A Study on P - Fuzzy Near Rings and Its Properties

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Abstract In this paper, we define the new concept of P – Fuzzy near ring on them Algebra A and study the relation between P – Fuzzy near ring and P – Fuzzy subalgebra on the Algebra A, Also we state and prove the properties of P – Fuzzy near rings.

Keywords — P – Fuzzy set, P – Fuzzy Group, P– Fuzzy subalgebra, P – Fuzzy rings, Fuzzy near rings and P – Fuzzy near rings.

I. INTRODUCTION

In 1965 Zadeh [144] mathematically formulated the fuzzy subset concept. He defined fuzzy subset of a non-empty set as a collection of objects with grade of membership in a continum, with each object being assigned a value between 0 and 1 by a membership function. Fuzzy set theory was guided by the assumption that classical sets were not natural, appropriate or useful notions in describing the real life problems, because every object encountered in this real physical world carries some degree of fuzziness.

Further the concept of grade of membership is not a probabilistic concept. A fuzzy set A is defined as a map from A to the real unit interval I = [0,1]. It has grown by leaps and bounds and innumerable numbers have appeared in various journals. Applications of fuzzy sets and fuzzy logic were introduced by Mamdani in 1975. A group is an algebraic system with one binary operation. Generalizing Z along with two operations + and .using the properties distributive and associative under . , the concept of Ring arises.

The definitions given to the concept of P – Fuzzy set, P – fuzzy subalgebra on the algebra A and fuzzy rings also involve different operations. Now we unify and generalize these definitions using P – fuzzy Birings. The theorems proved also highly generalize the existing ones.

II. PRELIMINARIES

Definition 3.1 If $(P, \leq)$ is a partially ordered set and X is a nonempty set, then any mapping $\mu : A \to P$ is a P-fuzzy subset of A or a P-fuzzy set on A, denoted by P^A

Definition 3.2. Let A be a non-empty set and $P = (P, *, 1, \leq)$ a $(2, 0)$ type ordered algebra, i.e. Let

i) $(P, *)$ be a monoid, where 1 is the unity for $*$
ii) $(P, \leq)$be a Partially ordered set aith 1 as the greatest element.
iii) $*$be isotone in both variables.

P always denotes such a structure.

Definition 3.3. A P – fuzzy set $\mu \in P^A$ is called a P – fuzzy algebra or fuzzy subalgebra on the algebra A, if

- For any n – ary($n \geq 1$) operation $f \in F$
  $\mu(f(x_1 \ldots\ldots\ldots x_n)) \geq \mu(x_1) * \ldots\ldots\ldots\ldots * \mu(x_n)$
  for all $x_1 \ldots\ldots\ldots x_n \in A$
- For any constant (nullary operation) C
  $\mu(c) \geq \mu(x)$for all $x \in A$.
  (Note: If A is a group then nullary operation is consequence of $n - ary$ operation)

Definition 3.4. Let G be a group. A fuzzy subset μ of a group G is called a fuzzy subgroup of the group G if

- $\mu(xy) \geq min\ (\mu(x), \mu(y))$ for every$x, y \in G$ and
- $\mu\ (x^{-1}) = \mu\ (x)$ for every $x \in G$.

Definition 3.5. A fuzzy subset μ of a ring R is called a Fuzzy subring of R if for all $x, y \in R$ then

- $\mu(x - y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy) \geq min\ (\mu(x), \mu(y))$ for all $x, y \in R$.

