Stronger Forms of Fuzzy #rg -Continuous Functions in Fuzzy Topological Spaces

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Abstract **In this paper the concepts of strongly fuzzy #rg -continuous, perfectly fuzzy #rg -continuous and completely fuzzy #rg -continuous have been introduced and some interesting properties of these spaces are investigated.**

Keywords — strongly fuzzy #rg -continuous, perfectly fuzzy #rg -continuous and completely fuzzy #rg -continuous.

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I. Introduction

Ever since the introduction of fuzzy sets by Zadeh [12] and fuzzy topological spaces by Chang [4] various notions in classical topology have been extended to fuzzy topological spaces (fts for short). The concepts of #rg -closed sets is introduced and its properties are studied by S. Syed et. al [8] in the year 2011 for general topological spaces. For the fuzzy topological spaces, g - continuous fuzzy maps were introduced and studied by [6] and [5]. Recently A. Vadivel [9, 10] introduced the concept of fuzzy #rg -closed sets, fuzzy #rg -open sets and fuzzy #rg -continuous mappings in fts. In this paper the concepts of strongly fuzzy #rg -continuous, perfectly fuzzy #rg -continuous and completely fuzzy #rg -continuous have been introduced and studied. Also it is proved that every strongly fuzzy #rg -continuous function is f -continuous function and also every strongly f -continuous function is a strongly fuzzy #rg -continuous function. And every perfectly fuzzy #rg -continuous function is f -continuous function and also every perfectly fuzzy #rg –continuous function is a perfectly f -continuous function. And every completely fuzzy #rg -continuous function is f -continuous function and also every completely fuzzy #rg -continuous function is a completely f -continuous function.

Let X , Y and Z be sets. Throughout the present chapter (X, α) , (Y, β) and (Z, γ) (or simply X , Y and Z) mean fuzzy topological spaces on which no separation axioms is assumed unless explicitly stated.

Before entering into our work we recall the following definitions, which are due to various authors.

II. Preliminaries

If A is a subset of X with a topology τ,then the closure of A is denoted by τ - cl(A) or cl(A) , the interior of A is denoted by τ-int(A) or int(A) and the complement of A in X is denoted by A^c .

Definition 2.1. A fuzzy set λ in a fts (X, τ) is called

(i) fuzzy regular open set [1] if $intcl(\lambda) = \lambda$ and a fuzzy regular closed set if $clint(\lambda) = \lambda$,

(ii) fuzzy regular semi open [13] if there exists fuzzy regular open set μ in X such that $\mu \le \lambda \le cl(\mu)$.

Definition 2.2. A fuzzy set λ in a fts (X,τ) is called

(i) a fuzzy generalized closed set (briefly, f g -closed set)[3] if $cl(\lambda) \le \mu$ whenever $\lambda \le \mu$ and μ is fuzzy open in X ,

(ii) a fuzzy regular weakly closed set (briefly, f rw -closed) [11] if $cl(\lambda) \le \mu$, whenever $\lambda \le \mu$ and μ is fuzzy regular semiopen in X ,

(iii) fuzzy # regular generalized closed (briefly fuzzy #rg -closed) [9] if $cl(\lambda) \le \mu$ whenever $\lambda \le \mu$ and μ is frw -open in X .

The complements of the above mentioned closed sets are respective open sets.

Definition 2.3. A fuzzy topological space (X, τ) is called

(i) a fuzzy $T^{\#rg}$ -space [9] if every fuzzy #rg -closed set is a fuzzy closed set,

(ii) a fuzzy ${}^{\#rg}T$ -space [9] if every f g -closed set is fuzzy #rg -closed set.

