Intuitionistic Q-Fuzzy Strong Bi-ideals of Near-rings

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Abstract : **In this paper, we introduce intuitionistic Q-fuzzy strong bi-ideals of a near-rings and investigate some of their related properties.**

Keywords — Fuzzy Bi-ideal, Q-Fuzzy Bi-ideals, Intuitionistic fuzzy bi-ideals and Intuitionistic Q-fuzzy strong bi-ideals.

I. INTRODUCTION

In 1965, Zadeh [13] took a collection of objects which fail to satisfy the condition of a set and he called it as fuzzy set. After the emergence of fuzzy concept, it evolved in all branches of Science and Engineering and Rosenfeld [10] the one who first used this idea in group theory. As a generalization Liu [6] extended this fuzzy concept to ideal theory in ring. Abou-Zaid [1] fuzzified near-ring and applied it to ideals. One of the important generalization of fuzzy set is intuitionistic fuzzy set which was introduced by Atanassov [2].

Biswas [3] introduced intuitionistic fuzzy subgroups of a group. Jun, Kim and Yon [5] studied intuitionistic fuzzy R-subgroups of near-rings. Fuzzy bi-ideals of near-ring is presented in [9]. In this paper, we come across IQFS (in briefly intuitionistic Q-fuzzy strong) bi-ideal of a near-ring and obtain the characterization of a strong bi-ideal in terms of a IQFS- bi-ideal of a near-ring.

II. PRELIMINARIES

A nonempty set N together with two binary operations "$+$" and "$.$" is said to be a near-ring, if

- N is a group under the operation "$+$", it is a semigroup under the operation "$.$" and "$.$" right distributive over addition.
- Let A and B be two subsets of N, the product of A and B is defined as $AB = \{ab / a \in A, b \in B\}$. Also we define another product "$*$" as $A * B = \{a(b+i) - ab / a, b \in A, i \in B\}$.
- $0x = 0$. In general $x0 \neq 0$, for some x in N. If $x0 = 0$, for all x in N, then N is zero symmetric. A subgroup A of $(N,+)$ is called a bi-ideal of near-ring N if $ANA \cap (AN) * A \subseteq A$.

A function A from a non-empty set X to the unit interval $[0,1]$ is called a fuzzy subset of N. Let Q be a non-empty set. An intuitionistic Q-fuzzy subset μ of N is an object having the form $\mu = \{(x, A_\mu(x,q), B_\mu(x,q)) / x \in N\}$, where the functions $A_\mu : N \times Q \to [0,1]$ and $B_\mu : N \times Q \to [0,1]$ denote the degree of membership and degree of non membership of each element $x \in N$ to the set μ, respectively, and $0 \le A_\mu(x,q) + B_\mu(x,q) \le 1$ for all $x \in N$. In brief, we shall use the symbol $\mu = (A_\mu, B_\mu)$ for the intuitionistic fuzzy subset $\mu = \{(x, A_\mu(x,q), B_\mu(x,q)) / x \in X\}$. For the definition of the Q-fuzzy subgroup, subnear-ring and intuitionistic Q-fuzzy ideals of near-ring refer [7].

Let $\mu = (A_\mu, B_\mu)$ and $\lambda = (A_\lambda, B_\lambda)$ be two intuitionistic fuzzy subsets of N. We define an intuitionistic Q-fuzzy "$*$" product in near-ring as follows.

$$(\mu * \lambda)(x,q) = \begin{cases} \{ \sup\limits_{x=a(b+i)-ab} \{\min\{A_\mu(a,q), A_\mu(b,q), B_\mu(i,q)\}\}, \\ \inf\limits_{x=a(b+i)-ab} \{\max\{A_\lambda(a,q), A_\lambda(b,q), B_\lambda(i,q)\}\}\}; \\ \quad if\ x = a(b+i) - ab, a, b, i \in N \\ and\ q \in Q. \\ \quad [0,1], \quad Otherwise. \end{cases}$$

III. IQFS- BI-IDEAL OF NEAR-RING

Definition 3.1 An intuitionistic Q-fuzzy bi-ideal $\mu = (A_\mu, B_\mu)$ of N is called an IQFS- bi-ideal of N, if

(i) $N \circ A_\mu \circ A_\mu \subseteq A_\mu$

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(ii) $N \circ B_\mu \circ B_\mu \supseteq B_\mu$

Example 3.2 Let $N = \{0, x, y, z\}$ be the klein's four group. Define "$+$" and "$\cdot$" is defined as follows.