Proposition 3.6. Let μ be a fuzzy subring of $(R, +, .)$ then

- μ is a fuzzy subgroup of $(R, +)$
- μ is a fuzzy subgroupoid of $(R,\ .)$

III. FUZZY NEAR - RING

Definition 4.1. Let R be a Near-Ring and N a fuzzy set in R. Then N is called a Fuzzy Near-Ring in R if

- $N(x + y) \geq \min(N(x), N(y))$
- $N(-x) \geq N(x)$
- $N(xy) \geq \min\{N(x), N(y)\}$for all$x, y \in R$

Definition 4.2. Let R be a Near-Ring and N a Fuzzy Near-Ring in R. Let Y be a Near -Ring module over R and M a fuzzy set in Y. Then M is called a Fuzzy Near - Ring module in Y if

(i) $M(x + y) \geq \min\{M(x), M(y)\}$
(ii) $M(\lambda x) = \min\{N(\lambda), M(x)\}$ for all $x, y \in Y, \lambda \in R$
(iii) $M(0) = 1$

If N is an ordinary near ring then the condition (ii) is replaced by (ii)

$M(\lambda x) > M(x) for\ all\ \lambda \in N\ and\ for\ all\ x \in Y$.

Theorem 4.1 Let Y be a Near Ring module over a fuzzy near – ring N in R. Then M is a Fuzzy Near – Ring Module in Y if and only if $M(\lambda(x) + \mu(x)) \geq \min\{\min\{N(\lambda), M(x)\}, \min\{N(\mu), M(y)\}\}$ for all $\lambda, \mu \in N\ and\ for\ all\ x, y \in Y$.

If N is an ordinary Near – Ring then the above condition is replaced by $M(\lambda x + \mu y) \geq \min\{M(x), N(y)\}$ for all $x, y \in Y$.

Theorem 4.2 Let Y be a Near Ring module over a near – ring R with identity. If M is a P - Fuzzy Near – Ring Module in Y if λ∈ R is invertible then $M(\lambda x) = M(x)$ for all $x \in Y$.

Proof: If λ∈ M is invertible then for all x ∈Y, sine Y is a Near Ring Module,

$M(x) = M(\lambda - 1\ \lambda x) \geq M(\lambda x) \geq M(x)$ *and so* $M(\lambda x) = M(x)$.

Definition 4.3. A P - fuzzy subset $\mu \in P^A$ is called a P - Fuzzy subring of $R$of the algebra A if for all $x, y \in R$ then

- $\mu(x - y) \geq \min(\mu(x), \mu(y))$
- $\mu(xy) \geq min\ (\mu(x), \mu(y))$ for all $x, y \in R$

Definition 4.4. A non – empty set (R, +, .) with two binary operations '+' and '.' is called P – Fuzzy biring if $R = R_1 \cup R_2$ where R_1 and R_2 are proper subsets of R and

- $(R_1, +, .)$ is a P - Fuzzy ring
- $(R_2, +, .)$ is a P - Fuzzy ring

Note: A P – Fuzzy biring R is called finite if R contains only finite number of elements. If R has infinite number of elements then R is of infinite order.

Definition 4.5. A P – Fuzzy biring $R = R_1 \cup R_2$ is called P – Fuzzy commutative biring if both R_1 and R_2 are commutative rings. Even if one of R_1 or R_2 is not a commutative biring then the biring is a non – commutative biring. Also the biring R has a monounit if a unit exists which is common to both R_1 and R_2. If R_1 and R_2 are rings which has separate unit then the biring $R = R_1 \cup R_2$ is a bi-ring with unit.

Definition 4.6. Let $R = (R_1 \cup R_2, +, .)$ be a biring. The map μ: R→ [0, 1] is called P – Fuzzy subbiring of the ring R if there exists two fuzzy subsets μ_1(of R_1) and μ_2(of R_2) such that

- $(\mu_1, +, .)$ is a P – Fuzzy Subring of $(R_1, +, .)$
- $(\mu_2, +, .)$ is a P – Fuzzy Subrinng of $(R_2, +, .)$
- $\mu = \mu_1 \cup \mu_2$

IV. PROPERTIIES OF P - FUZZY NEAR RING

Definition 5.1: A set N together with two binary operation + and . is called a P - near – ring if:

1. N is a P - Group under addition
2. Multiplication is associative in P
3. Multiplication on the P - right distributes over addition.

