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A Fuzzy Multi-Item Production Inventory Model With Deterioration Rate Using Regular Weighted Point Of Lotus Petal Fuzzy Number

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Abstract - In this paper, a multi-item fuzzy production inventory model with deterioration rate and two warehouse under two constraints have been considered. In this model the deterioration rate and demand rate considered as random variable and the production rate depends directly on demand rate. The lotus petal fuzzy number is defined and its properties are given. The parameters involved in this model are represented by lotus petal fuzzy number. The expected average total cost is defuzzified by the regular weighted point technique. The analytical expressions for expected inventory level in temporary warehouse, maximum inventory level and average total cost are derived for the proposed model by using nonlinear programming technique. A numerical example is presented to illustrate the results.

Keywords —lotus petal fuzzy number, machine time constraint, regular weighted point technique, temporary warehouse cost constraint.

AMS Subject Classification (2010): 90B05, 03E72.

I. Introduction

Inventory problems are common in manufacturing, service and business operations in general. Some inventory models were formulated in a static environment where the demand is assumed that constant and steady over a finite planning horizon. Many items of inventory such as electronic products, fashionable clothes, tasty food products etc., generate increasing sales after gaining consumer's acceptance. The sale for the other products may decline drastically due to the introduction of more competitive products or due to the change of consumer's preference. Therefore the demand of the product during its growth and decline phases can be taken as continuous time dependent function such as non-linear.

Most of the existing inventory models in the literature assume that items can be stored indefinitely to meet the future demands. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Therefore, if the rate of deterioration is not sufficiently low, its impact on modelling of such an inventory model cannot be ignored. In this connections, inventory problems for deteriorating items have been studied extensively by many researchers [1],[4],[5] and [7] from time to time. Research in this area started with the work of Whitin [8], who considered fashion goods deteriorating at the end of prescribed storage period. Goyal and Giri [3] gave recent trends of modelling in deteriorating item inventory. Samantha and Ajanta roy work based on realistic production lot-size inventory model for deteriorating items is given in [6].

In conventional inventory models, uncertainties are treated as randomness and arehandled by probability theory. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [9] is applicable. A. FarithaAsma and E.C. Henry Amirtharajanalized multi objective inventory model of deteriorating items with two constraints using fuzzy optimization technique [2].

In the real situation, at the time of production the deterioration rate and demand rate are varied. So that the deterioration rate and demand rate are considered as a random variable which follows rayleigh distribution and bounded pareto distribution respectively.

In the existing inventory models, they used only rented warehouse and own warehouse. When using the rented warehouse the transportation cost, rental and maintenance cost are all high. Here, the model newly constructed with permanent warehouse and temporary warehouse.

This paper is organised as follows:

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In section 2, assumptions and notations for the fuzzy production inventory model under consideration are given. The mathematical formulation for the proposed model is explained in crisp environment under section 3. In section 4, lotus petal fuzzy number is defined and its properties are given. To defuzzify the model, the regular weighted point of lotus petal fuzzy number is determined in section 5. In section 6, the mathematical model is explained in fuzzy environment. In section 7, an application of this model is given in both crisp and fuzzy environment.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used throughout this paper:

Assumptions:

1. Demand rate is taken as a random variable which follows bounded pareto distribution.
2. Production rate is demand dependent that is $p_i = c_i d_i$, $0 < c_i < n$, i=1,2,3…, n is a finite number.
3. Deterioration rate is taken as a random variable which follows reyleigh distribution
4. First, the goods are stored in permanent warehouse, then the remaining goods are stored in temporary warehouse.
5. The goods of permanent warehouse are consumed only after consuming the goods kept in temporary warehouse.
6. The permanent warehouse has a fixed capacity of I_{Pi} units.
7. The inventory level depleted due to demand and deterioration.
8. Shortages are not allowed.

Notations:

n – theno.of items. The following notations for i[th] item.

$I_{1i}(t)$ – the level of inventory at time t, $0 \le t \le t_1$.

$I_{2i}(t)$ – the level of inventory in temporary warehouse at time t, $t_1 \le t \le t_2$.

$I_{3i}(t)$ – the level of inventory in permanent warehouse at time t, $t_1 \le t \le t_3$.

I_{Pi}– fixed capacity for permanent warehouse at time t_1.

