. A vertex v of a graph G is said to be a superior eccentric vertex of a vertex u if d_D(u, v) = e_D(u). A vertex u is superior eccentric vertex of G if it is a superior eccentric vertex of some vertex v.

Definition 1.7[2]: A superior dominating set S of vertices of G is called a superior eccentric dominating set if every vertex of V(G) – S has some superior eccentric vertex in S.

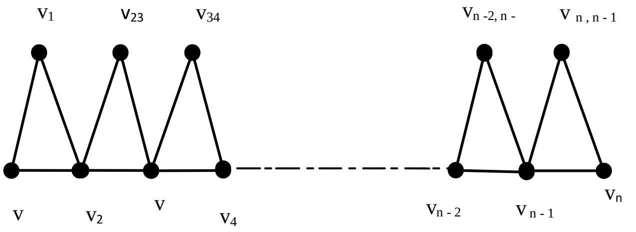
SUPERIOR ECCENTRIC DOMINATION IN SOME GRAPHS:

In this section, we find the superior eccentric domination number of some triangular snake graphs.

II. TRIANGULAR SNAKE GRAPH

The Triangular Snake Graph T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. The minimum cardinality of a superior eccentric domination in triangular snake graph is $\gamma_{sed}(T_n)$.

 DOI : 10.18231/2454-9150.2018.0853



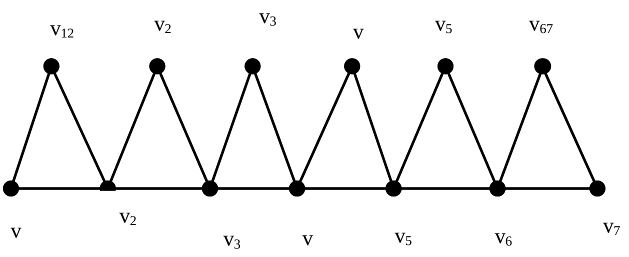
Let $v_1, v_2, \dots, v_n$ be the vertices of the underlying path in a triangular snake graph and let $v_{i, i+1}$, i = 1, 2, 3, … , n -1 be the vertex which is joined to v_i and v_{i+1} in the triangular snake graph. Then

Superior distance $d_D(v_1, v_{12}) = 2$,
Superior distance $d_D(v_{12}, v_1) = 2$,
Superior distance $d_D(v_1, v_2) = 5$,
Superior distance $d_D(v_1, v_3) = 7$,
Superior distance $d_D(v_1, v_4) = 8$,
Superior distance $d_D(v_i, v_{i+1}) = 8$ for $1 < i < n-1$,
Superior distance $d_D(v_i, v_{i, i+1}) = d_D(v_i, v_{i+1, i}) = 5$,
Superior distance $d_D(v_{i, i+1}, v_{i+1, i+2}) = 4$
Superior distance $d_D(v_1, v_n) = 2 + d(v_2, v_{n-1}) + 2 = 2 + (n-3) + 2 = n+1$,
Superior distance $d_D(v_{12}, v_n) = n+1$,
Superior distance $d_D(v_1, v_{n-1, n}) = n+1$,
Superior distance $d_D(v_{12}, v_{n-1, n}) = n+1$,
Superior distance $d_D(v_1, v_{n-1}) = 2 + d(v_2, v_{n-2}) + 5 = 2 + (n-4) + 5 = n+3$,
Superior distance $d_D(v_{12}, v_{n-1}) = n+3$,
Superior distance $d_D(v_i, v_j) = 3+2+d(v_{i+1}, v_{j-1}) + 5 = 10 + (j-i-2) = 8 + (j-i)$ for $i > 2$, $j < n-1$,
Superior distance $d_D(v_i, v_n) = 5 + d(v_{i+1}, v_{n-1}) + 2 = 7 + (n-1-i-1) = 5 + (n-i)$,
Superior distance $d_D(v_i, v_{i, i+1}) = d_D(v_i, v_{i+1, i}) = 5$,
Superior distance $d_D(v_{i, i+1}, v_{i+1, i+2}) = 4$.

