A Queueing - Inventory Model with Three Classes of Customers Using Different Fuzzy Number

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Abstract - This paper consider a fuzzy queueing – inventory model with three classes of customers and the services of customers are considered to be selected. The first class of customer are getting service at a probability p. The steady state probability distribution and the other performance measures are derived by using the inventory level state transitions equations. The eye shape fuzzy number is defined and its properties are given. Our proposed model have been considered in fuzzy environment. The parameters involved in this model are represented by using different fuzzy number. The total cost is defuzzified by the weighted canonical representation of different fuzzy number. Then by using the MATLAB software the global optimum total cost are found.

Keywords-Inventory, Queue, Customers, Performance measures, Eye shape fuzzy number ,weighted canonical representation technique.

AMS Subject Classification:60K25,90B05

I. INTRODUCTION

The inventory modelling is also very important because the decision making affects the total business. ie., input and output of products, cash handling , minimization of cost and maximization of profit. In most of the literature on inventory models it is assumed that all demand for a single item is equally important. However, in practice, the demand for an item can often be categorised into classes of different priority.

Queueing system is the most applications of alternating renewal process and has been widely applied to many practical problems. The finite-capacity queueing systems have been widely studied by many researchers such as Gouweleeuw and Tijms[4], Bretthauer and Cote [1], Wagner [16], Grossand Harris [5], M. Schwarz, C. Sauer, H. Daduna, R. Kulik, R. Szekli, [20] and N. Zhao, Z. T. Lin, [21]. In classic queueing systems, the interarrival times of customers and service times of servers are characterized as random variables.

Research on queueing - inventory systems has captured much attention over the last decades. A queueing – inventory system is different from the traditional queueing system because of the way the attached inventory influences the service. If there is no inventory on hand the service will be interrupted.

Berman and Kim (1999) analyzed a queueing-inventory system with Poisson arrivals, exponential service times and zero lead times. The authors proved that the optimal policy is ‘‘never to order when the system is empty’’. Berman and Sapna (2000) studied queueing-inventory systems with Poisson arrivals, arbitrary distribution service times and zero lead times. The optimal value of the maximum allowable inventory which minimizes the long-run expected cost rate has been obtained. Berman and Sapna (2001) discussed a finite capacity system with Poisson arrivals, exponential distributed lead times and service times. The existence of a stationary optimal service policy has been proved. Berman and Kim (2001) studied internet-based supply chains with Poisson arrivals, exponential service times and the Erlang lead times and found that the optimal ordering policy has a monotonic threshold structure. Berman and Kim (2004) addressed an infinite capacity queuing-inventory system with Poisson arrivals, exponential service times and exponential lead times. The authors identified a replenishment policy which maximized the system profit. Schwarz et al. (2006) derived stationary distributions of joint queue length and inventory processes in explicit product form for M/M/1 queuing-inventory system with lost sales under various inventory management policies such as (r,Q) policy and (r,S) policy. The M/M/1 queueing-inventory system with backordering was investigated by Schwarz and Daduna (2006). Heetal. (2002a) quantified the value of information used in inventory control. All the above studies about queueing-inventory systems are limited to two classes of customers. A Queueing-inventory system with two classes demand and subjected to selected service discussed by Hong Chen, Zhongfang Zhou, Mingwu Liu. To our knowledge, we have not found any literature studying a queueing-inventory system with three or more classes customers with different priority.But all these problems are solved with the assumption that the coefficient or cost parameters are specified in a precious way. In real life there are many diverse situations due to uncertainty. Here inventory costs are imprecise. That is fuzzy in nature. Early works in using fuzzy concept in decision making were done by Zadeh and Bellman through introducing fuzzy goals, cost and constraints. This research paper discuss with a fuzzy queueing – inventory model with three classes of customers are priority, special and ordinary the services of customers are subjected to a selected service. The eye shape fuzzy number is defined and its properties are given. Our proposed model have been considered in crisp and fuzzy environment. The parameters involved in this model are represented by eye shape fuzzy number. The total cost is defuzzified by the weighted canonical representation of eye shape fuzzy number. Then by using the MATLAB software the global optimum of the total cost are found.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used throughout this paper.

Assumptions:

A single – server queueing inventory system.

Three classes of customers and subjecting to selected service.

The arrival process for three classes is state independent.

Customers arrival follows poisson process.

Service discipline FCFS (First come first service)

Lead-time is exponentially distributed with parameter μ.

Notations:

Q - odering quantity

s - Reorder point

I(t) - on-hand inventory level at time t

λ_1 - Intensity of priority customers

λ_2 - Intensity of special customers

λ_3 - Intensity of ordinary customers

$\overline{R}$ - the mean reorder rate

$\overline{\psi}_1$ - the mean shortage rate for the priority customers

$\overline{\psi}_2$ - the mean shortage rate for the special customers

$\overline{\psi}_3$ - the mean shortage rate for the ordinary customers.

h - the inventory holding cost per unit per unit time.

c_l - the replenishment price per unit.

g_1 - the shortage cost per unit for the priority customer lost.

g_2 - the shortage cost per unit for the special customer lost.

g_3 - the shortage cost per unit for the ordinary customer lost.