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

·	0	x	y	z
0	0	0	0	0
x	0	0	x	0
y	0	0	y	0
z	0	0	z	0

Define a fuzzy subset $\mu = (A_\mu, B_\mu)$ where

$A_\mu : N \times Q \to [0,1]$ by

$A_\mu(0,q) = 0.8$ $A_\mu(x,q) = 0.6$ $A_\mu(y,q) = 0.3 = A_\mu(z,q)$

. Then

$(A_\mu \circ N \circ A_\mu)(0,q) = 0.3,\ (A_\mu \circ N \circ A_\mu)(x,q) = 0.3,$

$(A_\mu \circ N \circ A_\mu)(y,q) = 0.3,\ (A_\mu \circ N \circ A_\mu)(z,q) = 0.3,$

$(N \circ A_\mu \circ A_\mu)(0,q) = 0.3,\ (N \circ A_\mu \circ A_\mu)(x,q) = 0.3,$

$(N \circ A_\mu \circ A_\mu)(y,q) = 0.3,\ (N \circ A_\mu \circ A_\mu)(z,q) = 0.3,$

and so A_μ is an IQFS-bi-ideal of N and

$B_\mu : N \times Q \to [0,1]$ by

$B_\mu(0,q) = 0.2,\ B_\mu(x,q) = 0.7,\ B_\mu(y,q) = 0.9 = B_\mu(z,q)$

. Then

$B_\mu \circ N \circ B_\mu(0,q) = 0.9,\ B_\mu \circ N \circ B_\mu(x,q) = 0.9,$

$B_\mu \circ N \circ B_\mu(y,q) = 0.9,\ (B_\mu \circ N \circ B_\mu)(z,q) = 0.9,$

$(N \circ B_\mu \circ B_\mu)(0,q) = 0.9,\ (N \circ B_\mu \circ B_\mu)(x,q) = 0.9,$

$(N \circ B_\mu \circ B_\mu)(y,q) = 0.9,\ (N \circ B_\mu \circ B_\mu)(z,q) = 0.9$

,

and so B_μ is an IQFS-bi-ideal of N. Thus $\mu = (A_\mu, B_\mu)$ is an IQFS-bi-ideal of N.

Theorem 3.3. Let $\mu_i = \{(A_{\mu_i}, B_{\mu_i}) : i \in I\}$ be any collection of IQFS-bi-ideals in a near-ring N. Then $\cap_{i \in I} \mu_i$ is an IQFS-bi-ideal of N where $\cap_{i \in I} \mu_i = \{(\cap_{i \in I} A_{\mu_i}, \cup_{i \in I} B_{\mu_i})\}$.

Proof. Let $\{\mu_i : i \in I\}$ be any collection of IQFS-bi-ideals of N.

Now for all $x, y \in N$ and $q \in Q$,

$\cap_{i \in I} A_{\mu_i}(x-y,q) = min\{A_{\mu_i}(x-y,q) / i \in I, q \in Q\}$

$\geq min\{min\{A_{\mu_i}(x,q), A_{\mu_i}(y,q)\ i \in I, q \in Q\}\}$

(since A_{μ_i} is an IQF-subgroup of N.)

$= min\{\cap_{i \in I} A_{\mu_i}(x,q), \cap_{i \in I} A_{\mu_i}(y,q) / i \in I,$

$q \in Q\} \cup_{i \in I} B_{\mu_i}(x-y,q)$

$= max\{B_{\mu_i}(x-y,q) / i \in I, q \in Q\} \leq max\{max\{B_{\mu_i}(x),$

$B_{\mu_i}(y,q)\} / i \in I, q \in Q\}$

(since B_{μ_i} is an IQF-subgroup of N)

$= max\{\cup B_{\mu_i}(x,q), \cup_{i \in I} B_{\mu_i}(y,q) / i \in I\}$

Therefore $\cap_{i \in I} \mu_i$ be an IQF-subgroup of N.

To prove: $\cap_{i \in I} \mu_i$ be an IQF-bi-ideal of N.

Since $A_\mu = \cap_{i \in I} A_{\mu_i} \subseteq A_{\mu_i}$, for every $i \in I$, we have for all $x \in N$.

$((A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu))(x,q)$

$\leq ((A_{\mu_i} \circ N \circ A_{\mu_i}) \cap ((A_{\mu_i} \circ N) * A_{\mu_i}))(x,q)$

(since A_{μ_i} be an IQF-bi-ideal of N).