Let $S = \{v_1, v_2, v_{3,4}, v_{6,7}, \dots\dots\dots\dots, v_{n-3, n-2}, v_{n-1}, v_n\}$. S is the superior eccentric dominating set of the triangular snake graph. S is also minimum with this property and $|S| = \lceil \frac{n+9}{3} \rceil$. If n is a multiple of 3, $\gamma_{sed}(T_n) = \frac{n+9}{3}$. If n is not a multiple of 3, $\gamma_{sed}(T_n) = \lceil \frac{n+9}{3} \rceil$.

Example:

If n = 7



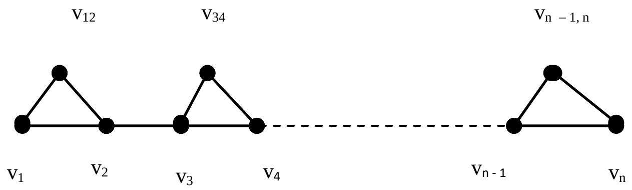
Let $S = \{v_1, v_2, v_{3,4}, v_{5,6}, v_6, v_7\}$, $V-S = \{v_3, v_4, v_{12}, v_{23}, v_{45}, v_5\}$. S is a minimum superior eccentric dominating set of the triangular snake graph. Therefore, $\gamma_{sed}(T_n) = \lceil \frac{n+9}{3} \rceil$.

Alternate Triangular Snake Graph:

An Alternate Triangular snake $A(T_n)$ is obtained from a path $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} (alternatively) to new vertex v_{i+1}. That is every alternate edge of a path is replaced by C_3. The minimum cardinality of a superior eccentric domination in alternate triangular snake graph is $\gamma_{sed}(AT_n)$.

When n is even, two types is alternate triangular snake graphs arise.

(i) n is even (type 1)



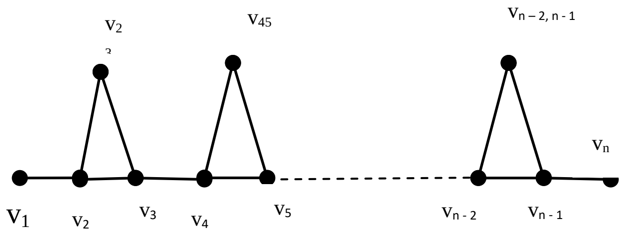
Superior distance $d_D(v_1, v_2) = 2$,
Superior distance $d_D(v_{12}, v_1) = 2$,
Superior distance $d_D(v_1, v_n) = 2 + d(v_2, v_{n-1}) + 2 = 2 + n-3 + 2 = n+1$,
Superior distance $d_D(v_1, v_{n-1,n}) = n+1$,
Superior distance $d_D(v_{12}, v_n) = n+1$,
Superior distance $d_D(v_{12}, v_{n-1,n}) = n+1$,
Superior distance $d_D(v_1, v_{n-1}) = 2 + d(v_2, v_{n-1}) + 3 = 2 + n-3 + 3 = n+2$,
Superior distance $d_D(v_{12}, v_{n-1}) = n+2$,
Superior distance $d_D(v_{12}, v_{n-1}) = n+2$,
Superior distance $d_D(v_1, v_4) = 7$.
Superior distance $d_D(v_i, v_{i+1}) = 6$ (i is even),
Superior distance $d_D(v_i, v_{i+1}) = 5$(i is odd) for $i > 2$,
Superior distance $d_D(v_i, v_j) = 4 + d(v_{i+1}, v_j) + 3 = 7 + (j-i-1) = 6 + j - i$,
Superior distance $d_D(v_i, v_j) = 3 + d(v_i, v_j) + 3 = j-i+6$, (for i is even, j is odd),
Superior distance $d_D(v_i, v_j) = 4 + d(v_{i+1}, v_{j-1}) + 4 = 8 + (j-1-i-1) = 6+j-i$, (for i is odd, j is even).
Superior distance $d_D(v_i, v_j) = 3 + d(v_i, v_{j-1}) + 4 = 7 + (j - 1 - i) = 6 + (j - i)$
Superior
$d(v_i, v_{i+2}) + 1 = 3+2+1 = 6$,
Superior distance $d_D(v_i, v_{i, i+1}) = 4$ when i is odd,
Superior distance $d_D(v_i, v_{j, j+1}) = 2 + d(v_i, v_j) + 2 = 4 + d(v_i, v_j) = 4 + j - i$ (for i is odd, j is odd),
Superior distance $d_D(v_i, v_{j, j+1}) = 3 + d(v_i, v_j) + 2 = 5 + (j - i)$ (for i is even , j is odd),
Superior distance $d_D(v_{i, i+1}, v_{j, j+1}) = 1 + d(v_i, v_{j+1}) + 1 = 2 + (j + 1 - i) = j - i + 3$ (for i and j are odd).