I_{mi} – expected maximum inventory level at time t_1. (decision variable)

I_{Ti}– expected maximum inventory level of temporary warehouse at time t_1..(decision variable)

p_i–production rate.

d_i–demand rate.

θ_i- deterioration rate.

t_1 -running time of machine for production.

TC– expected average total cost.

$\tilde{S}_{ci}$ - fuzzy setup cost per cycle for production.

$\tilde{h}_{ci}, \tilde{d}_{ci}$ - fuzzy holding cost, deteriorating cost per unit per unit time.

$\tilde{T}_{wci}$ - fuzzy cost for 1 square feet of temporary warehouse.

$\tilde{T}_{wsi}$ - fuzzy total space for temporary warehouse.

$\tilde{T}_{twci}$ - fuzzy total temporary warehouse cost.

$\tilde{T}_{mi}$ - fuzzy machine working time consumed per month.

$\tilde{m}_i$ - fuzzy production time per cycle.

III. MATHEMATICAL FORMULATION

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost and temporary warehouse cost. The rate of change of the inventory during the following periods are governed by the following differential equations.

$$\frac{dI_{1i}(t)}{dt} + \theta I_{1i}(t) = p_i - d_i \qquad 0 \le t \le t_1 \qquad \text{----- (3.1)}$$

with boundary condition $I_{1i}(0) = 0$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \qquad t_1 \le t \le t_2 \qquad \text{----- (3.2)}$$

with boundary conditions $I_{2i}(t_2) = 0$ and $I_{2i}(t_1) = I_{Ti}$

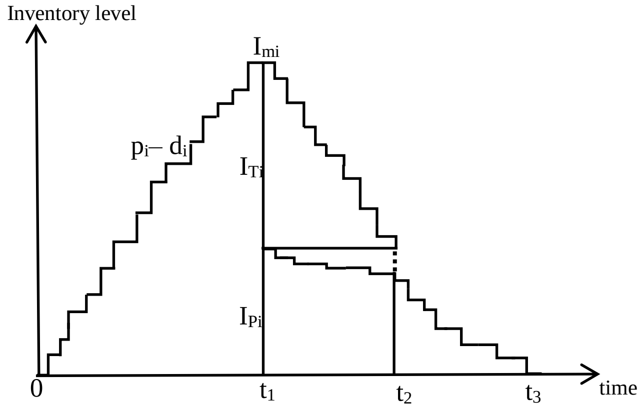

Figure 1: Graphical representation of the state of inventory system

$$\frac{dI_{3i}(t)}{dt} + \theta_i I_{3i}(t) = 0 \qquad t_1 \le t \le t_2 \text{----- (3.3)}$$

with boundary conditions $I_{3i}(t_1) = I_{Pi}$

$$\frac{dI_{3i}(t)}{dt} + \theta_i I_{3i}(t) = -d_i \qquad t_2 \le t \le t_3 \qquad \text{----- (3.4)}$$

with boundary condition $I_{3i}(t) = 0$

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From (3.1) $I_{1i}(t)=\frac{p_i-d_i}{\theta_i}\left(1-e^{-\theta_i t}\right)$ ----- (3.5)

From (3.2) $I_{Ti}=\frac{d_i}{\theta_i}\left(e^{\theta_i(t_2-t_1)}-1\right)$ ----- (3.6)

From (3.3) $I_{3i}(t)=I_{pi}\,e^{\theta_i(t_1-t)}$ ----- (3.7)

From (3.4) $I_{3i}(t)=\frac{d_i}{\theta_i}\left(e^{\theta_i(t_3-t)}-1\right)$ ----- (3.8)

Maximum inventory level per cycle is $I_{mi}=I_{Ti}+I_{Pi}$-----(3.9)

Expected demand rate is

$$E(d_i)=\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}}\left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i-1}}-\frac{1}{H_i^{\alpha_i-1}}\right)$$