$\leq A_{\mu_i}(x,q)$ for every $i \in I$ and $q \in Q$.

It follows that

$((A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu))(x,q)$

$= inf\{A_{\mu_i}(x,q) : i \in I\}$

$= (\cap_{i \in I} A_{\mu_i})(x,q)$

$= A_\mu(x,q)$

Thus $((A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu) \subseteq A_\mu$

Since $B_\mu = \cup_{i \in I} B_{\mu_i} \supseteq B_{\mu_i}$ for some $i \in I$, we have for all $x \in N$.

$((B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu))(x,q)$

$\geq ((B_{\mu_i} \circ N \circ B_{\mu_i}) \cup ((B_{\mu_i} \circ N) * B_{\mu_i}))(x,q)$

$\geq B_{\mu_i}(x,q)$ for some $i \in I$ and $q \in Q$.

It follows that

$((B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu))(x,q)$

$\geq sup\{B_{\mu_i}(x,q) : i \in I, q \in Q\}$

$= \cup_{i \in I} B_{\mu_i}(x,q)$

$= B_\mu(x,q).$

Thus $((B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu)) \supseteq B_\mu$

Thus $\cap_{i \in I} \mu_i$ be an IQF-bi-ideal of N.

To prove: $\cap_{i \in I} \mu_i$ be an IQFS-bi-ideal of N.

Now for all $x \in N$, since $A_\mu = \cap_{i\in I} A_{\mu_i} \subseteq A_{\mu_i}$, for every $i \in I$, we have

$$(N \circ A_\mu \circ A_\mu)(x,q) \quad \leq \quad (N \circ A_{\mu_i} \circ A_{\mu_i})(x,q)$$
$$\leq \quad A_{\mu_i}(x,q) \ \ for\ every\ i \in I, q \in Q.$$

It follows that,

$$(N \circ A_\mu \circ A_\mu)(x,q) \quad \leq \quad inf\{A_{\mu_i}(x) : i \in I, q \in Q\}$$
$$= \quad (\cap_{i\in I} A_{\mu_i})(x,q)$$
$$= \quad A_\mu(x,q).$$

Thus $N \circ A_\mu \circ A_\mu \subseteq A_\mu$.

Now for all $x \in N$, since $B_\mu = \cup_{i\in I} B_{\mu_i} \supseteq B_{\mu_i}$, for some $i \in I, q \in Q$, we have

$$(N \circ B_\mu \circ B_\mu)(x,q) \quad \geq \quad (N \circ B_{\mu_i} \circ B_{\mu_i})(x,q)$$
$$\geq \quad B_{\mu_i}(x,q) \ \ for\ every\ i \in I, q \in Q.$$

It follows that,

$$(N \circ B_\mu \circ B_\mu)(x,q) \quad \geq \quad sup\{B_{\mu_i}(x,q) : i \in I, q \in Q\}$$
$$= \quad (\cup_{i\in I} B_{\mu_i}(x,q))$$
$$= \quad B_\mu(x,q).$$

Thus $N \circ B_\mu \circ B_\mu \supseteq B_\mu$.

$\cap_{i\in I} \mu_i$ be an IQFS-bi-ideal of N.

Theorem 3.4 Every IQF-left N-subgroup of N is an IQFS-bi-ideal of N.

Proof. Let $\mu = (A_\mu, B_\mu)$ be an IQF-left N-subgroup of N.

To prove: μ is an IQFS-bi-ideal of N.

First we prove: μ is an IQF-bi-ideal of N.

Choose $l,m,n,s,t,i,n_1,n_2,s_1,s_2,t_1,t_2$ in N such that $l = mn = s(t+i) - st, n = n_1n_2, s = s_1,s_2$ and $t = t_1t_2$.