 DOI : 10.18231/2454-9150.2018.0853

S = { v_1, v_2, v_{34}, v_{57}, v_{78}, ….., $v_{n-3,\ n-2}$, v_{n-1}, v_n}. S is the superior eccentric dominating set of the alternate triangular snake graph. S is also minimum with this property .

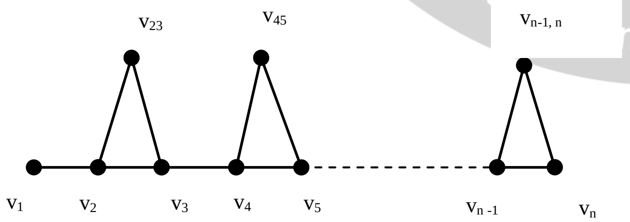
Therefore, $\gamma_{sed}(AT_n) = (\frac{n}{2} - 2) + 4 = \frac{n+4}{2}$

(ii) **n is even (type 2)**



Superior distance $d_D(v_1, v_2) = 4$,
Superior distance $d_D(v_1, v_{23}) = 3$,
Superior distance $d_D(v_2, v_n) = 4 + d(v_3, v_n) = 4 + n - 3 = n+1$,
Superior distance $d_D(v_2, v_{n-1}) = 4 + d(v_3, v_{n-2}) + 4 = 8 + (n - 5) = n + 3$,
Superior distance $d_D(v_1, v_n) = n - 1$,
Superior distance $d_D(v_{23}, v_n) = 1 + d(v_2, v_n) = 1 + n - 2 + 3 = n + 2$,
Superior distance $d_D(v_i, v_j) = 3 + d(v_i, v_j) + 1 + 1 = 5 + j + 1 - i = 6 + (j - i)$ (for i is odd, j is odd),
Superior distance $d_D(v_i, v_j) = 3 + d(v_i, v_j) + 3 = 6 + (j - i)$(for j is odd, i is even),
Superior distance $d_D(v_i, v_j) = 3 + d(v_i, v_j) + 3 = 6 + (j - i)$(for i is odd, j is even),
Superior distance $d_D(v_{m,\ m+1}, v_j) = 1 + d(v_m, v_j) + 3 = 4 + (j - m)$. (j is odd),
Superior distance $d_D(v_{m, m+1}, v_j) = 2 + d(v_{m+1}, v_j) + 3 = 5 + j - m - 1 = 4 + (j - m)$(j is even).

Hence v_2 and v_{n-1} are superior eccentric vertices of G. S = {v_1, v_2, v_{23}, v_{45}, ….., $v_{n-2,\ n-1}$, v_{n-1}}.
S is the superior eccentric dominating set of the alternate triangular snake graph. S is also minimum with this property. Therefore, $\gamma_{sed}(AT_n) = \frac{3n-2}{2}$