Expected deterioration rate is $E(\theta_i)=\sigma_i\sqrt{\frac{\pi_i}{2}}$

Expected average total cost

$$=\sum_{i=1}^{n}\frac{1}{t_3}\left[\begin{array}{l}setup\ \cos t+E(holding\cos t)\\+E(\det erioration\ \cos t)+temporary\ warehouse\ \cos t\end{array}\right]$$

where,

Setup cost = S_{ci}

Holding cost

$$=h_{ci}\left\{\int_0^{t_1}I_{1i}(t)\,dt+\int_{t_1}^{t_2}\left(I_{2i}(t)+I_{3i}(t)\right)dt+\int_{t_2}^{t_3}I_{3i}(t)\,dt\right\}$$

$$=h_{ci}\left\{\begin{array}{l}\int_0^{t_1}\frac{p_i-d_i}{\theta_i}\left(1-e^{-\theta_i t}\right)dt\\+\int_{t_1}^{t_2}\left[\left(\frac{d_i}{\theta_i}\left(e^{\theta_i(t_2-t)}-1\right)\right)+I_{pi}e^{\theta_i(t_1-t)}\right]dt\\+\int_{t_2}^{t_3}\left(\frac{d_i}{\theta_i}\left(e^{\theta_i(t_3-t)}-1\right)\right)dt\end{array}\right\}$$

on simplification

$$=h_{ci}\left\{\begin{array}{l}\frac{p_i-d_i}{\theta_i^2}\left(e^{-\theta_i t_1}-1\right)+\frac{d_i}{\theta_i^2}\left(\begin{array}{l}e^{\theta_i(t_2-t_1)}+e^{\theta_i(t_3-t_2)}-2\\+\theta_i t_1-\theta_i t_3\end{array}\right)\\+I_{p_i}\left(\frac{1-e^{\theta_i(t_1-t_2)}}{\theta_i}\right)\end{array}\right\}$$

Deterioration cost = $d_{ci}\left\{\int_0^{t_1}I_{1i}(t)\,dt-\int_{t_1}^{t_3}d_i\ dt\right\}$

$$=d_{ci}\left\{\int_0^{t_1}\left(\frac{p_i-d_i}{\theta_i}\left(1-e^{-\theta t}\right)\right)dt-\int_{t_1}^{t_3}d_i\,dt\right\}$$

on simplification

$$\Rightarrow d_{ci}\left\{\frac{p_i-d_i}{\theta_i^2}\left(e^{-\theta t_1}+\theta_i t_1-1\right)-d_i\left(t_3-t_1\right)\right\}$$

Temporary warehouse cost $=T_{wci}\times T_{wsi}$

$$Min\ E(TC(t_1,t_2,t_3))$$

$$=\sum_{i=1}^{n}\frac{1}{t_3}\left[\begin{array}{l}S_{ci}\\+h_{ci}\left\{\begin{array}{l}\frac{c_iE(d_i)-E(d_i)}{E(\theta_i)^2}\left(e^{-E(\theta_i)t_1}-1\right)\\+\frac{E(d_i)}{E(\theta_i)^2}\left(\begin{array}{l}e^{E(\theta_i)(t_2-t_1)}+e^{E(\theta_i)(t_3-t_2)}\\-2+E(\theta_i)t_1-E(\theta_i)t_3\end{array}\right)\\+I_{pi}\left(\frac{1-e^{E(\theta_i)(t_1-t_2)}}{E(\theta_i)}\right)\end{array}\right\}\\+d_{ci}\left\{\begin{array}{l}\frac{c_iE(d_i)-E(d_i)}{E(\theta_i)^2}\left(e^{-E(\theta_i)t_1}+E(\theta_i)t_1-1\right)\\-E(d_i)\left(t_3-t_1\right)\end{array}\right\}\\+T_{wci}\,T_{wsi}\end{array}\right]$$

Subject to : $T_{wci}\,T_{wsi}\le T_{twci}$; $T_{mi}\,m_i\le t_1$ ------ (3.10)