Then

$$((A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu))(l,q) = min\{(A_\mu \circ (N \circ A_\mu))(l,q), ((A_\mu \circ N) * A_\mu)(l,q)\}$$

$$= min\{\sup_{l=mn} min\{A_\mu(m,q), (N \circ A_\mu)(n,q)\},$$
$$((A_\mu \circ N) * A_\mu)(s(t+i) - st)\}$$

$$= \quad min\{\sup_{l=mn} min\{A_\mu(m,q), \ \sup_{n=n_1n_2} min\{N(n_1),$$
$$A_\mu(n_2)\}, ((A_\mu \circ N) * A_\mu)(s(t+i) - st)\}\}$$

$$= \quad min\{\sup_{l=mn} min\{A_\mu(m,q), \ \sup_{n=n_1n_2} min\{N(n_1),$$
$$A_\mu(n_2)\}, ((A_\mu \circ N) * A_\mu)(s(t+i) - st)\}\}$$

$$\leq min\{\sup_{l=mn} min\{N(m,q), A_\mu(mn,q)\},$$
$$N(s(t+i) - st)\}$$
$$= A_\mu(mn,q) = A_\mu(l,q)$$

Thus $(A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu) \subseteq A_\mu$.

Choose $l,m,n,s,t,i,m,n_1,n_2,s_1,s_2,t_1,t_2$ in N such that $(l,q) = (mn,q)$

$= s(t+i) - st, n = n_1,n_2, s = s_1s_2$ and $t = t_1t_2$

Then

$$(B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu)(l,q)$$
$$= max\{(B_\mu \circ N \circ B_\mu)(l,q), ((B_\mu \circ N) * B_\mu)(l,q)\}$$

$$= max\{\inf_{l=mn} max\{(B_\mu(m,q), (N \circ B_\mu)(n,q))\},$$
$$((B_\mu \circ N) * B_\mu)(s(t+i) - st)\}$$

$$= max\{\inf_{l=mn} max\{B_\mu(m,q), \ \inf_{n=n_1n_2} max\{N(n_1), B_\mu(n_2)\}\},$$
$$((B_\mu \circ N) * B_\mu)(s(t+i) - st, q)\}$$
$$= max\{\inf_{l=mn} max\{B_\mu(m,q), \ \inf_{n=n_1n_2} B_\mu(n_2)\},$$
$$((B_\mu \circ N) * B_\mu)((s(t+i) - st), q)\}$$
$$\geq max\{\inf_{l=mn} max\{N(m,q), B_\mu(mn,q)\},$$
$$N(s(t+i) - st)\} = B_\mu(st,q) = B_\mu(l,q)$$

Thus $(B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu) \supseteq B_\mu$.

Next we prove: μ is an IQFS-bi-ideal of N.

Choose $l,m,n,n_1,n_2 \in N$ and $q \in Q$ such that $l = mn$ and $n = n_1n_2$. Then

$$(N \circ A_\mu \circ A_\mu)(l,q)$$
$$= \quad \sup_{l=mn} min\{N(m,q), (A_\mu \circ A_\mu)(n,q)\}$$
$$= \quad \sup_{l=mn} min\{N(m,q), \ \sup_{n=n_1n_2} min\{A_\mu(n_1,q), A_\mu(n_2,q)\}\}$$
$$= \quad \sup_{l=mn} min\{1, \ \sup_{n=n_1n_2} min\{A_\mu(n_1,q), A_\mu(n_2,q)\}\}$$
$$= A_\mu(mn_1n_2,q)$$
$$= \quad A_\mu((mn_1)n_2,q)$$
$$> A_\mu(n_2,q)$$
$$\leq \quad \sup_{l=mn} min\{N(n_1,q), A_\mu(mn,q)\}$$
$$= \quad \sup_{l=mn} min\{1, A_\mu(mn,q)\}$$
$$= \quad A_\mu(mn,q)$$
$$= \quad A_\mu(l,q)$$

Therefore $N \circ A_\mu \circ A_\mu \subseteq A_\mu$.

Choose $l, m, n, n_1, n_2 \in N$ and $q \in Q$ such that $l = mn$ and $n = n_1, n_2$. Then

$$(N \circ B_\mu \circ B_\mu)(l,q) = \inf_{l=mn} max\{(N(m), B_\mu \circ B_\mu)(n,q)\}$$
$$= \inf_{l=mn} max\{B_\mu(n_1), B_\mu(n_2)\}$$

$$B_\mu(mn,q) = B_\mu(mn_1n_2,q)$$
$$= B_\mu((mn_1)n_2,q) \le B_\mu(n_2,q))$$
$$\ge \inf_{l=mn} max\{N(n_1,q), B_\mu(mn,q)\}$$
$$= \inf_{l=mn} max\{0, B_\mu(mn)\}$$
$$= B_\mu(mn,q)$$
$$= B_\mu(l,q)$$

Thus $\mu = (A_\mu, B_\mu)$ be an IQFS-bi-ideal of N.