P - ordinary customer get service at probability p

$\tilde{h}$ - fuzzy inventory holding cost per unit per unit time.

$\tilde{c}_1$ - fuzzy replenishment price per unit.

$\tilde{g}_1$ - fuzzy shortage cost per unit for the priority customer lost.

$\tilde{g}_2$ - fuzzy shortage cost per unit for the special customer lost.

$\tilde{g}_3$ - fuzzy shortage cost per unit for the ordinary customer lost.

III. MATHEMATICAL MODEL IN CRISP ENVIRONMENT

3.1 Problem Formulation and Analysis

Consider an inventory system in which there are three classes of customers (priority customers, special customers and ordinary customers) are subjected to selected service. The arrival process for three classes of customers is state independent and each customer needs exactly one item from the inventory. Priority customers, Special customers, Ordinary customers arrive according to a poisson process with rate λ_1, λ_2, λ_3 when the on hand inventory drops to a fixed level s, an order for Q(>s) units is placed. Hence the maximum on hand inventory Q+s. The condition Q>s ensures that there are no perpetual shortages. The lead time for the order is exponentially distributed with parameter $\mu(>0)$. As orders are usually placed when stock levels get low, ordinary customers who arrive when an order is pending are not served and hence these demands are lost. Also , when inventory level is zero, demands due to three types of customers are assumed to be lost.

Let I(t) denote the on-hand inventory level at time t. Since Q>s, at any given point of time there is atmost one order pending, and as such from our assumptions it is clear that the inventory level process $\{I(t);t\geq 0\}$ with state space E={0,1,2,...Q+s} is a markov process.

Let P(i,j,t)=Pr[I(t) = j/I(0) = i], i,j $\in$ E.

In the steady state, let $P(j) = \lim_{t\to\infty} P(i,j,t)$. Then P(j) satisfies the following balance equations

$$(\lambda_1+\lambda_2+\lambda_3)P(Q+s)=\mu P(s) \qquad \text{------- (1)}$$

$$(\lambda_1+\lambda_2+\lambda_3)P(j)=(\lambda_1+\lambda_2+\lambda_3)P(j+1)+\mu P(j-Q) \qquad \text{-------- (2)}$$

$$j = Q, Q+1, \ldots\ldots\ldots, Q+s-1$$

$$(\lambda_1 + \lambda_2 + \lambda_3) P(j) = (\lambda_1 + \lambda_2 + \lambda_3) P(j+1)$$

$$j = s+1, s+2, \ldots\ldots\ldots, Q-1 \qquad \text{------- (3)}$$

$$(\lambda_1 + \lambda_2 + p\lambda_3 + \mu) P(s) = (\lambda_1 + \lambda_2 + p\lambda_3) P(s+1)$$

-------- (4)

$$(\lambda_1 + \lambda_2 + p\lambda_3 + \mu) P(s) = (\lambda_1 + \lambda_2 + p\lambda_3) P(j+1)$$

$$j = 1,\ 2, \ldots\ldots\ldots, s-1 \qquad \text{-------- (5)}$$

$$\mu P(0) = (\lambda_1 + \lambda_2 + p\lambda_3) P(1) \qquad \text{------- (6)}$$

Solve the equation by the means of recursive process and get.

From equation (6)

$$P(1) = \frac{\mu P(0)}{\lambda_1 + \lambda_2 + p\lambda_3}$$

From equation (5)

$$P(j+1) = \frac{p\lambda_3 + \lambda_2 + \lambda_1 + \mu}{\lambda_1 + \lambda_2 + p\lambda_3} P(j)\ ; \quad j = 1, 2, \ldots s-1$$

j=1,

$$P(2) = 1 + \frac{\mu}{\lambda_1 + \lambda_2 + p\lambda_3} P(1) \ = \left(1 + \frac{\mu}{\lambda_1 + \lambda_2 + p\lambda_3}\right)\left(\frac{\mu P(0)}{\lambda_1 + \lambda_2 + p\lambda_3}\right)$$

j=2,

$$P(3) = 1 + \frac{\mu}{\lambda_1 + \lambda_2 + p\lambda_3} P(2) \qquad = \left(1 + \frac{\mu}{\lambda_1 + \lambda_2 + p\lambda_3}\right)^2 \left(\frac{\mu P(0)}{\lambda_1 + \lambda_2 + p\lambda_3}\right)$$

$\vdots$

$\vdots$

$\vdots$

$$P(j) = \left(1 + \frac{\mu}{\lambda_1 + \lambda_2 + p\lambda_3}\right)^{j-1} \left(\frac{\mu P(0)}{\lambda_1 + \lambda_2 + p\lambda_3}\right) \ j = 1, 2, \ldots\ldots\ldots\ldots, s \qquad \text{------- (7)}$$