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$$
\begin{aligned}
&Min\, E(TC(t_1,t_2,t_3)) \\
&= \sum_{i=1}^{n} \frac{1}{t_3} \left[\begin{aligned}
&S_{ci} \\
&+ h_{ci} \left\{ \begin{aligned}
&\frac{\left(\dfrac{L_i^{\alpha_i}}{1-\left(\dfrac{L_i}{H_i}\right)^{\alpha_i}} \left(\dfrac{\alpha_i}{\alpha_i-1}\right) \left(\dfrac{1}{L_i^{\alpha_i-1}} - \dfrac{1}{H_i^{\alpha_i-1}} \right) \right)(c_i-1)}{\left(\sigma_i \sqrt{\dfrac{\pi_i}{2}}\right)^2} \left(e^{-\left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right) t_1} - 1 \right) \\
&+ \frac{\left(\dfrac{L_i^{\alpha_i}}{1-\left(\dfrac{L_i}{H_i}\right)^{\alpha_i}} \left(\dfrac{\alpha_i}{\alpha_i-1}\right) \left(\dfrac{1}{L_i^{\alpha_i-1}} - \dfrac{1}{H_i^{\alpha_i-1}} \right) \right)}{\left(\sigma_i \sqrt{\dfrac{\pi_i}{2}}\right)^2} \left(\begin{aligned} &e^{\left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right)(t_2-t_1)} + e^{\left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right)(t_3-t_2)} \\ &-2 + \left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right) t_1 - \left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right) t_3 \end{aligned} \right) \\
&+ I_{pi} \left(\frac{1 - e^{\left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right)(t_1-t_2)}}{\left(\sigma_i \sqrt{\dfrac{\pi_i}{2}}\right)} \right)
\end{aligned} \right\} \\
&+ d_{ci} \left\{ \begin{aligned}
&\frac{\left(\dfrac{L_i^{\alpha_i}}{1-\left(\dfrac{L_i}{H_i}\right)^{\alpha_i}} \left(\dfrac{\alpha_i}{\alpha_i-1}\right) \left(\dfrac{1}{L_i^{\alpha_i-1}} - \dfrac{1}{H_i^{\alpha_i-1}} \right) \right)(c_i-1)}{\left(\sigma_i \sqrt{\dfrac{\pi_i}{2}}\right)^2} \left(e^{-\left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right) t_1} + \left(\sigma_i \sqrt{\frac{\pi_i}{2}}\right) t_1 - 1 \right) \\
&- \left(\dfrac{L_i^{\alpha_i}}{1-\left(\dfrac{L_i}{H_i}\right)^{\alpha_i}} \left(\dfrac{\alpha_i}{\alpha_i-1}\right) \left(\dfrac{1}{L_i^{\alpha_i-1}} - \dfrac{1}{H_i^{\alpha_i-1}} \right) \right)(t_3-t_1)
\end{aligned} \right\} \\
&+ T_{wci}\, T_{wsi}
\end{aligned} \right]
\end{aligned}
$$

$$\text{subject to} : T_{wci}\, T_{wsi} \le T_{twci},\ T_{mi}\, m_i \le t_1 \text{ ------- (3.11)}$$

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the differential equations using MATLAB software, the values of t_1^*, t_2^* and t_3^* are obtained. Substituting t_1^*, t_2^* and t_3^* in (3.6), (3.9) and (3.11) the optimum values I_T^*, I_m^* and TC^* are obtained.

IV. LOTUS PETAL FUZZY NUMBER AND ITS PROPERTIES

Definition: Lotus petal fuzzy number

A Lotus petal fuzzy number $\tilde{A}$ described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

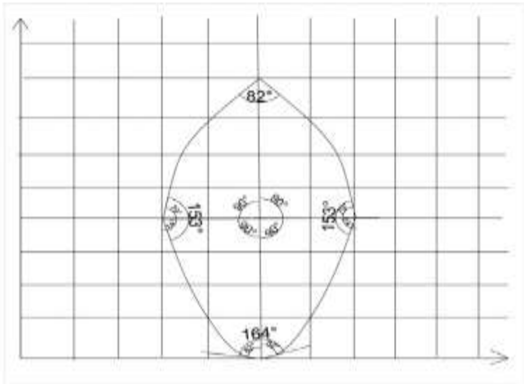

Figure 2: Graphical representation of lotus petal fuzzy number

$$
\mu_{\tilde{A}}(x) = \begin{cases}
\dfrac{1}{2} & at \quad x = a, c \\
\dfrac{1}{2}\left[1 + \sqrt{\dfrac{x-a}{b-a}}\right] & at \quad a \le x \le b \\
\dfrac{1}{2}\left[1 + \sqrt{\dfrac{c-x}{c-b}}\right] & at \quad b \le x \le c \\
\dfrac{1}{2}\left[\left(1 - \dfrac{c-x}{c-b}\right)^2\right] & at \quad c \ge x \ge b \\
\dfrac{1}{2}\left[\left(1 - \dfrac{x-a}{b-a}\right)^2\right] & at \quad b \ge x \ge a \\
0 \,\&\, 1 & at \quad x = b
\end{cases}
$$

This type of fuzzy number be denoted as $\tilde{A}$ = [a, b, c], whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0,1]
2. $\mu_{\tilde{A}}(x)$ is a convex function.
3. $\mu_{\tilde{A}}(x) = 0$ & 1 at x=b.
4. $\mu_{\tilde{A}}(x) = \dfrac{1}{2}$ at x= a & c.
5. $\mu_{\tilde{A}}(x)$ is strictly decreasing as well as increasing and continuous on [a,b] and [b,c].