Theorem 3.5. Every IQF-left ideal of N is an IQFS-bi-ideal of N.

Proof. Let $\mu = (A_\mu, B_\mu)$ is an IQF-left ideal of N.

To prove: μ is an IQFS-bi-ideal of N.

First we prove: μ is an IQF-bi-ideal of N.

Choose $l, m, n, s, t, i, m_1, m_2, s_1, s_2, t_1, t_2$ in N and $q \in Q$ such that

$$l = mn = (s(t+i) - st, q), m = (m_1m_2, q), s = s_1s_2$$

and $t = t_1t_2$. Then

$$((A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu))(l,q) = min\{A_\mu \circ N \circ A_\mu(l,q), ((A_\mu \circ N) * A_\mu)(l,q)\}$$
$$= min\{\sup_{l=mn} min\{(A_\mu \circ N)(m,q), A_\mu(n,q)\},$$
$$((A_\mu \circ N) * A_\mu)(s(t+i) - st, q)\}$$
$$= min\{\sup_{l=mn} min\{(A_\mu \circ N)(m_1m_2), A_\mu(n,q),$$
$$\sup_{l=s(t+i)-st} min((A_\mu \circ (s), (A_\mu \circ N)(t,q), A(i,q)))\}\}$$

(Since, $A_\mu((s(t+i) - st), q) \ge (A_\mu(i,q)))$

$$\le min\{\sup_{l=mn} min\{N(m_1m_2,q), N(n,q)\}\},$$
$$\sup_{l=s(t+i)-st} min\{N(s,q), N(t,q), A_\mu((s(t+i) - st), q)\}$$
$$= A_\mu((s(t+i) - st), q) = A_\mu(l,q)$$

Thus $(A_\mu \circ N \circ A_\mu) \cap ((A_\mu \circ N) * A_\mu) \subseteq A_\mu$.

Choose $l, m, n, s, t, i, m_1, m_2, s_1, s_2, t_1, t_2$ in N and $q \in Q$ such that $l = mn = (s(t+i) - st, q)$ $m = m_1m_2, s = s_1s_2$ and $t = t_1t_2$. Then

$$((B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu))(l,q)$$
$$= max\{B_\mu \circ N \circ B_\mu(l,q),$$
$$((B_\mu \circ N) * B_\mu)(l,q)\}$$
$$= max\{\inf_{l=mn} max(B_\mu \circ N)(m,q), B_\mu(n,q),$$
$$((B_\mu \circ N) * B_\mu)((s(t+i) - st), q)\}$$
$$= max\{\inf_{l=mn} max(B_\mu \circ N)(m_1m_2,q), B_\mu(n,q)\},$$
$$\inf_{l=s(t+i)-st} max\{(B_\mu \circ N)(s,q), (B_\mu \circ N)(t,q), B_\mu(i,q)\}$$
$$B_\mu(s(t+i) - st) \le B_\mu(i,q)) \ge$$
$$max\{\inf_{l=mn} max(N(m_1m_2), N(n,q)\},$$
$$\inf_{l=s(t+i)-st} max\{N(s,q), N(t,q)\}$$
$$B_\mu(s(t+i) - st) \le B_\mu(i,q))\}.$$
$$= B_\mu(s(t+i) - st, q) = B_\mu(l,q).$$

Therefore $(B_\mu \circ N \circ B_\mu) \cup ((B_\mu \circ N) * B_\mu) \supseteq B_\mu$.

Thus $\mu = (A_\mu, B_\mu)$ be IQF-bi-ideal of N.

Next we prove: μ be an IQFS-bi-ideal of N.

Choose $a, b, c, b_1, b_2 \in N$ and $q \in Q$ such that $a = bc = ((b(n+c) - bn), q)$. Then

$$(N \circ A_\mu \circ A_\mu)(a,q)$$
$$= \sup_{a=bc} min\{(N \circ A_\mu)(b,q), A_\mu(c,q)\}$$
$$= \sup_{a=bc}\{min\{\sup_{b=b_1b_2} N(b_1), A_\mu(b_2), q)\},$$
$$A_\mu(c,q)\}$$
$$= \sup_{a=bc}\{min\{\sup_{b=b_1b_2} A_\mu(b_2), q)\}, A_\mu(c,q)\}$$
$$A_\mu(l,q) = A_\mu(mn,q)$$
$$= A_\mu((b(n+c) - bn), q) > A_\mu(c,q)$$

and $N(b_2 \ge A_\mu(b_2)))$

$$\le \sup_{a=bc}\{min\{N(b_2), A_\mu((b(n+c) - bn), q)\},$$
$$= A_\mu(bc,q) = A_\mu(a,q).$$

Therefore $N \circ A_\mu \circ A_\mu \subseteq A_\mu$.