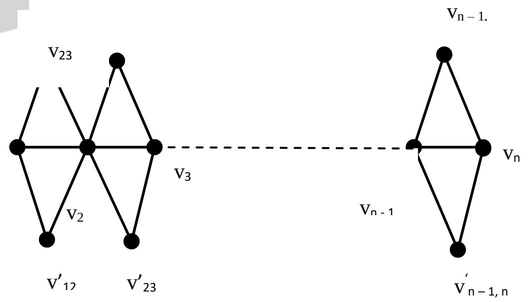
(iii) **n is odd**

Superior distance $d_D(v_1, v_2) = 4$,
Superior distance $d_D(v_2, v_3) = 6$,
Superior distance $d_D(v_{23}, v_2) = 4$,
Superior distance $d_D(v_1, v_n) = 1 + d(v_2, v_{n-1}) + 2 = 3 + n - 3 = n$,
Superior distance $d_D(v_i, v_n) = 3 + d(v_i, v_{n-1}) + 2 = 5 + n - i - 1 = n - i + 4$ (i < 1 & i is odd)
Superior distance $d_D(v_i, v_n) = 4 + d(v_{i+1}, v_{n-1}) + 2 = 4 + n - 1 - i - 1 = n - i + 4$(i < 1 & i is even)
Superior distance $d_D(v_i, v_j) = 1 + d(v_i, v_{j-1}) + 3 = 4 + j - i - 1 = 3 + j - i$($i \geq 2, j \leq n - 1$),
Superior distance $d_D(v_{23}, v_{n-1}) = 2 + d(v_3, v_{n-1}) + 3 = 5 + n - 1 - 3 = n + 1$,
Superior distance $d_D(v_2, v_n) = 4 + d(v_3, v_{n-1}) + 2 = 4 + n - 1 - 3 + 2 = n + 2$,
Superior distance $d_D(v_{23}, v_n) = 2 + d(v_3, v_{n-1}) + 2 = 4 + n - 1 - 3 = n$,
Superior distance $d_D(v_1, v_i) = d(v_1, v_i) + 3 = i - 1 + 3 = i + 2$(i is even),
Superior distance $d_D = d(v_1, v_{i-1}) + 2 = i - 1 - 1 + 2 = i$(i is even),
Superior distance $d_D(v_1, v_{n-1}) = d(v_1, v_{n-1}) + 3 = n - 1 - 1 + 3 = n + 1$,
Superior distance $d_D(v_i, v_{n-1}) = 2 + 2 + d(v_{i+1}, v_{n-1}) + 3 = 4 + n - 1 - i - 1 + 3 = n - i + 5$(1< i, i is even),
Superior distance $d_D(v_i, v_{n-1}) = 3 + d(v_i, v_{n-1}) + 3 = 6 + n - 1 - i = n - i + 5$(1<i, i is odd),
Superior distance $d_D(v_2, v_i) = 4 + d(v_3, v_i) + 3 = 7 + i - 3 = i + 4$(i is even),
Superior distance $d_D((v_2, v_i) = 4 + d(v_3, v_{i-1}) + 3 = 7 + i - 1 - 3 = i + 3$.

Hence v_2 and v_{n-1} are superior eccentric vertices of G. S = {v_2, v_{23}, v_{45}, …, $v_{n-3,\ n-2}$, v_{n-1}}.
S is the superior eccentric dominating set of the alternate triangular snake graph. S is also minimum with this property. Therefore , $\gamma_{sed}(AT_n) = \frac{n+1}{2}$.

III. DOUBLE TRIANGULAR SNAKE GRAPH

A double triangular snake graph DT_n consist of two triangular snakes that have a common path. That is, a double triangular snake graph is obtained from a path v_1, v_2,……, v_n by joining v_i and v_{i+1} to a new vertex v_{i+1}($1 \leq i \leq n$-1) and to a new vertex v_{i+1} ($1 \leq i \leq n$-1). The minimum cardinality of a superior eccentric domination in Double triangular Snake Graph is $\gamma_{sed}(DT_n)$.



Superior distance $d_D(v_1, v_{12}) = 3$,
Superior distance $d_D(v_1, v'_{12}) = 3$,
Superior distance $d_D(v_2, v_{23}) = 6$,
Superior distance $d_D(v_2, v_3) = 12$,
Superior distance $d_D(v_i, v_n) = 4 + d(v_{i-1}, v_n) + 4 = 8 + n - i - 1 = 7 + n - i$,