From equation (3)

$$(\lambda_1 + \lambda_2 + \lambda_3) P(j) = (\lambda_1 + \lambda_2 + \lambda_3) P(j+1)$$
$$P(j) = P(j+1), \quad j = s+1,\ s+2, \ldots., Q-1$$
$$P(s+1) = P(s+2) = P(s+3) = \ldots\ldots\ldots = P(Q)$$

From equation (4)

$$P(s+1) = \frac{(p\lambda_3 + \lambda_2 + \lambda_1 + \mu)}{\lambda_1 + \lambda_2 + p\lambda_3} P(s)$$

From equation (7)

$$P(s+1) = \frac{(p\lambda_3 + \lambda_2 + \lambda_1 + \mu)}{\lambda_1 + \lambda_2 + p\lambda_3} \left(1 + \frac{\mu}{\lambda_1 + \lambda_2 + p\lambda_3}\right)\left(\frac{\mu P(0)}{\lambda_1 + \lambda_2 + p\lambda_3}\right)$$

$$=\left[\frac{\lambda_1+\lambda_2+\lambda_3}{\lambda_1+\lambda_2+p\lambda_3}+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right]\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)\left(\frac{\mu P(0)}{\lambda_1+\lambda_2+p\lambda_3}\right)$$

$$=\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s-1}\left(\frac{\mu P(0)}{\lambda_1+\lambda_2+p\lambda_3}\right)$$

$$P(s+1)=\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}\left(\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)P(0)$$

In general,

$$P(j)=\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}\left(\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)P(0) \text{ ------- (8)}$$

$$j=s+1, s+2, \ ....Q$$

From equation (3)

$$(\lambda_1+\lambda_2+\lambda_3)P(j)=(\lambda_1+\lambda_2+\lambda_3)P(j+1)+\mu P(j-Q)$$

$$j=Q, Q+1, ...,Q+s-1$$

$$P(j)=\frac{(\lambda_1+\lambda_2+\lambda_3)P(j+1)}{\lambda_1+\lambda_2+\lambda_3}+\frac{\mu P(j-Q)}{\lambda_1+\lambda_2+\lambda_3}$$

$$P(j)-\frac{\mu P(j-Q)}{\lambda_1+\lambda_2+\lambda_3}=P(j+1)$$

$$P(j+1)=P(j)-\frac{\mu P(j-Q)}{\lambda_1+\lambda_2+\lambda_3}$$

For j = Q

$$P(Q+1)=P(Q)-\frac{\mu P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

$$=\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}-\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

$$=\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}-1\right]\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

For j=Q+1

$$P(Q+2)=P(Q+1)-\frac{\mu P(1)}{\lambda_1+\lambda_2+\lambda_3}$$

$$=\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}-1\right]\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}-\frac{\mu}{\lambda_1+\lambda_2+\lambda_3}\left(\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)P(0)$$

$$=\left\{\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}-1\right]-\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right\}\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

$$=\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}-\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)\right]\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

For j=Q + s - 1

$$P(Q+s)=\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}-\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s-1}\right]\frac{\mu\ P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

Q + s = j
s = j - Q

$$P(j)\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}-\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{j-Q-1}\right]\frac{\mu\,P(0)}{\lambda_1+\lambda_2+\lambda_3}$$

-------- (9)

$j=Q+1,\ Q+2,\ \ldots\ldots,\ Q+s$

The system must subject to the normalizing condition written as

$$\sum_{j=0}^{Q+s}P(j)=1$$

It follows from (7)-(9) that we obtain

$$P(0)+\sum_{j=1}^{s}P(j)+\sum_{j=s+1}^{Q}P(j)+\sum_{j=Q+1}^{Q+s}P(j)=1$$

$$P(0)\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}(\lambda_3(1-p)+\mu Q)\right]+(\lambda_1+\lambda_2+p\lambda_3)=\lambda_1+\lambda_2+\lambda_3$$

$$P(0)=\frac{\lambda_1+\lambda_2+\lambda_3}{(\lambda_1+\lambda_2+p\lambda_3)+(\lambda_3(1-p)+\mu Q)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}}$$

Inserting (10) in (7)-(9) respectively, we have the analytical steady state probability distributions of the inventory level.