Similarly, to define a left P - near – ring by replacing the right distributive law.

Definition 5.2. Let R be a P - Near-Ring and A be a fuzzy set in R. Then P* is called a P - Fuzzy Near-Ring in R if

- $A(x + y) \geq \min(A(x), A(y))$
- $A(-x) \geq A(x)$
- $A(xy) \geq \min\{A(x), A(y)\}$ for all $x, y \in R$

Definition 5.3. Let R be a Near-Ring and P* be a Fuzzy Near-Ring in R. Let Y be a P - Near-Ring module over R and A is a fuzzy set in Y. Then A is called a P - Fuzzy Near-Ring module in Y if

(iv) $A(x + y) \geq \min\{A(x), A(y)\}$

(v) $A(\lambda x) = \min\{A(\lambda), A(x)\}$ for all $x, y \in Y, \lambda \in R$

(vi) $A(0) = 1$

If N is an ordinary P - near ring then the condition (ii) is replaced by (ii) $A(\lambda x) > A(x) for\ all\ \lambda \in A\ and\ for\ all\ x \in Y$.

Theorem 5.1 Let Y be a Near Ring module over a P - fuzzy near-ring N in R. Then A is a P - Fuzzy Near – Ring Module in Y if and only if

$$A(\lambda(x) + \mu(x)) \geq \min\{\min\{N(\lambda), A(x)\}, \min\{N(\mu), A(y)\}\}$$

for all $\lambda, \mu \in N\ and\ for\ all\ x, y \in Y$.

Proof:

Let Y be a Near Ring module over a P - fuzzy near – ring N in R. Using First condition of P – Fuzzy Algebra.

For any n – ary($n \geq 1$) operation $f \in F$

$\mu(f(x_1...x_n)) \geq \mu(x_1)*...*\mu(x_n)$ for all $x_1...x_n \in A$

and also N satisfies the conditions of an ordinary Near – Ring then the above condition is replaced by $A(\lambda x + \mu y) \geq \min\{A(x), N(y)\}$ for all $x, y \in Y$.

$$\Rightarrow \geq \min\{\min\{N(\lambda), A(x)\}, \min\{N(\mu), A(y)\}\}$$

Converse Part,

If $A(\lambda(x) + \mu(x)) \geq \min\{\min\{N(\lambda), A(x)\}, \min\{N(\mu), A(y)\}\}$

for all $\lambda, \mu \in N\ and\ for\ all\ x, y \in Y$. By P – Fuzzy Algebra

For any n – ary($n \geq 1$) operation $f \in F$

$\mu(f(x_1...x_n)) \geq \mu(x_1)*...*\mu(x_n)$ for all $x_1...x_n \in A$

Then, $A(\lambda x + \mu y) \geq \min\{A(x), N(y)\}$ for all $x, y \in Y$.

⇒ N is an ordinary Near – Ring.

Theorem 5.2 Let Y be a Near Ring module over a near – ring R with identity. If A is a P - Fuzzy Near – Ring Module in Y if λ∈ R is invertible then $A(\lambda x) = A(x)$ for all $x \in Y$.

Proof: If λ ∈ A is invertible, for all x ∈ Y and by using fuzzy subgroup of the group G, G ∈ A if

- $\mu(xy) \geq min\ (\mu(x), \mu(y))$ for every $x, y \in G$ and
- $\mu\ (x^{-1})\ =\ \mu\ (x)$ for every $x \in G$.

A(x) = A (λ–1, λx) ≥ A(λx) ≥ A(x)

A(λx) = A(x).

Theorem 5.3: Let { Ai / i∈ I} be a family of P - fuzzy near – ring modules in Y then ∩Ai is a P - fuzzy near - ring module in Y
i∈I

Proof: Consider P – fuzzy Algebra on the Algebra A.