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Definition 2.4. Let X , Y be two fuzzy Topological spaces. A function f : X → Y is called

(i) *fuzzy continuous* (briefly, f -continuous) [2] if $f^{-1}(\lambda)$ is fuzzy open set in X , for every fuzzy open set λ of Y ,

(ii) *fuzzy strongly continuous* (briefly, f s -continuous) function [2] if $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed set in X , for every fuzzy set λ in Y ,

(iii) *fuzzy strongly g -continuous* (briefly, fsg -continuous) function [2] if $f^{-1}(\lambda)$ is fuzzy open (closed) set in X , for every fuzzy g -open (g - closed) set λ in Y ,

(iv) *fuzzy perfectly continuous* (briefly, f p -continuous) function [2] if $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed set in X , for every fuzzy open set λ in Y ,

(v) *fuzzy perfectly g-continuous* (briefly, fpg -continuous) function [2] if $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed set in X , for every fuzzy g- open set (g -closed) λ in Y ,

(vi) *fuzzy completely continuous* (briefly, fc -continuous) function [7] if $f^{-1}(\lambda)$ is fuzzy regular open set in X , for every fuzzy open set λ in Y ,

(vii) *fuzzy #rg -continuous* function [10] if $f^{-1}(\lambda)$ is fuzzy #rg -open set in X , for every fuzzy open set λ in Y ,

(viii) *fuzzy #rg -irresolute* [10] if $f^{-1}(\lambda)$ is fuzzy #rg -open set in X , for every fuzzy #rg -open set λ in Y .

III. STRONGER FORMS OF FUZZY # RG - CONTINUOUS FUNCTIONS

Now, the stronger forms of fuzzy #rg -continuous functions namely strongly fuzzy #rg -continuous, perfectly fuzzy #rg -continuous and completely fuzzy #rg -continuous have been introduced and studied.

Definition 3.1. A functions f : X → Y is called strongly fuzzy #rg - continuous if the inverse image of every fuzzy #rg -open set in Y is fuzzy open set in X .

Theorem 3.1. A function f : X → Y is called strongly fuzzy #rg - continuous iff the inverse image of every fuzzy #rg -closed set in Y is fuzzy closed set in X .

Proof . Assume that f is strongly fuzzy #rg -continuous. Let λ be fuzzy #rg -closed set in Y .

Then $1-\lambda$ is fuzzy #rg -open set. Since f is strongly fuzzy #rg -continuous, $f^{-1}(1-\lambda)$ is fuzzy open set in X . But

$f^{-1}(1-\lambda) = 1 - f^{-1}(\lambda)$ and so $f^{-1}(\lambda)$ is fuzzy closed set in X.

Conversely, suppose that the inverse image of every fuzzy #rg -closed set in Y is fuzzy closed set in X . Let μ be fuzzy #rg -open set in Y , then $1-\mu$ is fuzzy #rg -closed set in Y . By hypothesis, $f^{-1}(1-\mu)$ is fuzzy closed set in X . Now $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$ and so $f^{-1}(\mu)$ is fuzzy open set in X . Hence f is strongly fuzzy #rg -continuous.

Theorem 3.2. Every strongly fuzzy #rg -continuous function is a f - continuous function.

Proof Let f : X → Y be strongly fuzzy #rg -continuous. Let μ be fuzzy open set in Y , and so μ is fuzzy #rg -open set in Y . Then $f^{-1}(\mu)$ is fuzzy open set in X . Hence f is f -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.1 Let X = {a, b, c} and the fuzzy sets μ and γ be defined as follows. μ = 1/a +0.9 / b+ 0.8/b,

γ= 0.4/a+ 0.5/b+ 0.7/c. Consider τ = {0, 1, γ}

Define f : (X, τ) → (X, τ) by f (a) = a, f (b) = b and f (c) = c . Then f

is f -continuous but not strongly fuzzy #rg -continuous as the fuzzy set μ is fuzzy #rg -closed in Y and $f^{-1}(\mu) = \mu$ is not fuzzy closed set in X . Hence is f -continuous function.

Theorem 3.3. Every fs -continuous function is a strongly fuzzy #rg - continuous function.

Proof Let f : X → Y be f -strongly continuous function. Let μ be fuzzy #rg -open set in Y , And then $f^{-1}(\mu)$ is both fuzzy open and fuzzy closed set in X as f is f -strongly continuous function. Hence f is strongly fuzzy #rg -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.2. Let X = Y = {a, b, c} and the fuzzy sets λ , μ and γ be defined as follows. λ= 0.4/a + 0.5/b + 0.7/ c, μ = 1/a +0.9 / b+ 0.8/b, γ= 0/a+ 0.1/b+ 0.2/c.