Let I denote the average inventory level

Using $\bar{I}=\sum_{j=1}^{s+Q}j\ P(j)$

where $\bar{I}=I_1+I_2+I_3$

$$I_1=\sum_{j=1}^{s}j\ P(j),\ I_2=\sum_{j=s+1}^{Q}j\ P(j),I_3=\sum_{j=Q+1}^{s+Q}j\ P(j)$$

$$I_1=\sum_{j=1}^{s}j\ P(j)=1P(1)+2P(2)+\ldots..+s\,P(s)$$

$$I_1=\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\mu}\right)\left[1-(s+1)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}+s\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s+1}\right]$$

$$I_2=\sum_{j=s+1}^{Q}j\ P(j)$$

$$I_2=(s+1)P(s+1)+(s+2)P(s+2)+\ldots.+Q\,P(Q)$$

$$=\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}\frac{\mu}{\lambda_1+\lambda_2+\lambda_3}P(0)\left[(s+1)+(s+2)+\ldots.+Q\right]$$

$$I_2=\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s}\frac{\mu}{\lambda_1+\lambda_2+\lambda_3}P(0)\left[\frac{Q^2+Q-s^2-s}{2}\right]$$

$$I_3=\sum_{j=1}^{s}j\ P(j)$$

$$I_3 = (Q+1)P(Q+1) + (Q+2)P(Q+2) + ... + (Q+s)P(Q+s)$$

$$= Q+1\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s - 1\right]\frac{\mu}{\lambda_1+\lambda_2+\lambda_3}P(0) + (Q+2)\left[\begin{array}{l}\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)\\ -\left(1+\frac{\mu}{\lambda_1+\lambda_2+P\lambda_3}\right)\end{array}\right]$$

$$\frac{\mu}{\lambda_1+\lambda_2+\lambda_3}P(0) + ... + (Q+s)\left[\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s - \left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^{s-1}\right]\frac{\mu}{\lambda_1+\lambda_2+\lambda_3}P(0)$$

$$I = I_1 + I_2 + I_3$$

$$I = \left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s P(0)\left[\begin{array}{l}\left(\frac{s(1-p)\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)}\right)+\frac{\mu Q^2+Q+2Qs}{2(\lambda_1+\lambda_2+\lambda_3)}-\frac{Q(\lambda_1+\lambda_2+p\lambda_3)}{\lambda_1+\lambda_2+\lambda_3}\\ -\left\{\frac{(\lambda_1+\lambda_2+p\lambda_3(1-p)\lambda_3}{\mu(\lambda_1+\lambda_2+\lambda_3)}\right\}\end{array}\right]$$

$$+\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)\left[Q+\frac{\lambda_3(1-p)}{\mu}\right]P(0)$$

Definition:

Let us denote

$\overline{R}$ = the mean reorder rate

$\overline{\psi}_1$ = the mean shortage rate for the priority customers

$\overline{\psi}_2$ = the mean shortage rate for the special customers

$\overline{\psi}_3$ = the mean shortage rate for the ordinary customers.

Then,

$$\overline{R} = (\lambda_1+\lambda_2+\lambda_3)\,p(s+1) = \left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)\mu P(0)$$

$$\overline{\psi}_1 = \lambda_1\ P(0)$$

$$\overline{\psi}_2 = \lambda_2\ P(0)$$

$$\overline{\psi}_3 = (1-p)\lambda_3\sum_{j=1}^{s}P(j) + p\lambda_3 P(0)$$

------ (15)

$$= \left[(1-p)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s + p\right]\lambda_2 P(0)$$

3.3 Optimal cost Analysis

The problem of minimizing the steady state expected cost rate under the following cost structure

$$\text{Cost}\ (s,Q,p) = h\overline{I} + c_1 Q\overline{R} + g_1\overline{\psi}_1 + g_2\overline{\psi}_2 + g_3\overline{\psi}_3$$

$$= h\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s\left[\begin{array}{l}\left(\frac{s(1-p)\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)}\right)+\frac{\mu Q^2+Q+2Qs}{2(\lambda_1+\lambda_2+\lambda_3)}-\frac{Q(\lambda_1+\lambda_2+p\lambda_3)}{\lambda_1+\lambda_2+\lambda_3}\\ -\left\{\frac{(\lambda_1+\lambda_2+p\lambda_3)((1-p)\lambda_3)}{\mu(\lambda_1+\lambda_2+\lambda_3)}\right\}\end{array}\right]P(0)$$

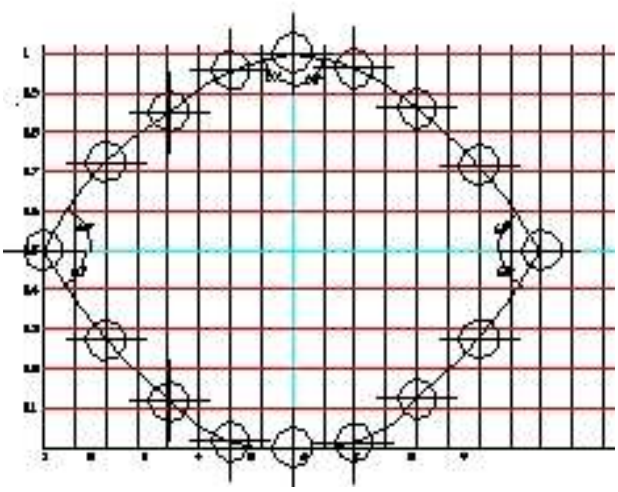
$$+\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)\left[Q+\frac{(1-p)\lambda_3}{\mu}\right]P(0) + C_1 Q\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s\mu P(0) + g_1\lambda_1 P(0) \quad \text{---------- (16)}$$

$$+ g_2\lambda_2 P(0) + g_3\left[(1-p)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s + p\right]\lambda_2 P(0)$$

Using MATLAB, values of s*,Q*,p* are obtained.