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Properties:

1. Left and right opposite angles are equal.
2. The horizontal and vertical diagonal bisect each other and meet at 90°.

V. REGULAR WEIGHTED POINT OF LOTUS PETAL FUZZY NUMBER

For the Lotus petal fuzzy number $\tilde{A}$=[a, b, c] , the α -cut is $\tilde{A}_{\alpha}=[L_{\tilde{A}}(\alpha),R_{\tilde{A}}(\alpha)]$ and the regular weighted point for $\tilde{A}$ is given by,

$$r_w(\tilde{A})=\frac{\int_0^1 \frac{L_{\tilde{A}}(\alpha)+R_{\tilde{A}}(\alpha)}{2} f_1(\alpha)\,d\alpha}{\int_0^1 f(\alpha)\,d\alpha}$$

$$=\int_0^1 \left[L_{\tilde{A}}(\alpha)+R_{\tilde{A}}(\alpha)\right] f_1(\alpha)\,d\alpha$$

$$\text{where, } f(\alpha)=\begin{cases}(1-2\alpha) & when \quad \alpha\in\left[0,\frac{1}{2}\right]\\ (2\alpha-1) & when \quad \alpha\in\left[\frac{1}{2},1\right]\end{cases}$$

$$f_1(\alpha)=\begin{cases}\omega(1-2\alpha) & when \quad \alpha\in\left[0,\frac{1}{2}\right],\\ (1+\omega)(2\alpha-1) & when \quad \alpha\in\left[\frac{1}{2},1\right]\end{cases}$$

$0<\omega<1$.

The regular weighted point of a lotus petal fuzzy number is of the form

$$r_w(\tilde{A})=\omega\left(\frac{31a+58b+31c}{120}\right)+\left(\frac{a+2b+c}{8}\right)$$

VI. INVENTORY MODEL IN FUZZY ENVIRONMENT

The proposed inventory model in fuzzy environment is

$$Min\ E(\tilde{T}C(t_1,t_2,t_3))$$

$$=\sum_{i=1}^{n}\frac{1}{t_3}\left[\begin{array}{l}\tilde{S}_{ci}\\ +\tilde{h}_{ci}\left\{\begin{array}{l}\frac{c_i E(d_i)-E(d_i)}{E(\theta_i)^2}\left(e^{-E(\theta_i)t_1}-1\right)+\frac{E(d_i)}{E(\theta_i)^2}\\ \left(e^{E(\theta_i)(t_2-t_1)}+e^{E(\theta_i)(t_3-t_2)}-2\right.\\ \left.+E(\theta_i)t_1-E(\theta_i)t_3\right)\\ +I_{pi}\left(\frac{1-e^{E(\theta_i)(t_1-t_2)}}{E(\theta_i)}\right)\end{array}\right\}\\ +\tilde{d}_{ci}\left\{\begin{array}{l}\frac{c_i E(d_i)-E(d_i)}{E(\theta_i)^2}\left(e^{-E(\theta_i)t_1}+E(\theta_i)t_1-1\right)\\ -E(d_i)(t_3-t_1)\end{array}\right\}\\ +\tilde{T}_{wci}\ \tilde{T}_{wsi}\end{array}\right]$$

subject to : $\tilde{T}_{wci}\ \tilde{T}_{wsi}\le\tilde{T}_{twci}$, $\tilde{T}_{mi}\ \tilde{m}_i\ \le t_1$ ------ (6.1)