Consider τ = {0, 1, λ,γ} and σ = {0, 1, γ } . Then (X,τ) and (Y, σ) are fts. Define f : X → Y by f (a) = a, f (b) = b and f (c) = c . Then f is strongly fuzzy #rg -continuous but not f -strongly continuous as the fuzzy set μ is fuzzy #rg -closed set in Y and $f^{-1}(\mu) = \mu$ is fuzzy closed set in X but not fuzzy open set in X . Hence f is strongly fuzzy #rg -continuous.

Theorem 3.4. Let f : X → Y be f -continuous and Y be fuzzy $T^{\#rg}$ - space. Then f is strongly fuzzy #rg -continuous.

Proof Let be fuzzy #rg -open set in Y . Then μ is fuzzy open set in Y. Since Y is fuzzy $T^{\#rg}$ -space. And hence $f^{-1}(\mu)$ is fuzzy open set in X as f is f -continuous. Hence f is strongly fuzzy #rg -continuous.

Theorem 3.5. Let f : X → Y be strongly fuzzy #rg - continuous and Y be fuzzy $^{\#rg}T$ -space. Then f is f - strongly g -continuous.

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Proof Let μ be fuzzy g -open set in Y and so is fuzzy #rg -open set in Y , since Y is fuzzy $^{\#rg}$T -space. Since f is strongly fuzzy #rg -continuous, $f^{-1}(\mu)$ is fuzzy open set in X . Hence f is f -strongly g -continuous.

Theorem 3.6. If f : X $\rightarrow$ Y be strongly fuzzy #rg -continuous and g: Y $\rightarrow$ Z is strongly fuzzy #rg-continuous, then the composition map $g \circ f : X \rightarrow Z$ is strongly fuzzy #rg -continuous.

Proof Let μ be fuzzy #rg -open set in Z . Then $g^{-1}(\mu)$is fuzzy open set in Y , since g is strongly fuzzy #rg -continuous. And therefore $g^{-1}(\mu)$is fuzzy #rg -open set in Y . Also since f is strongly fuzzy #rg -continuous, $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$ is fuzzy open set in X . Hence $g \circ f$ is strongly fuzzy #rg -continuous.

Theorem 3.7. Let $f : X \rightarrow Y$, $g : Y \rightarrow Z$ be maps such that f is strongly fuzzy #rg -continuous and g is fuzzy #rg -continuous then $g \circ f : X \rightarrow Z$ is f-continuous.

Proof Let λ be fuzzy closed set in Z . Then $g^{-1}(\lambda)$ is fuzzy #rg -closed set in Y . Since g is fuzzy #rg -continuous. And since f is strongly fuzzy #rg -continuous, $f^{-1}(g^{-1}(\lambda)) = (g \circ f)^{-1}(\lambda)$ is fuzzy closed set in X . Hence $g \circ f$ is f -continuous.

Theorem 3.8. If f : X $\rightarrow$ Y be strongly fuzzy #rg -continuous and g : Y $\rightarrow$ Z is fuzzy #rg -irresolute. Then the composition map $g \circ f : X \rightarrow Z$ is strongly fuzzy #rg -continuous.

Proof Let μ be fuzzy #rg -open set in Z . Then $g^{-1}(\mu)$ is fuzzy #rg - open set in Y . Since g is fuzzy #rg -irresolute. And then $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$ is fuzzy open set in X .

Also since f is strongly fuzzy #rg - continuous. Hence $g \circ f$ is strongly fuzzy #rg -continuous.

Definition 3.2. A function $f : X \rightarrow Y$ is called perfectly fuzzy #rg - continuous if the inverse image of every fuzzy #rg -open set in Y is both fuzzy open and fuzzy closed set in X .

Theorem 3.9. A map $f : X \rightarrow Y$ is perfectly fuzzy #rg -continuous iff the inverse image of every fuzzy #rg -closed set in Y is both fuzzy open and fuzzy closed set in X .

Proof Assume that f is perfectly fuzzy #rg -continuous. Let λ be fuzzy #rg -closed set in Y . Then $1 - \lambda$ is fuzzy #rg -open set in Y . And therefore $f^{-1}(1-\lambda)$ is both fuzzy open and fuzzy closed set in X . But $f^{-1}(1 - \lambda) = 1 - f^{-1}(\lambda)$ and so $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed set in X.