IV. EYE SHAPE FUZZY NUMBER

Definitions 4.1: The Eye shape fuzzy number Ã described as a normalised convex fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows



$$\mu_{\tilde{A}}(x) = \begin{cases} 1-\frac{1}{2}\left(\frac{x-b}{a-b}\right)^2 & for\ \ a \le x \le b \\ 1-\frac{1}{2}\left(\frac{x-b}{c-b}\right)^2 & for\ b \le x \le c \\ 1-\frac{1}{2}\left(\frac{x-b}{b-a}\right)^2 & for\ a \le x \le b \\ 1-\frac{1}{2}\left(\frac{x-b}{b-c}\right)^2 & for\, b \le x \le c \end{cases}$$

where a,b and c are real numbers.

This type of fuzzy number be denoted as Ã = $(a,b,c)_{EFN}$ where $\mu_{\tilde{A}}(x)$ satisfies the following conditions.

1. Opposites angles are equal in the horizontal line.

2. The horizontal and vertical diagonal bisect each other and meet at 90^o.

3. In the horizontal diagonal, the base of the adjacent angles are equal.

Weighted Canonical Representation of Eye shape Fuzzy Number:

To simplify the representation of fuzzy numbers consider two parameters – value and ambiguity which represent some basic features of fuzzy numbers and hence they were called a canonical representation of fuzzy numbers.

$$WCR_E(\tilde{A}) = wVal_E(\tilde{A}) + (1+w)\, Amb_E(\tilde{A})$$

W = weighted value

Where, $Val_E(\tilde{A}) = \int_0^1 \alpha \left[L_\mu(\alpha) + R_\mu(\alpha) \right] d\,\alpha$

$$= \int_0^{0.5} \alpha \left[b + (b-a)\sqrt{2\alpha} \right] d\alpha + \int_{0.5}^{1} \alpha \left[b + (a+b)\sqrt{2(1-\alpha)} \right] d\alpha$$

$$+ \int_0^{0.5} \alpha \left[b + (b-c)\sqrt{2\alpha} \right] d\alpha + \int_{0.5}^{1} \alpha \left[b + (c-b)\sqrt{2(1-\alpha)} \right] d\alpha$$

$$= \frac{11b+4c}{30} + \frac{11b+49}{30} = \frac{11b+2c+2a}{15}$$

$$Amb_E(\tilde{A}) = \int_0^1 \alpha \left[R_\mu(\alpha) - L_\mu(\alpha) \right] d\alpha$$

$$= \left(\int_0^{0.5} \alpha \left[b + (b-c)\sqrt{2\alpha} \right] d\alpha + \int_{0.5}^{1} \alpha \left[b + (c-b)\sqrt{2(1-\alpha)} \right] d\alpha \right)$$

$$- \left(\int_0^{0.5} \alpha \left[b + (b-a)\sqrt{2\alpha} \right] d\alpha + \int_{0.5}^{1} \alpha \left[b + (a+b)\sqrt{2(1-\alpha)} \right] d\alpha \right)$$

$$= \frac{11b+4c}{30} - \frac{11b+49}{30} = \frac{2(c-a)}{15}$$

$$WCR_E(\tilde{A}) = w\frac{11b+2c+2a}{15} + (1+w)\frac{2(c-a)}{15}$$

Where w = 1

$$= \frac{11b+2c+2a}{15} + \frac{4(c-a)}{15} = \frac{-2a+6c+11b}{15}$$

Weighted Canonical Representation of Triangular Fuzzy Number:

$$WCR_T(\tilde{A}) = wVal_T(\tilde{A}) + (1+w)\,Amb_T(\tilde{A})$$

W = weighted value

Where,

$$Val_E(\tilde{A}) = \int_0^1 \alpha \left[L_\mu(\alpha) + R_\mu(\alpha) \right] d\alpha$$

$$= \int_0^1 \alpha \left[\left(c - \alpha(c-b)\right) + \left(a + \alpha(b-a)\right) \right] d\alpha$$

$$= \frac{2b-2a+3a+2b-2c+3c}{6} = \frac{4b+a+c}{6}$$

$$Amb_E(\tilde{A}) = \int_0^1 \alpha \left[R_\mu(\alpha) - L_\mu(\alpha) \right] d\alpha$$

$$= \int_0^1 \alpha \left[\left(a + \alpha(b-a)\right) - \left(c - \alpha(c-b)\right) \right] d\alpha$$

$$WCR_E(\tilde{A}) = w = \frac{4b+a+c}{6} + (1+w)\frac{(c-a)}{6}$$

Where w = 1

$$== \frac{4b+a+c}{6} + \frac{2(c-a)}{6} = \frac{4b+4c-a}{6}$$

Weighted Canonical Representation of Trapezoidal Fuzzy Number:

$$WCR_T(\tilde{A}) = wVal_T(\tilde{A}) + (1+w)\,Amb_T(\tilde{A})$$

W = weighted value

Where,

$$Val_E(\tilde{A}) = \int_0^1 \alpha\,\left[L_\mu(\alpha) + R_\mu(\alpha)\right]\,d\alpha$$

$$= \int_0^1 \alpha\,\left[\left(d-\alpha(d-c)\right) + \left(a+\alpha(b-a)\right)\right]d\alpha$$

$$= \frac{3d-2a+3a+2b-2d+2c}{6}$$

$$= \frac{2b+a+d+2c}{6}$$

$$Amb_E(\tilde{A}) = \int_0^1 \alpha\,\left[R_\mu(\alpha) - L_\mu(\alpha)\right]\,d\alpha$$

$$= \int_0^1 \alpha\,\left[\left(a+\alpha(b-a)\right) - \left(d-\alpha(d-c)\right)\right]d\alpha$$

$$= \frac{3d-2d+2c-3a-2b+2a}{6} \quad WCR_E(\tilde{A}) = w\frac{a+2b+d+2c}{6} + (1+w)\frac{d+2c-a-2b}{6}$$

$$= \frac{d+2c+3c-2b-a}{6}$$

Where w = 1

$$= \frac{a+2b+d+2c}{6} + \frac{2(d+2c-a-2b)}{6}$$

$$= \frac{-2b+6c-a+3d}{6}$$

V. THE PROPOSED INVENTORY MODEL IN FUZZY ENVIRONMENT

If the cost parameters are characterized by different types fuzzy number then the equation (16) is transformed to

$$\text{Cost}\ \ (s,Q,p) = \tilde{h}\,\bar{I} + \tilde{c}_1\,Q\,\bar{R} + \tilde{g}_1\,\bar{\psi}_1 + \tilde{g}_2\ \bar{\psi}_2 + \tilde{g}_3\ \bar{\psi}_3$$

$$= \tilde{h}\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \left[\begin{array}{l} \left(\dfrac{s(1-p)\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)}\right)+\dfrac{\mu Q^2+Q+2Qs}{2(\lambda_1+\lambda_2+\lambda_3)} \\ -\dfrac{Q(\lambda_1+\lambda_2+p\lambda_3)}{\lambda_1+\lambda_2+\lambda_3} \\ -\left\{\dfrac{(\lambda_1+\lambda_2+p\lambda_3)((1-p)\lambda_3)}{\mu(\lambda_1+\lambda_2+\lambda_3)}\right\} \end{array}\right] P(0)+\tilde{h}\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)\left[Q+\frac{(1-p)\lambda_3}{\mu}\right]P(0)$$

$$+\tilde{c}_1 Q\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \mu P(0)+\tilde{g}_1\lambda_1 P(0)+\tilde{g}_2\lambda_2 P(0)+\tilde{g}_3\left[(1-p)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s+p\right]\lambda_2 P(0)$$

In our proposed model, we have consider the cost parameters h,c_1,g_1,g_2,g_3 as a eye shaped fuzzy number as follows $\tilde{h}=(h_1,h_2,h_3)$, $\tilde{c}=(c_1,c_2,c_3)$, $\tilde{g}_1=(g_{11},g_{12},g_{13})$, $\tilde{g}_2=(g_{21},g_{22},g_{23})$ and $\tilde{g}_3=(g_{31},g_{32},g_{33})$.Now by using the defuzzification technique the proposed model is reduced

Case(I) :

$$WCR_{TR}(C(s,Q))=WCR_{TR}(h)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \left[\begin{array}{l} \left(\dfrac{s(1-p)\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)}\right)+\dfrac{\mu Q^2+Q+2Qs}{2(\lambda_1+\lambda_2+\lambda_3)}-\dfrac{Q(\lambda_1+\lambda_2+p\lambda_3)}{\lambda_1+\lambda_2+\lambda_3} \\ -\left\{\dfrac{(\lambda_1+\lambda_2+p\lambda_3)((1-p)\lambda_3)}{\mu(\lambda_1+\lambda_2+\lambda_3)}\right\} \end{array}\right] P(0)$$

$$+h\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)\left[Q+\frac{(1-p)\lambda_3}{\mu}\right]P(0)+WCR_{TR}c_1 Q\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \mu P(0)$$

$$+WCR_{TR}g_1\lambda_1 P(0)+WCR_{TR}g_2\lambda_2 P(0)+WCR_{TR}g_3\left[(1-p)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s+p\right]\lambda_2 P(0)$$