Choose $a, b, c, b_1, b_2 \in N, q \in Q$ such that $(a,q) = (bc,q)$ and $(b,q) = (b_1b_2,q)$.

Then,

$$(N \circ B_\mu \circ B_\mu)(a,q)$$
$$= \inf_{a=bc} max\{(N \circ B_\mu)(b,q), B_\mu(c,q)\}$$
$$=$$
$$\inf_{a=bc} max\{\inf_{b=b_1b_2} max\{N(b_1,q), B_\mu(b_2,q), B_\mu(c,q)\}$$
$$= \inf_{a=bc} max\{\inf_{b=b_1b_2} \{B_\mu(b_2,q), B_\mu(c,q)\}\}$$

$$B_\mu(a,q) = B_\mu(bc,q)$$
$$= B_\mu(b(n+c)-bn) \leq B_\mu(c,q)$$

and

$$= \inf_{a=bc} max\{N(b_2,q), B_\mu(b(n+c)-bn)\}$$
$$= \inf_{a=bc} max\{0, B_\mu(bc,q)\}$$
$$= B_\mu \text{ (bc,q)}$$
$$= B_\mu \text{ (a,q)}$$

Therefore $(N \circ B_\mu \circ B_\mu) \supseteq B_\mu$.

Thus $\mu = (A_\mu, B_\mu)$ is an IQFS-bi-ideal of N.

Theorem 3.6. Let $\mu = (A_\mu, B_\mu)$ be any IQFS-bi-ideal of a near ring N. Then A_μ (lst,q) ≥ min{ A_μ (s,q), A_μ (t,q) *and* B_μ (lst,q) ≤ max{ B_μ (s,q), B_μ (t,q) } for all l,s,t∈ N *and* q∈Q.

Proof. Assume that (A_μ, B_μ) N. Then $N \circ A_\mu \circ A_\mu \subseteq A_\mu$ *and* $(N \circ B_\mu \circ B_\mu) \supseteq B_\mu$. .*Let* l,s,t *and* q ∈Q be any element N. Then A_μ (lst,q) ≥ $(N \circ A_\mu \circ A_\mu)$ (lst,q)

$$= \sup_{lst=pq} min\{(N \circ A_\mu)(p), A_\mu(q)\}$$
$$\geq \min (N \circ A_\mu \circ A_\mu) \text{ (ls,q)}, \; A_\mu \text{ (t,q)}$$

$$= min\{ \sup_{ls=z_1z_2} min\{N(z_1,q), A_\mu(z_2,q)\}, A_\mu(t,q)\}$$
$$\geq \min\{\min\{N(l,q), \; A_\mu \text{ (s,q)}, \; A_\mu \text{ (t,q)}$$
$$= min\{min\{1, A_\mu(s,q), A_\mu(t,q)\}\}$$
$$= min\{A_\mu(s,q), A_\mu(t,q)\}$$

This shows that $A_\mu(lst,q) \geq min\{A_\mu(s,q), A_\mu(t,q)\}$ for all s,t∈N *and*

$$B_\mu(lst,q) \leq (N \circ B_\mu \circ B_\mu)(lst,q)$$
$$= \inf_{lst=pq} max\{(N \circ B_\mu)(p), B_\mu(q)\}$$
$$= max\{(N \circ B_\mu)(ls,q), B_\mu(t,q)\}$$

$$= \max\{ \inf_{ls=z_1z_2} \max\{N(z_1,q), B_\mu(z_2,q)\}, B_\mu(t,q)\}$$

$$\leq max\{max\{N(l), B_\mu(s,q)\}, B_\mu(t,q)\}$$
$$= \max\{\max\{1, \; B_\mu \text{ (s,q)}, \; B_\mu \text{ (t,q)}\}$$
$$= max\{A_\mu(s,q), B_\mu(t,q)\}$$

This shows that

$B_\mu(lst,q) \leq max\{B_\mu(s,q), B_\mu(t,q)\} \; \forall \, l,s,t \in N$ and $q \in Q$.

IV. Conclusion

In this article, the notion of IQFS- bi-ideals of a near-rings and derived some properties of these ideals.

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