Conversely, suppose that the inverse image of every fuzzy #rg -closed set in Y is both fuzzy open and fuzzy closed set in X . Let μ be fuzzy #rg -open set in Y . Then $1- \mu$ is fuzzy #rg -closed set in Y . By hypothesis, $f^{-1}(1-\mu)$ is both fuzzy open and fuzzy closed set in X . But $f^{-1}(1-\mu) = 1 - f^{-1}(\mu)$. Therefore $f^{-1}(\mu)$ is both fuzzy open and fuzzy closed set in X . Hence f is perfectly fuzzy #rg -continuous.

Theorem 3.10. Every perfectly fuzzy #rg -continuous function is a f - continuous function.

Proof Let $f : X \rightarrow Y$ be perfectly fuzzy #rg -continuous. Let μ be fuzzy open set in Y , and so is fuzzy #rg -open set in Y . Since f is perfectly fuzzy #rg -continuous, then $f^{-1}(\mu)$ is both fuzzy open and fuzzy closed set in X . That is $f^{-1}(\mu)$ is fuzzy open set in X . Hence f is f -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3. Let X = Y = {a, b, c} and the fuzzy sets λ , μ and γ be defined as follows. $\lambda = 0.4/a + 0.5/b + 0.7/c$, $\mu = 1/a + 0.9/b + 0.8/b$, $\gamma = 0/a + 0.1/b + 0.2/c$.

Consider $\tau = \{0, 1, \lambda,\gamma\}$ and $\sigma = \{0, 1, \gamma\}$. Then (X,τ) and (Y, σ) are fts. Define $f : X \rightarrow Y$ by f (a) = a, f (b) = b and f (c) = c . Then f is f -continuous but not perfectly fuzzy #rg -continuous as the fuzzy set μ is fuzzy #rg -closed set in Y and $f^{-1}(\mu) = \mu$ is not fuzzy open set in X but fuzzy closed set in X . Hence f is f -continuous.

Theorem 3.11. Every perfectly fuzzy #rg -continuous function is a f p - continuous function.

Proof Let $f : X \rightarrow Y$ be perfectly fuzzy #rg -continuous. Let μ be fuzzy open set in Y then μ be fuzzy #rg -open set in Y . Since f is perfectly fuzzy #rg -continuous. Then $f^{-1}(\mu)$ is both fuzzy open and fuzzy closed set in X and hence f is f p -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.4 Let X = Y = {a, b, c} and the fuzzy sets λ, μ, γ, δ and α be defined as follows. $\lambda = 0.4/a + 0.5/b + 0.7/c$, $\mu = 0.6/a + 0.5/b + 0.3/c$, $\gamma = 0.6/a + 0.5/b + 0.7/c$, $\delta = 0.4/a + 0.5/b + 0.3/c$ and $\alpha = 1/a + 0.9/b + 0.8/c$. Consider $\tau = \{0, 1, \lambda, \gamma, \mu, \delta\}$and $\sigma = \{0, 1, \lambda\}$. Then (X, τ) and (Y, σ) are fts. Define $f : X \rightarrow Y$ by f (a) = a, f (b) = b and f (c) = c . Then f is f p -continuous but not perfectly fuzzy #rg -continuous as the fuzzy α set is fuzzy #rg -closed in Y and $f^{-1}(\alpha) = \alpha$ is not both fuzzy open and fuzzy closed set in X . Hence f is not perfectly fuzzy #rg -continuous.

Theorem 3.12. If $f : X \rightarrow Y$ is f -perfectly g -continuous and Y is fuzzy $^{\#rg}$T -space then f is perfectly fuzzy #rg -continuous function.

Proof Let μ be fuzzy g -open set in Y . Then μ is fuzzy #rg -open set in Y as Y is fuzzy $^{\#rg}$T -space. Since f is f -perfectly g -continuous, $f^{-1}(\mu)$ is both fuzzy open and fuzzy closed set in X . And therefore f is perfectly fuzzy #rg -continuous function.