$$WCR_{TP}(C(s,Q))=WCR_{TP}(h)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \left[\begin{array}{l} \left(\dfrac{s(1-p)\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)}\right)+\dfrac{\mu Q^2+Q+2Qs}{2(\lambda_1+\lambda_2+\lambda_3)}-\dfrac{Q(\lambda_1+\lambda_2+p\lambda_3)}{\lambda_1+\lambda_2+\lambda_3} \\ -\left\{\dfrac{(\lambda_1+\lambda_2+p\lambda_3)((1-p)\lambda_3)}{\mu(\lambda_1+\lambda_2+\lambda_3)}\right\} \end{array}\right] P(0)$$

$$+h\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)\left[Q+\frac{(1-p)\lambda_3}{\mu}\right]P(0)+WCR_{TP}c_1 Q\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \mu P(0)$$

$$+WCR_{TP}g_1\lambda_1 P(0)+WCR_{TP}g_2\lambda_2 P(0)+WCR_{TP}g_3\left[(1-p)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s+p\right]\lambda_2 P(0)$$

$$WCR_{E}(C(s,Q))=WCR_{E}(h)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \left[\begin{array}{l} \left(\dfrac{s(1-p)\lambda_3}{(\lambda_1+\lambda_2+\lambda_3)}\right)+\dfrac{\mu Q^2+Q+2Qs}{2(\lambda_1+\lambda_2+\lambda_3)}-\dfrac{Q(\lambda_1+\lambda_2+p\lambda_3)}{\lambda_1+\lambda_2+\lambda_3} \\ -\left\{\dfrac{(\lambda_1+\lambda_2+p\lambda_3)((1-p)\lambda_3)}{\mu(\lambda_1+\lambda_2+\lambda_3)}\right\} \end{array}\right] P(0)$$

$$+h\left(\frac{\lambda_1+\lambda_2+p\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\right)\left[Q+\frac{(1-p)\lambda_3}{\mu}\right]P(0)+WCR_{E}c_1 Q\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s \mu P(0)$$

$$+WCR_{E}g_1\lambda_1 P(0)+WCR_{E}g_2\lambda_2 P(0)+WCR_{E}g_3\left[(1-p)\left(1+\frac{\mu}{\lambda_1+\lambda_2+p\lambda_3}\right)^s+p\right]\lambda_2 P(0)$$

Using MATLAB, values of s*,Q* are obtained

VI. NUMERICAL EXAMPLE

Let us consider a situation where c=20, l_1=0.5, l_2=0.5, l_3=0.5, m=0.5, g_1=8, g_2=10, g_3=10, h=20, p=0, 0.2, 0.4, 0.6, 0.8, 1 and $\tilde{h}$ = (10,20,30), $\tilde{g}_1$ = (5,10,15), $\tilde{g}_2$ = (5,10,15), $\tilde{g}_3$=(5,10,15), $\tilde{C}$ = (10,20,30)

Table 1: Optimal Solutions of crisp and fuzzy model:

			C_1	h_1	g_1	g_2	g_3	Q^*	S^*	$C(S^*,Q^*)$
P=0	**Crisp**		20	20	8	10	10	18.6138	4.0892	20,051
	Fuzzy	*Eye shape*	14	14	5.6	7	7	15.3819	4.2450	11.479
		Triangular	15.8333	15.8333	12.667	15.8333	15.8333	15.5280	4.3717	17.772
		Trapezoidal	15.5000	15.5000	11.3333	7	7	15.965	4.4891	14,806
P=0.2	**Crisp**		20	20	8	10	10	17.9350	4.8624	26,957
	Fuzzy	*Eye shape*	14	14	5.6	7	7	14.3835	4.9933	21,715
		Triangular	15.8333	15.8333	12.667	15.8333	15.8333	14.2774	5.2319	24,760
		Trapezoidal	15.5000	15.5000	11.3333	7	7	14.3854	5.1108	23,273
P=0.4	**Crisp**		20	20	8	10	10	16.4359	5.4405	32,410
	Fuzzy	*Eye shape*	14	14	5.6	7	7	13.9682	6.0250	23,283
		Triangular	15.8333	15.8333	12.667	15.8333	15.8333	14.1032	5.4704	26,188
		Trapezoidal	15.5000	15.5000	11.3333	7	7	14.2998	6.2541	30,130
P=0.6	**Crisp**		20	20	8	10	10	15.4337	5.9659	36,385
	Fuzzy	*Eye shape*	14	14	5.6	7	7	13.5743	6.4025	25,650
		Triangular	15.8333	15.8333	12.667	15.8333	15.8333	13.8219	6.0689	33,781
		Trapezoidal	15.5000	15.5000	11.3333	7	7	13.0146	6.9210	35,547
P=0.8	**Crisp**		20	20	8	10	10	14.2854	6.4992	40,719
	Fuzzy	*Eye shape*	14	14	5.6	7	7	13.1779	6.7893	28,347
		Triangular	15.8333	15.8333	12.667	15.8333	15.8333	12.8605	6.5823	38,867
		Trapezoidal	15.5000	15.5000	11.3333	7	7	11.9280	7.4468	39,696
P=1	**Crisp**		20	20	8	10	10	13.4592	7.0319	44,179
	Fuzzy	*Eye shape*	14	14	5.6	7	7	12.8931	7.1786	31,678
		Triangular	15.8333	15.8333	12.667	15.8333	15.8333	11.9915	6.9885	42,142
		Trapezoidal	15.5000	15.5000	11.3333	7	7	12.2131	7.6849	43,182