A P – fuzzy set $\mu \in P^A$ is called a P – fuzzy algebra or fuzzy subalgebra on the algebra A, if

- For any n – ary($n \geq\ 1$) operation $f \in F$
  $\mu(f(x_1 \ldots\ldots\ldots x_n))\ \geq \mu(x_1) * \ldots\ldots\ldots\ldots\ldots * \mu(x_n)$
  for all $x_1 \ldots\ldots\ldots x_n \in A$
- For any constant (nullary operation) C
  $\mu(c)\ \geq \mu(x)$ for all $x \in A$.

Let A = ∩Ai then λ∈ R and for all x, y ∈Y
i∈I

A(x+y) = infAi(x + y)
i∈I
≥ inf { min {Ai(x), Ai(y)}
i∈I
= min {infAi(x), infAi(y)}
i∈I
= min{ A(x), A(y)}

and A(λx) = infAi(λx)
i∈I
≥ inf {min{N(λ), Ai(x)}
i∈I
= min {N(λ), inf A(x)}
i∈I
= min{N(λ), A(x)}is a P - fuzzy near - ring module in Y
i∈I

Theorem 5.4: Let Y and W be near – ring modules over a P -fuzzy near – ring N is a near – ring R and θ a homomorphism of Y into W. Let A be a fuzzy near ring module in W. Then the inverse image $\theta - 1$ (A) of A is a P - fuzzy near – ring module in Y.

Proof: For all x, y ∈Y and for all λ, μ∈ R

$\theta^{-1}(A)(\lambda x+\mu y) = A(\theta(\lambda x+\mu y))$

$= A(\lambda\theta(x)+\mu\theta(y))$

$\geq \min\{ \min\{N(\lambda), A(\theta(x))\}, \min\{N(\mu), A\theta(y))\}\}$

$= \min\{\min\{N(\lambda), \theta^{-1}(A)(x)\}, \min\{N(\mu), \theta^{-1}(A)(y)\}\}$

Theorem 5.5: Let Y and W be near – ring modules over a P -fuzzy near – ring N in a near – ring R and θ a homomorphism of Y into W. Let W be a fuzzy near – ring module in Y that has the supremum property then the image θ(M) of M is a P - fuzzy near ring module in W.

Proof: Let μ, ν ∈ W. It either $\theta - 1$ (M) or $\theta - 1$ (v) is empty.

Suppose that neither $\theta - 1$ (M) or $\theta - 1(v)$ is empty then

θ(M)(λ(u) + μ(v)) = sup M(w)

≥ min{min{N(λ), θ(M)(u)}, min{N(μ), θ(M)(v)}}, ω ∈ $\theta^{-1}(\lambda u+\mu v)$.

V. Conclusion

In this paper, the concept of P – Fuzzy near-ring on the Algebra A is defined and some of its theorems are discussed. In Future we can develop this P –Fuzzy near-ring as P – Fuzzy near-field.

REFERENCES

[1] Dr.S.Priyadarshini, "A study on P - fuzzy rings", International journal of Current research, vol 2, issue 5, pp 380 to 383.

[2] Dr.S.Priyadarshini, "A study on P - Fuzzy Field using P-Fuzzy Algebra and its properties", International Journal for Research in Engineering Application & Management, vol 04, issue 03,P.No. 703 to 705.

[3] Dr. S.Priyadarshini,"A Comparative study on P-Fuzzy birings using birings", International Journal of Mathematical Sciences and Engineering, vol 12, No 1, April 2018, UGC approved Journal.

[4] Dr. S.Priyadarshini," P-Fuzzy bifield using P-Fuzzy Algebra on the Algebra A", Journal of Emerging Technologies and Innovation Research, October 2018, vol 5, issue 10,p.no. 65 to 69. UGC approved Journal.

[5] Laszlo filep.Study of fuzzy algebras and relations from a general viewpoint,acta Mathematica Academiae Paedagogicae Nyrlregyhaziensis,Tonus 14(1998), 49 - 55.

[6] L. A. Zadeh. Fuzzy sets. Information Control, 8:338-353, 1965.