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Theorem 3.13. If f : X → Y is f -perfectly continuous and Y T #rg is fuzzy -space, then f is perfectly fuzzy #rg -continuous function

Proof Let μ be fuzzy #rg -open set in Y . Then μ is fuzzy open set in Y as Y is fuzzy T $^{\#rg}$ -space. Since f is f -perfectly continuous, f^{-1}(μ) is both fuzzy open and fuzzy closed set in X . And therefore f is perfectly fuzzy #rg -continuous function.

Theorem 3.14. Every perfectly fuzzy #rg -continuous function is strongly fuzzy #rg -continuous function.

Proof Let f : X → Y be perfectly fuzzy #rg -continuous. Let μ be fuzzy #rg -open set in Y . And then f^{-1}(μ) is both fuzzy open and fuzzy closed set in X . Therefore f^{-1}(μ) is fuzzy open set in X . Hence f is strongly fuzzy #rg -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5 In the Example 3.2, then f is strongly fuzzy #rg -continuous but not perfectly fuzzy #rg -continuous as the fuzzy set is fuzzy #rg -closed in Y and f^{-1}(μ) = μ is not fuzzy open set in X but fuzzy closed set in X . Hence f is strongly fuzzy #rg -continuous.

Theorem 3.15. Let f : X → Y , g : Y → Z be two perfectly fuzzy #rg - continuous functions then g ∘ f : X → Z is perfectly fuzzy #rg -continuous function.

Proof Let μ be fuzzy #rg -open set in Z . And then g^{-1}(μ) is both fuzzy open and fuzzy closed in Y . Since g is perfectly fuzzy #rg -continuous, and therefore g^{-1} (μ) is fuzzy #rg -open set in Y . Also since f is perfectly fuzzy #rg -continuous f^{-1} (g^{-1} (μ)) = (g ∘ f)$^{-1}$ (μ) is both fuzzy open and fuzzy closed set in X . Hence g ∘ f is perfectly fuzzy #rg -continuous function.

Theorem 3.16. Let f : X → Y is perfectly fuzzy #rg -continuous and g : Y → Z is fuzzy #rg -irresolute function then g ∘ f : X → Z is perfectly fuzzy #rg -continuous function.

Proof Let μ be fuzzy #rg -open set in Z . And then g−^{1}(μ) is fuzzy #rg - open set in Y . Since g is fuzzy #rg -irresolute function. Also since f is perfectly fuzzy #rg -continuous. f^{-1} (g^{-1} (μ)) = (g ∘ f)$^{-1}$ (μ) is both fuzzy open and fuzzy closed set in X . Hence g∘f is perfectly fuzzy #rg -continuous function.

Definition 3.3. A map f : X → Y is said to be completely fuzzy #rg - continuous if the inverse image of every fuzzy #rg -open set in Y is fuzzy regular open set in X .

Theorem 3.17. A map f : X → Y is completely fuzzy #rg -continuous iff the inverse image of every fuzzy #rg -closed set in Y is fuzzy regular closed set in X .

Proof Assume that f is completely fuzzy #rg -continuous. Let λ be fuzzy #rg -closed set in Y . Then 1 − λ is fuzzy #rg -open set in Y . And therefore

f^{-1}(1−λ) is both fuzzy regular open set in X . But f^{-1}(1 − λ) =1−f^{-1}(λ) and so f^{-1}(λ) is both fuzzy regular closed set in X .

Conversely, assume that the inverse image of every fuzzy #rg -closed set in Y is fuzzy regular closed set in X . Let μ be fuzzy #rg -open set in Y . Then 1 − μ is fuzzy #rg -closed set in Y . By hypothesis, f^{-1}(1 −μ) is fuzzy regular closed set in X . Now f^{-1}(1 − μ) = 1 − f^{-1}(μ) . And therefore f^{-1}(μ) is fuzzy regular open set in X . Hence f is completely fuzzy #rg -continuous function.

Theorem 3.18. Every completely fuzzy #rg -continuous function is a f - continuous function.

Proof Let f : X → Y be completely fuzzy #rg -continuous function. Let μ be fuzzy open set in Y . Then μ is fuzzy #rg -open set in Y . And then f^{-1}(μ) is both fuzzy regular open set in X , and therefore f^{-1}(μ) is fuzzy open set in X .