VII. OBSERVATION

Table 1 shows the calculation for economic reorder quantity, reorder safety level and total cost. The total cost has been calculated in both crisp and fuzzy environment. By using eye shape fuzzy number defuzzification has been carried out and the corresponding changes have been observed.After long-run inventory cost is increasing in accordance to probability p. The optimal reorder point s* is increasing and Q* is decreasing in accordance to p. It is observed that the Inventory cost in fuzzy is less than the crisp model,and the reorder quantity and reorder level is higher in fuzzy while compared with crisp model also the results obtained in three cases, namely, when using triangular fuzzy number, trapezoidal fuzzy number and Eye shape fuzzy number, the fuzzy value is obtained in the Eye shape fuzzy number is best one.

REFERENCES

[1] Gouweleeuw, F. N., Tijms, H. C.: A simple heuristic for buffer design in finite-capacity queues. European Journal of Operational Research. 88 (1996) 592-598

[2] Gross, D., Harris, C. M.: Fundamentals of Queueing Theory. Wiley, New York (1998)

[3] Kao, C., Li, C. C., Chen, S. P.: Parametric programming to the analysis of fuzzy queues. Fuzzy Sets and Systems. 107 (1999) 93-100

[4] Berman, O., Kim, E., 1999. Stochastic models for inventory management at service facilities. Stochastic Models 15 (4), 695–718. Berman, O., Kim, E., 2001.

[5] Dynamic order replenishment policy in internet-based supply chains. Mathematical Methods of Operations Research 53 (3), 371–390. Berman, O., Kim, E., 2004.

[6] Dynamic inventory strategies for profit maximization in a service facility with stochastic service, demand and lead time. Mathematical Methods of Operations Research 60 (3), 497–521. Berman, O., Sapna, K.P., 2000.

[7] Inventory management at service facilities for systems with arbitrarily distributed service times. Communications in Statistics. Sto-chastic Models 16 (3–4), 343–360. Berman, O., Sapna, K.P., 2001.

[8] Optimal control of service for facilities holding inventory. Computers & Operations Research 28 (5), 429–441. Bright, L., Taylor, P., 1995.

[9] Performance measures of a make-to-order inventory-production system. IIE Transactions 32 (5), 409–419. He Q., Jewkes, E., Buzacott, J., 2002a.

[10] The value of information used in inventory control of a make-to-order inventory-production system. IIE Transactions 34 (11), 999–1013. He, Q., Jewkes, E.M., Buzacott, J., 2002b.

[11] Optimal and near-optimal inventory control policies for a make-to-order inventory-production system. *European Journal of Operational Research* 141 (1), 113–132. Jaiswal, N., 1968.

[12] Priority Queues. Academic Press. Krishnamoorthy, A., Deepak, T.G., Narayanan, V.C., Vineetha, K., 2006a.

[13] Effective utilization of idle time in an (s,S) inventory with positive service time. *Journal of Applied Mathematics and Stochastic Analysis*, 1–13. Krishnamoorthy, A., Narayanan, V.C., Deepak, T.G., Vineetha, P., 2006b.

[14] O. Berman, E. Kim, Stochastic models for inventory management at service facilities, Stochastic Models, 15(4), 1999, 695-718

[15] O. Berman, K. P. Sapna, Optimal control of service for facilities holding inventory, *Computers &Operations Research*, 28(5), 2001, 429-441

[16] R. Dekker, R. M. Hill, M. J. Kleijn, On the (S-1, S) lost sales inventory model with priority demand classes, Naval Res. Logist., 49(6), 2002, 593-610

[17] A. Y. Cohen, Stock rationing policy for a make-to-stock production system with two priority classes and backordering, Naval Res. Logist., 44(5), 1997, 457-472

[18] P. Melchiors, R. Dekker, M. Kleijn, Inventory rationing in an (s, Q) inventory model with lost sales and two demand classes, J. Oper. Res. Soc., 51(1), 2000, 11-122

[19] S. Nahmias, S. Demmy, Operating characteristics of an inventory system with rationing, Management Sci., 27(11), 1981, 1236-1245

[20] M. Schwarz, C. Sauer, H. Daduna, R. Kulik, R. Szekli, M/M/1 queueing systems with inventory, Queueing Systems, 54(1), 2006, 55-78 *H. Chen et al. / Journal of Information & Computational Science 9: 11 (2012) 3081{3089* 3089

[21] N. Zhao, Z. T. Lin, A queueing-inventory system with two classes of customers, *Int. J. Production Economics*, 129(1), 2011, 225-231.