$$Min\ E(\tilde{T}C(t_1,t_2,t_3))$$

$$=\sum_{i=1}^{n}\frac{1}{t_3}\left[\begin{array}{l}\tilde{S}_{ci}\\ +\tilde{h}_{ci}\left\{\begin{array}{l}\frac{\left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}}\left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i}-1}-\frac{1}{H_i^{\alpha_i}-1}\right)\right)(c_i-1)}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)^2}\\ \left(e^{-\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1}-1\right)\\ +\frac{\left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}}\left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i}-1}-\frac{1}{H_i^{\alpha_i}-1}\right)\right)}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)^2}\\ \left(\begin{array}{l}e^{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)(t_2-t_1)}+e^{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)(t_3-t_2)}\\ -2+\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1-\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_3\end{array}\right)\\ +I_{pi}\left(\frac{1-e^{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)(t_1-t_2)}}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)}\right)\end{array}\right\}\\ +\tilde{d}_{ci}\left\{\begin{array}{l}\frac{\left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}}\left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i}-1}-\frac{1}{H_i^{\alpha_i}-1}\right)\right)(c_i-1)}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)^2}\\ \left(e^{-\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1}+\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1-1\right)\\ -\left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}}\left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i}-1}-\frac{1}{H_i^{\alpha_i}-1}\right)\right)(t_3-t_1)\end{array}\right\}\\ +\tilde{T}_{wci}\ \tilde{T}_{wsi}\end{array}\right]$$

subject to : $\tilde{T}_{wci}\ \tilde{T}_{wsi}\le\tilde{T}_{twci}$, $\tilde{T}_{mi}\ \tilde{m}_i\ \le t_1$ ------ (6.2)

where ~ represents for fuzzification of the parameters.

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$\tilde{S}_{ci} = (S_{c1i}, S_{c2i}\ S_{c3i})$, $\tilde{h}_{ci} = (h_{c1i}, h_{c2i}, h_{c3i})$,
$\tilde{d}_{ci} = (d_{c1i}, d_{c2i}, d_{c3i})$, $\tilde{T}_{wci} = (T_{wc1i}, T_{wc2i}\ T_{wc3i})$,
$\tilde{T}_{wsi} = (T_{ws1i}, T_{ws2i}, T_{ws3i})$, $\tilde{T}_{twci} = (T_{twc1i}, T_{twc2i}, T_{twc3i})$,
$\tilde{T}_{mi} = (T_{m1i}, T_{m2i}, T_{m3i})$, $\tilde{m}_i = (m_{1i}, m_{2i}, m_{3i})$.

Now using the technique, regular weighted point of lotus petal fuzzy number, the above model is defuzzified as follows

$$
\begin{aligned}
&Min\, E(r_w(TC)(t_1,t_2,t_3)) \\
&= \sum_{i=1}^{n} \frac{1}{t_3} \Bigg[r_w(S_{ci}) + r_w(h_{ci}) \Bigg\{ \frac{\left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}} \left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i-1}} - \frac{1}{H_i^{\alpha_i-1}}\right) \right)(c_i-1)}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)^2} \left(e^{-\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1} - 1 \right) \\
&\quad + \frac{\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}} \left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i-1}} - \frac{1}{H_i^{\alpha_i-1}}\right)}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)^2} \left(e^{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)(t_2-t_1)} + e^{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)(t_3-t_2)} - 2 + \left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1 - \left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_3 \right) \\
&\quad + I_{pi} \left(\frac{1 - e^{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)(t_1-t_2)}}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)} \right) \Bigg\} \\
&\quad + r_w(d_{ci}) \Bigg\{ \frac{\left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}} \left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i-1}} - \frac{1}{H_i^{\alpha_i-1}}\right) \right)(c_i-1)}{\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)^2} \left(e^{-\left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1} + \left(\sigma_i\sqrt{\frac{\pi_i}{2}}\right)t_1 - 1 \right) \\
&\quad - \left(\frac{L_i^{\alpha_i}}{1-\left(\frac{L_i}{H_i}\right)^{\alpha_i}} \left(\frac{\alpha_i}{\alpha_i-1}\right)\left(\frac{1}{L_i^{\alpha_i-1}} - \frac{1}{H_i^{\alpha_i-1}}\right) \right)(t_3-t_1) \Bigg\} \\
&\quad + r_w(T_{wci})\, r_w(T_{wsi}) \Bigg]
\end{aligned}
$$

Subject to : $r_w(T_{wci})\ r_w(T_{wsi}) \le r_w(T_{twci})$, $r_w(T_{mi})\ r_w(m_i)\ \le t_1$ -------- (6.3)

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the differential equations using MATLAB software, the values of t_1^*, t_2^* and t_3^* are obtained. Substituting t_1^*, t_2^* and t_3^* in (3.6), (3.9) and (6.3), the optimum values I_T^*, I_m^* and TC^* are obtained.