 DOI : 10.18231/2454-9150.2018.0853

Superior distance $d_D(v_i, v_{n-1}) = 4 + d(v_i, v_{n-2}) + 8 = 12 + n - 2 - i = 10 + n - i$,
Superior distance $d_D(v_{12}, v_n) = 2 + d(v_2, v_{n-1}) + 2 = 4 + n - 3 = n + 1$,
Superior distance $d_D(v_1, v_{n-1, n}) = 5 + d(v_2, v_{n-1}) + 2 = 7 + n - 3 = 4 + n$,
Superior distance $d_D(v_{12}, v_{n-1, n}) = 2 + d(v_2, v_{n-1}) + 2 = 4 + n - 3 = n + 1$,
Superior distance $d_D(v_1, v_{n-1}) = 5 + d(v_2, v_{n-2}) + 8 = n - 4 + 8 + 5 = n + 9$,
Superior distance $d_D(v_{ij}, v_i) = 7$,
Superior distance $d_D(v_{ij}, v_j) = 7$,
Superior distance $d_D(v_i, v_{i+1}) = 12$,
Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_{j-1}) + 8 = 12 + (j - 1 - i) = 11 + j - 1$, (when $i > 2, j < n - 1$)
Superior distance $d_D(v_i, v_j) = 4 + d(v_{i+1}, v_j) + 8 = 12 + (j - 1 - i) = 11 + j - i$(for i and j is even),
Superior distance $d_D(v_i, v_j) = 4 + d(v_{i+1}, v_j) + 8 = 12 + (j - i - 1) = 11 + j - i$(for i and j is odd),
Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_{j-1}) + 8 = 11 + j - i$(for i is even, j is odd),

Superior distance $d_D(v_i, v_j) = 4 + d(v_{i+1}, v_{j-1}) + 8 = 12 + (j - 1 - i - 1) = 12 + j - i - 2 = 10 + (j - i)$ (for i is odd, j is even),

Hence v_1 and v_n are the superior eccentric vertices of other vertices.

$S = \{v_1, v_{12}, v'_{12}, v'_{23}, \ldots, v_{n-2, n-1}, v'_{n-2, n-1, \ldots}, v_{n-1, n}, v_n\}$. S is the superior eccentric dominating set of the double triangular snake graph. S is also minimum with this property. Therefore, $\gamma_{sed}(DT_n) = 2(n - 1)$.

IV. Double Alternate Triangular Snake Graph

A Double Alternate Triangular Snake Graph $DA(T_n)$ consists of two alternate triangular snake graph that have a common path. That is, a double alternate triangular snake graph is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i. The minimum cardinality of a superior eccentric domination in double alternate triangular snake graph is $\gamma_{sed}(DAT_n)$.

When n is even, two types are alternate triangular snake graphs arise.

(i)**n is even (type 1)**

Superior distance $d_D(v_1, v_{12}) = 3$,

Superior distance $d_D(v_1, v_2) = 6$,

Superior distance $d_D(v_2, v_3) = 9$,

Superior distance $d_D(v_2, v_{12}) = 5$,

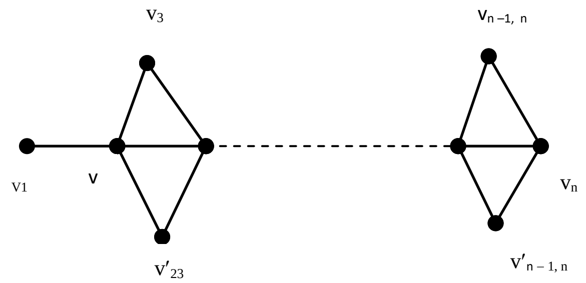
Superior distance $d_D(v_i, v_i + 1) = 5$,

Superior distance $d_D(v_{21}, v_1) = 3$,

Superior distance $d_D(v_1, v_n) = 5 + d(v_2, v_{n-1}) + 5 = n + 7$,

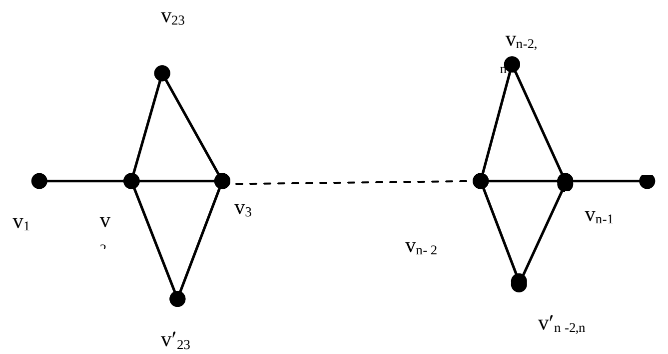
Superior distance $d_D(v_{12}, v_n) = n + 7$,