Hence f is f -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6 In the Example 3.1, then f is f -continuous but not completely fuzzy #rg -continuous as the fuzzy set μ is fuzzy closed in Y and f^{-1}(μ) = μ is not fuzzy regular open set in X but fuzzy open set in X . Hence f is f -continuous.

Theorem 3.19. Every completely fuzzy #rg -continuous function is a f - completely continuous function.

Proof Let f : X → Y be completely fuzzy #rg -continuous. Let μ be fuzzy open set in Y . Then μ be fuzzy #rg -open set in Y . And then f^{-1}(μ) is fuzzy regular open set in X . Hence f is f -completely continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7 In the Example 3.4, then f is f -completely continuous but not completely fuzzy #rg -continuous as the fuzzy set α is fuzzy #rg -closed in Y and f^{-1}(α) = α is not fuzzy regular open set in X . Hence f is not completely fuzzy #rg -continuous.

Theorem 3.20. Every completely fuzzy #rg -continuous function is strongly fuzzy #rg -continuous function.

Proof Let f : X → Y be completely fuzzy #rg -continuous. Let μ be fuzzy #rg -open set in Y . And then f^{-1}(μ) is fuzzy regular open set in X . And therefore f^{-1}(μ) is fuzzy

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open set in X . Hence f is strongly fuzzy #rg -continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.8 In the Example 3.2, then f is strongly fuzzy #rg -continuous but not completely fuzzy #rg -continuous as the fuzzy set μ is fuzzy #rg - closed in Y and $f^{-1}(\mu) = \mu$ is not fuzzy regular open set in X but fuzzy closed set in X Hence f is strongly fuzzy #rg -continuous.

Theorem 3.21. If f : X → Y is f -completely continuous and Y T #rg - is fuzzy space. Then f is completely fuzzy #rg -continuous function.

Proof Let μ be fuzzy #rg -open set in Y . Then μ is fuzzy open set in Y , since Y is fuzzy $T^{\#rg}$ -space. Also since f is f -completely continuous, $f^{-1}(\mu\ )$ is fuzzy regular open set in X . And hence f is completely fuzzy #rg -continuous function.

Theorem 3.22. Let f : X → Y is completely fuzzy #rg -continuous and g : Y → Z is fuzzy #rg -irresolute functions then g ∘f : X → Z is completely fuzzy #rg -continuous functions.

Proof Let μ be fuzzy #rg -open set in Z . And then $g^{-1}(\mu)$ is fuzzy #rg - open set in Y . Since g is fuzzy #rg -irresolute functions. Also since f is completely fuzzy #rg -continuous. And then $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu\ )$ is fuzzy regular open set in X. Hence g ∘ f is completely fuzzy #rg -continuous functions.

Theorem 3.23. If f : X → Y and g : Y → Z be two completely fuzzy #rg -continuous functions then g ∘ f : X → Z is completely fuzzy #rg - continuous functions.

Proof Let μ be fuzzy #rg -open set in Z . Then $g^{-1}(\mu)$ is fuzzy regular-open set in Y . Since g is completely fuzzy #rg -continuous. And so $g^{-1}(\mu\ )$ is fuzzy open set and then fuzzy #rg -open set in Y . Also since f is completely fuzzy #rg -continuous functions, $f^{-1}(g^{-1}(\mu\ )) = (g \circ f)^{-1}(\mu\ )$ is fuzzy regular-open set in X . And hence g ∘f is completely fuzzy #rg -continuous function.

Theorem 3.24. If f : X → Y , g : Y → Z completely fuzzy #rg -continuous functions and g fuzzy #rg -continuous then g ∘ f : X → Z is f -continuous functions.

Proof Let μ be fuzzy open set in Z . Then $g^{-1}(\mu)$ is fuzzy #rg -open set in Y . Since g is fuzzy #rg -continuous. And since f is completely fuzzy #rg -continuous, $f^{-1}(g^{-1}(\mu\ ))$ is fuzzy regular open set in X . That is $(g\circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ is fuzzy open set in X . Hence g∘f is f -continuous functions.

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