VII. NUMERICAL EXAMPLE

Develop a mathematical program to minimize the average total cost, subject to temporary warehouse cost and machine time.

a. Use I_{pi}= 45000 Ton. The following datum's are taken in proper units. (i = 1,2,3)

All costs are taken in rupees.

	i = 1	i = 2	i = 3
L_i	60	80	100
H_i	600	800	1000
α_i	0.2	0.3	0.5
σ_i	3	4	5
π_i	3.14	3.14	3.14
$\tilde{S}_{ci}$	(1000000, 1200000, 1400000)	(900000, 1100000, 1300000)	(800000, 1000000, 1200000)
$\tilde{h}_{ci}$	(3000, 4000, 5000)	(4000, 5000, 6000)	(5000, 6000, 7000)
$\tilde{d}_{ci}$	(300,400, 500)	(400,500,600)	(500,600,700)
$\tilde{T}_{wci}$	(100,150, 200)	(150,200,250)	(200,250,300)

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$\tilde{T}_{wsi}$ in sqft	(1000, 1050,1100)	(1500, 1550, 1600)	(2000, 2050, 2100)
$\tilde{T}_{twc\,i}$	(150000,200000,250000)	(300000,400000, 500000)	(500000,600000, 700000)
$\tilde{t}_{m\,i}$ in days	(23,25, 27)	(24,26,2)	(25,27,2)
m_i in months	(1, 1.5, 2)	(1.5,2,2.5)	(2,2.5, 3)

Using MATLAB software, the optimum values I_T^*, I_m^* and TC^* are tabulated.

Comparison table 7.1

Model	t_1^*/ month	t_2^*/ month	t_3^*/ month	I_T^*/ Ton	I_m^* /Ton	TC^* Rs
Crisp 1	2.8122	3.9317	5.0988	58,820	1,03,820	2,74,56,000
Crisp 2	3.0475	4.1120	5.1192	54,152	99,152	2,81,98,000
Crisp 3	3.0485	4.1154	5.1249	54,921	99,921	3,37,10,000
Fuzzy	3.1445	4.2243	5.2474	59,247	1,04,250	2,62,79,000

Observation:

From the above table, it should be noted that compared to crisp model, the fuzzy model is very effective method, because of the time consuming in fuzzy analysis and the optimal results are obtained easily.

i. The average total cost is obtained in fuzzy model is less than the crisp model.

ii. The optimal values I_T^*, I_m^* in fuzzy model are higher than the crisp model.

VIII. REFERENCES

[1] Chung KJ,Ting TS, A heuristic for replenishment for deteriorating items with a linear trend in demand, *J Oper Res Soc*, 44 (1993),1235-1241.

[2] Faritha.AAsma and Henry Amirtharaj.E.C, Solving multi objective inventorymodel of deteriorating items with two constraints using fuzzy optimization technique, *Intern. J. Fuzzy mathematical Archive*,10 (ISSN:2320-3242(P),2320-3250(online)) (2016),41-48.

[3] Goyal SK, Giri BC, recent trends in modelling of deteriorating inventory, *Eur J Oper Res*, 134 (2001),1-16.

[4] Mishra VK, Singh LS, Deteriorating inventory model with time dependent demand and partial backlogging, *Appl Math Sci*,4 (2010),3611-3619.

[5] Niketa D, Trivedi, NehalJ.Shukla and Nita H. Shah, Inventory model for deteriorating items with fixed life under quadratic demand and nonlinear holding cost, *IJEIT*, 3 (ISSN:2277-3754) (2014).

[6] Samanta.G.P, Ajanta roy,A Production inventory model with deteriorating items and shortages, *Yugoslav Journal or Operation Research*, 2 (2004), 219-230.

[7] Wee HM, A deterministic lot-size inventory model for deteriorating items with shortages and a declining market, *ComputOper*, 22 (1995)345-356.

[8] WhitinTM,The theory of inventory management, Princeton: *Princeton University Press* (1957).

[9] Zadeh.L.A.,Fuzzy sets, *Information and Control*,8 (1965), 338-353.

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