Superior distance $d_D(v_1, v_{n-1, n}) = n + 7$,
Superior distance $d_D(v_{12}, v_{n-1, n}) = n + 7$,
Superior distance $d_D(v_1, v_{n-1}) = 5 + d(v_2, v_{n-1}) + 4 = n + 6$,
Superior distance $d_D(v_{12}, v_{n-1}) = n + 6$,
Superior distance $d_D(v_1, v_3) = 10$,
Superior distance $d_D(v_1, v_4) = 12$,
Superior distance $d_D(v_i, v_{i+1}) = 8$(for $i > 2$) (when i is odd),
Superior distance $d_D(v_i, v_{i+1}) = 9$(for $i > 2$) (when i is even),
Superior distance $d_D(v_i, v_j) = 6 + d(v_{i+1}, v_j) + 4 = 10 + (j - i - 1) = 9 + (j - i)$(for i and j are odd),
Superior distance $d_D(v_i, v_j) = 5 + d(v_i, v_{j-i}) + 5 = 10 + j - i - 1 = 9 + j - i$(for i is even and j is odd),
Superior distance $d_D(v_i, v_j) = 6 + d(v_{i+1}, v_{j+1}) + 6 = 12 + (j - 1 - i - 1) = 10 + j - i$(for i and j are even),
Superior distance $d_D(v_i, v_j) = 5 + d(v_i, v_{j-i}) + 5$(for i is odd and j is even).



Hence v_1 and v_n are the superior eccentric vertices of other vertices. $S = \{v_1, v_2, v_4, \ldots, v_{n-2}, v_{n-1}, v_n\}$. S is the superior eccentric dominating set of the double alternate triangular snake graph. S is also minimum with this property. Therefore, $\gamma_{sed}(DAT_n) = \frac{n}{2} + 2$.

(ii) n is even (type 2)



Superior distance $d_D(v_1, v_2) = 5$,
Superior distance $d_D(v_1, v_{23}) = 4$,
Superior distance $d_D(v_1, v_n) = n - 1$,
Superior distance $d_D(v_1, v_{n-1}) = d(v_1, v_{n-2}) + 6 = n - 3 + 6 = n + 3$,

 DOI : 10.18231/2454-9150.2018.0853

Superior distance $d_D(v_{23}, v_n) = 1 + d(v_2, v_n) = 1 + n – 2 = n – 1$,

Superior distance $d_D (v_2, v_n) = 6 + d(v_3, v_n) = 6 + n – 3 = n + 3$,

Superior distance $d_D(v_2, v_{n-1}) = 6 + d(v_3, v_{n-2}) + 6 = 12 + n – 2 -3 = n + 7$, when $i \geq 2, j \leq n -1$

Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_{j+1}) + 4 = 8 + j + 1 – i = 9 + j – i$(for i and j are odd),

Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_j) + 4 = 8 + j – i$(for i is even, j is odd),

Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_j) + 4 = 8 + j – i$(for i is odd, j is even),when m is even , j is odd if $j = m + 1$

Superior distance $d_D(v_{m, m+1}, v_j) = 1 + d(v_m, v_{j-1}) + 6 = 7 + j – m – 1 = 6 + j – m$ (j is odd),

Superior distance $d_D(v_{m, m+1}, v_j) = 1 + d(v_m , v_{j-1}) + 5 = 5 + j – m$ (for j is even).

Hence $\{v_2, v_{n-1}\}$ are the superior eccentric vertices of G. S = $\{ v_2, v_4, v_6, v_8, \ldots\ldots\ldots, v_{n-4}, v_{n-2}, v_{n-1}, v_n\}$ are the superior eccentric dominating set of the double alternate triangular snake graph. S is also minimum with this property.

Therefore, $\gamma_{sed}(DAT_n) = \frac{n}{2} + 1$.

(iii) n is odd

Superior distance $d_D(v_1, v_2) = 5$,

Superior distance $d_D(v_1, v_{23}) = 3$,

Superior distance $d_D(v_1, v_n) = d(v_1,v_{n-1}) + 4 = 4 + n – 1 - 1 = n + 2$,

Superior distance $d_D(v_1, v_{n-1}) = d(v_1, v_{n-1}) + 4 = n – 2 + 4 = n + 2$,

Superior distance $d_D(v_{23}, v_{n-1}) = 2 + d(v_3,v_{n-1}) + 4 = 6 + n – 1 – 3 = n + 2$,

Superior distance $d_D(v_{23}, v_n) = 2 + d(v_3, v_{n-1}) + 4 = n – 4 + 6 = n + 2$,

Superior distance $d_D(v_2, v_n) = 5 + d(v_3, v_{n-1}) + 4 = 9 + n – 1 – 3 = n + 5$,

Superior distance $d_D(v_i, v_n) = 4 + d(v_i, v_{n-1}) + 4 = n – i + 7$(i<1, i is odd)

Superior distance $d_D(v_i, v_n) = 5 + d(v_{i+1}, v_n) + 4 = 9 + n – i – 1 = 8 + n – i$(i<1 i is even),

Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_j) + 4 = 8 + j – i$($i \geq 2, j \leq 2$)(for i is odd, j is even),

Superior distance $d_D(v_i, v_j) = 4 + d(v_i, v_{j-1}) + 4 = 8 + j – i – 1 = 7 + j - i$(for $i \geq 2, j \leq 2$)(i is odd, j is odd),

Superior distance $d_D(v_i, v_j) = 5 + d(v_{i+1}, v_{j-1}) + 4 = 9 + j – 1- i – 1= 7 + j - i$ ($i \geq 2, j \leq 2$)(whenw i is even, j is odd),

Superior distance $d_D(v_i, v_j) = 5 + d(v_{i+1}, v_j) + 4 = 9 + j – i - 1$($i \geq 2, j \leq 2$)(when i is even, j is even),

Superior distance $d_D(v_i, v_{n-1}) = 4 + d(v_i, v_{n-1}) + 4 = 8 + n – i – 1 = 7 + n - i$(for1< i, i is odd),

Superior distance $d_D(v_i, v_{n-1}) = 6 + d(v_{i+1}, v_{n-1}) + 4 = 10 + n – i – 1 - 1 = 8+ n - i$(for 1< i, i is odd),

Superior distance $d_D(v_2, v_i) = 6 + d(v_3, v_i) + 4 = 10 + i – 3 = 7 + i$(for $i \geq 2$, i is even),

Superior distance $d_D(v_2, v_i) = 6 + d(v_3, v_{i-1}) + 4 = 10 + i – 1 - 3 = 6 + i$(for $i \geq 2$, i is odd),

Hence v_2 and v_{n-1} are superior eccentric vertices of G. S = $\{ v_2, v_{23}, v_{45}, \ldots, v_{n-3, n-2}, v_{n-1}\}$.

S is the superior eccentric dominating set of the alternate triangular snake graph. S is also minimum with this property. Therefore, $\gamma_{sed}(DAT_n) = \frac{n+1}{2}$.

References

[1] F. Buckley and F. Harary, distance in graphs, Addison – Wesley, Reading 1990.

[2] E.J Cockayne, S.T Hedetniemi, 1977, Towards a theory of domination in graphs networks, pp 247-261.

[3] V. R.Kulli, Theory of Domination in Graphs, Vishwa international publication , Gulbarga, India 2010.

[4] T.W. Haynes, S.T.Hedetniemi and P.J slater, Fundamentals of domination in graphs, Marcel Dekkar, Newyork.

[5] T. N. Janakiraman, M. Bhanumathi and S. Muthammai, Eccentric Domination in Graphs, International Journal of Engineering Science, Advanced computing and Bio-technology vol. 1, No. 2, April- June 2010, pp. 55-70.

[6] KM. Kathiresan and G. Marimuthu, Superior Domination in Graphs, Jul 2008 Utilitas Mathematica.

[7] KM. Kathiresan and G. Marimuthu, Superior Distance in Graphs, Journal of combinatorial mathematics and combinatorial computing, May 2007.

[8] M.Bhanumathi and R.Meenal Abirami, Superior Eccentric Domination in Graphs, International Journal of Pure and Applied Mathematics, Dec 2017.

 DOI : 10.18231/2454-9150.2018.0853